A&AE 613
Spring 2000
Professor Steve Schneider
Problem Set 4
Handed Out: Friday, 18 February
Due: Monday, 6 March

1. Text, problem 4.42. Note that there is a typo in the equation, which should be
$$\epsilon u'' + u' + u = 0.$$

2. Consider 2D flow past an ellipse (sometimes called an elliptical cylinder). Use (x, y) coordinates, and let the flow be in the x direction. The equation for the ellipse is
$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1,$$
where $a > b$. Let $k = b/a$. The solution for steady potential flow over the ellipse yields a velocity distribution of
$$\frac{U_s}{U_\infty} = \frac{1+k}{\sqrt{1 + k^2 \cot^2 \phi}},$$
where U_s is the surface velocity tangent to the surface, and ϕ is defined by $x = a \cos\phi$ and $y = b \sin\phi$. This potential solution may be found in Schlichting (p. 418), who also gives a formula for the velocity gradient. Note that ϕ is *not* the polar angle. Except as otherwise indicated, assume that $Re_a = U_\infty a/\nu = 1000$. [3-14-00: check signs and directions in Schlichting's inviscid flow – Shin Matsumura points out that seems to be flowing right to left, since dU/ds ¿ 0 for 0 ¡ phi ¡ pi/2, where should be decelerating not accelerating... Schlichting's references are in German, so this check nontrivial. Elliptic cylinder is also in Milne-Thomson, Hydrodynamics, but non in a simple form, would have to be reduced; see also Schlichting p. 217ff]

   (a) Give a formula for the *steady flow* boundary layer momentum thickness on the ellipse, using Thwaites method. Assume that the boundary layer remains laminar. Also, give a formula for the pressure gradient parameter, λ, and the approximate location of separation. Simplify the formulas as much as is practical.

(b) Plot λ as a function of ϕ for $k = 1.0, 0.5, 0.1$. Discuss the effects of eccentricity, pressure gradient, and Reynolds number on the movement of the separation point. Discuss the effects of bluntness. Comment on the accuracy of the given potential flow solution for the three cases. Check your calculations by comparing the separation point for the $k = 1$ case to the well-known solution. Note: You will need to evaluate the formulas derived in (2a) using a numerical integration. Give references for any canned numerical integration routines you may use.

(c) Plot the momentum thickness θ for the same three values of k. Discuss the effects of eccentricity, pressure gradient, and Reynolds number on the boundary layer growth. How does your plot behave near the leading edge? What numerical difficulty might cause you to have problems near the leading edge?

(d) Discuss the effect of Reynolds number and bluntness on the boundary-layer thickness at the leading edge. How significant is this to the overall boundary-layer thickness?

(e) Discuss the limit $k \to 0$.