1. SUPersonic WIND TUNNEL

1.1 BACKGROUND

1.1.1 Objectives:
This handout is adapted from the one once used in AAE334L for the supersonic wind tunnel lab. It will aid you in the use of the wind tunnel, schlieren system, and apparatus for measuring pressure distribution.

1.1.2 Supersonic Flow Through A Nozzle

Figure 1 shows the flow through a converging-diverging nozzle. Assume that the pressure difference between Region I and Region II is great enough that the flow in Region I is subsonic and the flow in Region II is supersonic. Total quantities in the flow are designated by the subscript "o". The position along the nozzle with the smallest area is called the throat. Conditions where the flow is sonic (M = 1) are denoted with a "*" superscript.

From the continuity, momentum, and energy equations and the perfect gas laws the following relationships can be found:

\[
\frac{P_o}{P} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{\gamma}{\gamma-1}} \quad (1)
\]

\[
\frac{\rho_o}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\frac{1}{\gamma-1}} \quad (2)
\]

\[
\frac{T_o}{T} = 1 + \frac{\gamma - 1}{2} M^2 \quad (3)
\]
\[
\left( \frac{A}{A^*} \right)^2 = \frac{1}{M^2} \left[ \frac{2}{\gamma + 1} \left( 1 + \frac{\gamma - 1}{2} M^2 \right) \right]^{(\gamma + 1) / (\gamma - 1)} \tag{4}
\]

where

- \( p \) = pressure at a particular point along nozzle
- \( \rho \) = density at a particular point along nozzle
- \( T \) = temperature at a particular point along nozzle
- \( M \) = Mach number at a particular point along nozzle
- \( A \) = area at a particular point along nozzle
- \( \gamma \) = ratio of specific heats

These relationships are tabulated in textbooks.

### 1.1.3 Supersonic Flow Around Bodies

When a sharp-edged body (wedge) is in supersonic flow an oblique shock wave is formed which is attached to the wedge (Figure 2a). For the attached shock wave the relationship between the shock angle (\( \beta \)), wedge angle (\( \theta \)), and the Mach number ahead of the oblique shock wave (\( M_1 \)) is:

**Figure 2: Shock Wave Structure**

When a sharp-edged body (wedge) is in supersonic flow an oblique shock wave is formed which is attached to the wedge (Figure 2a). For the attached shock wave the relationship between the shock angle (\( \beta \)), wedge angle (\( \theta \)), and the Mach number ahead of the oblique shock wave (\( M_1 \)) is:
\[
\tan \theta = 2 \cot \beta \frac{M_1^2 \sin^2 \beta - 1}{M_1^2 (\frac{\gamma}{\gamma + \cos 2\beta}) + 2} \quad (5)
\]

The relationship between the shock angle \((\beta)\), wedge angle \((\theta)\), Mach number ahead of the oblique shock wave \((M_1)\) and the Mach number behind the oblique shock wave \((M_2)\) is:

\[
M_2^2 \sin^2 (\beta - \theta) = \frac{1 + \frac{\gamma - 1}{2} M_1^2 \sin^2 \beta}{\frac{\gamma M_1^2 \sin^2 \beta - \frac{\gamma - 1}{2}}{2}} \quad (6)
\]

These equations are given graphically in many textbooks.

The relations for the conditions (pressure, temperature, density, Mach number, etc.) before and after the oblique shock wave can be determined from the normal shock relations with the modification that the incoming Mach number be replaced by the normal component of the Mach number (see Figure 2a). Thus,

\[
M_{1\text{NORMAL}} = \frac{u_1}{a_1} = M_1 \sin \beta
\]

And therefore:

\[
M_{2a} = M_2 \sin(\beta - \theta)
\]

When a blunt body is placed in a supersonic flow a detached shock wave forms in front of the body (Figure 2b).

1.1.4 Supersonic Expansion by Turning

Consider the supersonic expansion of a flow through a convex turn such as the flow shown on the wedge of Figure 2a. For a sharp turn (Figure 3) there are a series of Mach lines emanating from a single point.
1.1.5 Prandtl-Meyer Function

The differential relation between $\theta$ and $M$ in an isentropic compression or expansion by turning is:

$$d\theta = \sqrt{M^2 - 1} \frac{dV}{V}$$

(7)

where:

$M$ = Mach Number
$V$ = Velocity
$\theta$ = Angle through which flow is turned
$d\theta$ = Change in $\theta$. Positive when expanding, negative when compressing.

