A&AE 613
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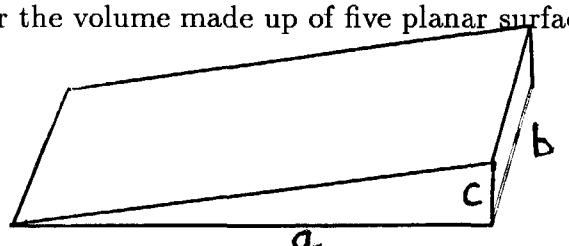
Exercise 1

1. Consider the volume made up of five planar surfaces as sketched:



With each of the five planes making up the surface one associates a vector equal to the surface area times the unit normal vector. Show that the sum of these vectors is zero.

2. Equilibrium between surface forces acting on the surface enclosing a volume and volume forces means:

$$\int_S \vec{\Pi}\, dA = \int_V \vec{f}\, dV.$$

For a sufficiently small volume the right hand side becomes negligible since a volume element vanishes faster than a surface element as the size goes to zero. Hence for a small volume

$$\int_S \vec{\Pi}\, dA = 0.$$

Apply this result to an infinitesimal volume of a shape like the one in problem 1 above, and show that the surface force $\vec{\Pi}$ must be a linear vector function of the normal vector $\hat{n}$.

Note: Exercises are aimed at supplementing the results obtained in class, and are not to be turned in. They give you a chance to prove results which were only sketched or stated in class. Part 1 is easy, but Part 2 is not as straightforward, although it can be done elegantly.
This exercise is taken almost verbatim from a handout of H. W. Liepmann's in Ae101a, Caltech, 1983.