Now $V$ may be written in terms of $M$ using the following equations:

$$V = aM$$

(8)

The relationship between $a_0$, $a$, and $M$ is:

$$\frac{a_0^2}{a^2} = 1 + \frac{\gamma - 1}{2} M^2$$

(9)

Using Equation (8) we have:

$$\frac{dV}{V} = \frac{dM}{M} + \frac{da}{a}$$

(10)
Differentiating (9) and using this in (10) we get:

\[ \frac{dV}{V} = \frac{dM}{M} \left[ \frac{1}{1 + \frac{\gamma - 1}{2} M^2} \right] \]  

(11)

Using Equation (11) in Equation (7) we have:

\[ d\theta = \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} dM \]  

(12)

From Equation (12) we have:

\[ \int d\theta = \int \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} dM \]  

(13)

Integrating Equation 13 we have:

\[ \theta = \left( \frac{\gamma + 1}{\gamma - 1} \right)^{\frac{1}{2}} \tan^{-1} \left[ \frac{\gamma - 1}{\gamma + 1} (M^2 - 1) \right]^{\frac{1}{2}} - \tan^{-1} \left[ M^2 - 1 \right]^{\frac{1}{2}} + \text{CONSTANT} \]  

(14)

Define the Prandtl-Meyer function (ν) as follows:

\[ \nu(M) = \left( \frac{\gamma + 1}{\gamma - 1} \right)^{\frac{1}{2}} \tan^{-1} \left[ \frac{\gamma - 1}{\gamma + 1} (M^2 - 1) \right]^{\frac{1}{2}} - \tan^{-1} \left[ M^2 - 1 \right]^{\frac{1}{2}} + \text{CONSTANT} \]  

(15)

Pick a value of θ and M so that the constant can be set. For M = 1 let θ = 0 (this value of θ was chosen arbitrarily); equation (14) then gives

\[ \text{CONSTANT} = 0. \]  

(16)

From equation (12) we have:

\[ \int_{\theta_1}^{\theta_2} d\theta = \int_{M_1}^{M_2} \frac{\sqrt{M^2 - 1}}{1 + \frac{\gamma - 1}{2} M^2} dM \]  

(17)

Thus from equation (17):

\[ \theta_2 - \theta_1 = \nu(M_2) - \nu(M_1) \]  

(18)

Let M_1 = 1, then θ_1 = 0 (Note: θ = 0 was chosen to correspond to M = 1, i.e. ν (1) = 0). Then from equation (18):
\[ \theta_2 = \nu(M_2) \]  

Thus \( \nu(M_2) \) is the angle through which the flow must be isentropically turned to obtain a Mach number of \( M_2 \) if it is initially at a Mach number of 1.

The value \( \theta = 0 \) was chosen to correspond to \( M = 1 \). Any value of \( \theta \) could be chosen to correspond to \( M = 1 \). For different values of \( \theta \) a different value for the CONSTANT would be obtained. Most values of the Prandtl-Meyer function are based on choosing \( \theta = 0 \) for \( M = 1 \), i.e. CONSTANT = 0.

**Example:**

Suppose we have a flow with \( M = M_1 \) and we turn the flow through an angle \( \Delta \theta \). Find the Mach number after the flow has been turned. NOTE: \( \Delta \theta \) is positive in this case since the flow is expanding.

![Figure 4: Prandtl-Meyer Expansion Fan](image)

For \( M_1 = 2.0 \) and \( \Delta \theta = 5 \)

\[ \theta_1 = \nu(M_1) = 26.38 \quad \text{(from tables)} \]

\[ \theta_2 = \theta_1 + \Delta \theta = 26.38 + 5 \]

\[ \theta_2 = 31.38 \]

Therefore, from tables,

\[ M_2 = 2.185 \]

**1.1.6 The Supersonic Wind Tunnel**

A schematic of the supersonic wind tunnel used in this laboratory is shown in Figure 5.
Compressed dry air is pumped into a storage tank with a total volume of 3000 gal. (about 400 cubic feet). This tank holds the large amount of air that is necessary for runs of any useful duration, as the mass flow through the tunnel is quite large. The air passes through several valves and is regulated down to the pressure desired. The regulated flow then passes through a knife valve, which can be opened very rapidly, and into the stilling or plenum chamber, where the turbulence from the pipe flow is reduced. Because the velocity of the flow in the plenum chamber is very small, the static and total values in the plenum chamber are nearly equal. Here they will be assumed to be equal.

The flow converges to the first throat where it reaches the sonic condition. The flow further expands and accelerates supersonically through the diverging nozzle until it reaches the test section, a region of constant area. In the test section, the flow is at a uniform velocity (at least nominally). As the air expands downstream of the first throat, the static temperature decreases considerably. This is why the air must be dry. If it were not, the dew point would be quickly reached and a condensation shock would form at the point where the temperature dropped below the critical value. The flow through the diverging nozzle is nearly isentropic due to the shape of the nozzle block. This shape may be computed by using the method of characteristics.

As the flow accelerates supersonically down the nozzle it encounters the model. The model is located in the test section so that the air is flowing at a uniform speed as it reaches the model. A series of shock waves and expansion fans will be formed due to the
disturbance of the airflow by the model. These shock waves and expansion fans are viewed using a schlieren system.

Immediately behind the test section is the second throat. The flow is compressed and decelerated as the throat necks down. The center flow does not decelerate to \( M = 1 \), however. Behind the second throat, the flow starts to expand and accelerate. However, since total pressure has been lost because of shocks in the test section, the flow cannot accelerate for long. A normal shock forms somewhere in the diverging portion after the second throat. The flow is subsonic behind the normal shock, which is what is desired. The purpose of the second throat is to control the position of the normal shock, so that it is downstream of the test section, and as weak as possible. The diffuser downstream of the second throat reduces the losses, so that less total pressure is required to run the tunnel. In our case of a "blow down" tunnel, longer runs are then permitted.

Small pressure taps are located along the walls of the nozzle and on the models. This allows the static pressure along the nozzle and models to be measured.

A vacuum tank and pump system is located downstream of the diffuser. This system allows running the tunnel at a high pressure ratio (for higher Mach number, with an appropriate nozzle block) or a lower total pressure (lower Reynolds number). If the vacuum system is not used, then the flapper cover outside the building above the exit pipe should be propped open with a piece of wood so that residual vacuum or pressure will not exist in the exit system. Note that since information travels only downstream in supersonic flow, lowering the downstream pressure has no effect for nominal cases. For low upstream pressures, the pressure ratio relative to atmospheric exit pressure is insufficient to obtain supersonic flow in the tunnel. For these low Reynolds number cases, lowering the downstream pressure enables starting the tunnel.

1.1.7 The Schlierene System

A schlieren photographic system is used to observe and photograph shock waves in the supersonic tunnel. The schlieren system is an optical method for locating shock waves, expansion fans, and other regions of high density variation. In the experiment, it will be used to measure the oblique shock angle, \( \beta \), on a wedge.

A top-view schematic of the schlieren system is shown in Figure 6. The knife edge is located at the source image and a camera is focused on the test section.
Figure 6: Schematic of a Schlieren System

With no flow through the test section the image of the light source is aligned so the light goes evenly past the knife edge giving a uniform image. When flow is started in the wind tunnel, regions of rapid density changes (shock waves, expansion fans, etc.) bend the light rays so that some rays no long go through. The shock waves etc. show up as variations in brightness. For a more detailed description see Liepmann and Roshko, *Elements of Gasdynamics*, Dover, 2003.

1.1.8 The Pressure Transducer System

A multi-channel pressure transducer system has been developed. When tubing is connected or disconnected, **IT IS CRITICAL NOT TO APPLY PRESSURES THAT OVERRANGE THE TRANSDUCERS!** Applying pressures above the rated range can damage the transducers!

There are 8 temperature-compensated pressure transducers, and a 10-channel unit with the electronics. The channels can be sampled with a voltmeter or oscilloscope.

1.2 EXPERIMENTAL PROCEDURE

1.2.1 Brief Description of the Experiment

In this lab, the supersonic wind tunnel will be operated in various configurations with various models. With the camera in position and focused, the tunnel will be started and oblique shock waves may form on the models. The pressure and temperature will be recorded for the plenum chamber for each case. For various cases, the surface pressures can be measured using the pressure transducers.

The schlieren image may also be recorded.
1.2.2 Apparatus

The equipment used in the experiment is expensive, so be very careful. The equipment being used is one of a kind and cannot be replaced on short notice. Following is a list of the equipment used and its use in this experiment:

1. **Blowdown Supersonic Wind Tunnel**
   Contains flow of high velocity air.

2. **Model in Tunnel**
   Creates a wave pattern in the flow.

3. **Test Bodies and Stands**
   Tests bodies to be used.

4. **Air Tanks**
   Contain high-pressure dry air used in creating supersonic flow.

5. **50-HP Compressor**
   Replenishes high-pressure air in tanks.

6. **Mirror**
   Reflects light from projection lamp across test body to TV camera. Reflective coating is on the outside, so don’t touch!

7. **Light Source**
   Continuous light source, used to illuminate shock wave.

8. **Carriage**
   Supports schlieren system.

9. **Knife Edge**
   Sets contrast for picture of shock wave.

10. **CCD Camera**
    Displays shock wave on computer.

11. **Computer**
    Records picture from CCD camera.

12. **Computer Screen**
    Displays picture from the CCD camera.

13. **Pressure Gauge**
    Measures static (assumed total) pressure in plenum (reservoir) chamber.

14. **Digital Thermometer**
    Measures static (assumed total) temperature in plenum chamber.

15. **Plenum Chamber**
    Damps out air turbulence. The velocity is very small in the plenum chamber, thus the total and static conditions are assumed to be equal.

16. **Pressure Transducers**
To obtain best accuracy for low pressure measurements in the supersonic region, the sensors provide pressures with respect to an internal full-vacuum reference, rather than with respect to atmospheric. The sensors have a range of 0-25 psia, and may be damaged if you apply pressures above 50 psia.

17. **Vessel and Gauge for Vacuum Reference**
Provides a source of nearly-zero absolute pressure for calibrating the pressure system. The gauge measures the pressure.

### 1.2.3 Procedure

The procedure consists of three parts to be done in the following order:

1. Preparing the air supply and vacuum reference tank,
2. Preparing the camera,
3. Running the tunnel.

### 1.2.4 Preparing the Air Supply and Vacuum Reference

1. The compressor will run continuously throughout the experiment. It is just replenishing the air supply in the storage tanks. Keep an eye on the outside tank pressure as shown in the gauge on the outside wall. Excessive use of compressed air will lower this pressure too far; this provides a limit on your ability to make repeated runs. Also, be considerate in your use of air if others are also using the compressed air supply.

2. Make sure **LEVER 2** is pushed completely up (south). If it is not up, the tunnel may start prematurely.

3. Open the air supply, Valve 3, (fully counterclockwise). Valve 1, the main air supply, should already be open.

4. The potentiometer controlling the pressure regulator should be adjusted to vary the stagnation pressure for the experiment. This pressure regulator controls the flow of air between the storage tanks and the plenum chamber, to maintain constant pressure downstream of the regulator (as long as there is sufficient pressure upstream). Adjust it carefully, and watch the gauges indicating stagnation pressure. Check the gauges again when the tunnel is running.

5. The tunnel is now ready for operation. To start the tunnel, pull **LEVER 2** completely down. To stop the tunnel push **LEVER 2** completely up.

### 1.2.5 Preparing the Cameras

1. Be careful with the camera system.

2. Turn on the power switch. The camera system and light source should come on. **DO NOT** touch any buttons unless instructed, as they have been preset.
3. Remove the mirror covers. There are two mirrors, one on the end of each arm of the carriage. **DO NOT TOUCH THE MIRROR SURFACES!**

4. Roll the carriage back and forth until a picture of the test body appears in the center of the TV screen. If a picture of the test body does not appear on the TV screen, obtain assistance.

### 1.2.6 Running the Tunnel

1. The air supply and all cameras should be ready.

2. The test body should be visible in the middle of the TV screen. **DO NOT** move the carriage or mirrors, or the light source will be misaligned.

3. Record the initial value of Heise Gauge, which should be the initial pressure of the plenum chamber.

4. Record the initial temperature of the plenum chamber.

5. The following must be done while the tunnel is running:
   A) Observe Shock Wave on TV screen.
   B) Record Heise Gauge value.
   C) Grab a single frame of the schlieren image
   D) Record Digital Thermometer value.
   E) Record the pressure data. The flow should stabilize a few seconds after you start the tunnel.
   F) Stop the tunnel as soon as you have the data you need.

6. Before continuing, know what each member of the group is going to do. A large amount of air will be used in a short period of time. If the tunnel runs longer than 45 seconds, it may go subsonic. **Be sure the pressure in the air tanks has returned to full pressure (above 110 psig) before making a run.** If the initial tank pressure is not high enough, the tunnel stagnation pressure may not remain reasonably constant during the run. Keep your tunnel runs as short as feasible (5-10 sec.?) in order to reduce the use of high-pressure air, and decrease the time you need to wait between runs.

7. The tunnel will make a loud noise when started, so put on ear protectors if desired.

8. If all runs are completed, continue with Shutting Down the Tunnel.

### 1.2.7 Shutting Down the Tunnel

Do step 5 even if another group is waiting to do the experiment:

1. Turn off **VALVE 3** (fully clockwise).
2. Pull LEVER 2 completely out, then close LEVER 2 after air has bled out.
3. Replace mirror covers.
4. Record the ambient temperature and pressure.
5. Remove the model and replace it with the model that was there when you started. Clean the tunnel if necessary.

1.2.8 Instructions for Changing the Test Body

1. Move carriage as far downstream as possible. It has automatic stops.
2. Disconnect the tygon tubing attached to the pressure taps in the bottom of the test stand (if any).
3. While holding the bottom of the test stand, unbolt the two wing bolts.
4. Carefully slide the model out of the tunnel. If you have any trouble, check to make sure there is no vacuum in the tunnel, and check with the TA.
5. Clean the plexiglas inside the tunnel test section with the liquid and wipes provided.
6. Carefully slide the model into the tunnel, and replace the two wing bolts. Make sure the body is pointed upstream.
7. Replace the tygon tubing on the taps of the base of the test stand.
1.2.9 Setup Information

Inside diameter of Plenum Chamber = 11.75 in.
Tunnel Width = 1.75 in.

NOTE: Diagram not to scale.

Figure 7: Sketch of Pressure Taps in Supersonic Wind Tunnel

The following table gives the locations of the pressure taps on the inside of the nozzle, along with the nozzle height at each tap.
<table>
<thead>
<tr>
<th>Station Number</th>
<th>Distance from throat to station, positive downstream, inches</th>
<th>Height from base to top of nozzle, inches</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>-3.0</td>
<td>3.230</td>
</tr>
<tr>
<td>2</td>
<td>-2.5</td>
<td>2.943</td>
</tr>
<tr>
<td>3</td>
<td>-2.0</td>
<td>2.752</td>
</tr>
<tr>
<td>4</td>
<td>-1.5</td>
<td>2.455</td>
</tr>
<tr>
<td>5</td>
<td>-1.0</td>
<td>1.717</td>
</tr>
<tr>
<td>6</td>
<td>-0.5</td>
<td>0.937</td>
</tr>
<tr>
<td>7 (throat)</td>
<td>0.0</td>
<td>0.796</td>
</tr>
<tr>
<td>8</td>
<td>1.0</td>
<td>0.926</td>
</tr>
<tr>
<td>9</td>
<td>2.0</td>
<td>1.100</td>
</tr>
<tr>
<td>10</td>
<td>3.0</td>
<td>1.281</td>
</tr>
<tr>
<td>11</td>
<td>4.0</td>
<td>1.447</td>
</tr>
<tr>
<td>12</td>
<td>5.0</td>
<td>1.628</td>
</tr>
<tr>
<td>13</td>
<td>6.0</td>
<td>1.767</td>
</tr>
<tr>
<td>14</td>
<td>7.0</td>
<td>1.883</td>
</tr>
<tr>
<td>15</td>
<td>8.0</td>
<td>1.986</td>
</tr>
<tr>
<td>16</td>
<td>9.0</td>
<td>2.074</td>
</tr>
<tr>
<td>17</td>
<td>10.0</td>
<td>2.140</td>
</tr>
<tr>
<td>18</td>
<td>11.0</td>
<td>2.189</td>
</tr>
<tr>
<td>19</td>
<td>12.0</td>
<td>2.213</td>
</tr>
<tr>
<td>20</td>
<td>13.0</td>
<td>2.213</td>
</tr>
<tr>
<td>21</td>
<td>14.0</td>
<td>2.213</td>
</tr>
<tr>
<td>22</td>
<td>15.0</td>
<td>2.213</td>
</tr>
<tr>
<td>23</td>
<td>20.0</td>
<td>2.138</td>
</tr>
</tbody>
</table>