Introduction to the Oscilloscope

10.1 Background

1.1.1 Objectives:
This handout describes:
1. The simpler skills involved in operating a digital oscilloscope.
2. Some basics about frequency-domain analysis.

It is adapted from a lab in AAE333L.

1.1.2 Specific Applications:
Unsteady flows are common. One of the most basic methods of observing unsteady phenomena is with the oscilloscope, perhaps the most fundamental modern electronic device

1.1.3 Introduction
Unsteady signals are commonly observed with an oscilloscope. Some of the theory behind the simple analog oscilloscope is described in the next section. The operation of a digital oscilloscope will then be discussed.

1.1.4 The Oscilloscope
An oscilloscope is a universal measuring instrument capable of measuring a wide variety of rapidly changing electrical phenomena, even if the phenomenon occurs only once and lasts only a fraction of a millionth of a second.

The oscilloscope graphs the changes in voltage in the event on its vertical axis, and its elapsed time on the horizontal axis. The user can determine whether the voltage is changing positively or negatively, measure the amplitude and duration of the event and observe the actual shape of the waveform.

The basic component of an oscilloscope is the cathode-ray tube (crt), Figure 1. The crt is a special type of vacuum tube in which electrons, emitted when the cathode is heated, are shaped into a narrow beam and accelerated to a high velocity before striking a phosphor-coated screen. The screen glows at the point where the electron beam strikes. A cylindrical grid surrounds the cathode and controls the beam intensity as electrons pass through the end opening in the grid. This control of beam intensity is obtained by varying the negative voltage of the grid.

After leaving the grid the electron beam passes through two or more cylindrical focusing anodes which narrow the beam. The first anode concentrates the electrons and the second anode increases their speed. This entire assembly is called an electron gun. The beam then passes between two sets of deflecting plates. The electron beam is attracted by the positive plate. Thus by varying the voltage on the horizontal and vertical plates the beam position can be controlled.

Figure 2 is a block diagram of a simplified analog oscilloscope, omitting power supplies. The waveform to be observed is fed into the vertical-amplifier input. The
calibrated VOLTS/DIV control sets the gain of this amplifier. The output of the vertical amplifier is fed through a delay line to the vertical-deflection plates of the cathode-ray tube. The purpose of the delay line is explained later.

**Figure 1. Cathode-Ray Tube**

The time-base generator or "sweep generator" develops a sawtooth wave that is used as a horizontal-deflection voltage. The rising or positive-going part of this sawtooth, called the "run-up" portion of the wave, is linear. That is, the waveform rises through a given number of volts during each unit of time. This rate of rise is set by the calibrated TIME/DIV control. The sawtooth voltage is fed to the time-base amplifier. The phase inverter within the amplifier allows this device to supply two output sawtooth waveforms simultaneously - a positive-going sawtooth is applied to the right-hand horizontal-deflection plate, and a negative-going sawtooth is applied to the left-hand deflection plate. As a result, the cathode-ray beam is swept horizontally to the right through a given number of graticule divisions during each unit of time - the sweep rate being controlled by the TIME/DIV control.

In order to maintain a stable display on the cathode-ray tube screen, each horizontal sweep must start at the same point on the waveform being displayed. To maintain a stable display, a sample of the displayed waveform is used to "trigger" a circuit producing a negative output voltage spike at some selected point on the displayed waveform. This triggering spike is used to start the run-up portion of the time-base sawtooth. Therefore "triggering" is synonymous with the starting of the horizontal sweep of the trace at the left-hand side of the graticule.

A rectangular "unblanking" wave derived from the time-base generator is applied to the grid of the cathode-ray tube. The duration of the positive part of this rectangular wave corresponds to the duration of the positive-going or run-up part of the time-base output, so that the beam intensity is switched on during its left-to-right travel and is switched off during its right-to-left retrace.
The leading edge of the waveform is used to actuate the trigger circuit. Yet we may want to observe this leading edge on the screen. Without additional circuitry this would be an impossibility, since the triggering and unblanking operations require a measurable time interval, often about 0.25 microsecond. This time interval is introduced by the delay line in the vertical-deflection channel after the point where the sample of the vertical signal is tapped off and fed to the trigger circuit, thus allowing the leading edge of the waveform to be observed. The delay line retards the waveform to the vertical-deflection plates until the trigger and time-base circuits have had an opportunity to get the unblanking and horizontal sweep operations underway. In this way, we can view the leading edge of that waveform even though it was used to trigger the horizontal sweep.
Figure 2. Block Diagram of Oscilloscope
1.1.5 Digital Oscilloscope

A digital oscilloscope is an oscilloscope that digitizes the input signal and stores the results. While a regular analog oscilloscope gives a continuous analog display of the signal, a digital oscilloscope samples the signal at set intervals chosen by the user using the time per division control. It then converts the voltage sampled and the time into numbers (digitizing) and stores these numbers as ordered pairs (time, voltage). The scope then represents the input signal by graphing these discrete points. If the display of a digital oscilloscope is magnified, these individual points can be seen, in contrast to the continuous curve traced by a regular oscilloscope.

The LeCroy 9304AM digital oscilloscope can acquire 250,000 samples in a single record. The number of samples represented on the screen is dependent on the time/division setting and on the setting of the “Record up to” control (maximum number of points). The sampling information can be displayed by pushing the “timebase setup” button.

A big advantage of the digital oscilloscope concerns the way the data is stored. Its memory can be configured in such a way that it will store a user-selected time record of data that actually precedes a triggering event. This capability is called pretriggering; it is possible because the scope is always sampling data, even before the trigger condition occurs. Also, since the input signal is represented digitally, as pairs of numbers, this information can be transferred directly to a printer, plotter, or another computer, for later processing. Slow signals can also be readily displayed.

Some of the important parameters of a digital scope are: 1) The time required to digitize a data point, 2) The minimum time between the acquisition of successive data points, 3) The precision with which the signal can be digitized (8 bit, 10 bit, 12 bit, etc.), 4) The maximum number of samples that can be digitized. If (1) and (2) are too slow compared to the maximum signal frequency, a phenomena known as "aliasing" may occur; a distorted and misleading signal can be obtained (see figure 3). This can happen if (2) is set too slow, even if the instrument is capable of sufficient speed. It is aliasing that makes wagon wheels appear to rotate backwards, in old Westerns.

![Figure 3 Aliasing](image-url)
1.1.6 Introduction to Power Spectra

An important tool for analyzing unsteady signals is the “power spectrum”, which is the average of a set of Fourier Transforms. The following section, taken from Vibrations and Waves, by A.P. French, W. W. Norton, 1971, recalls the fundamentals of Fourier Analysis.

1.1.7 Fourier Analysis in Action

(From A.P. French, , W.W. Norton, 1971.)

To put the Fourier analysis into practice, we must be able to determine the coefficients of the component sine or cosine functions. The process of doing this is called harmonic analysis, and the properties of sine and cosine functions make it a quite simple affair.

Consider the expansion for $y(x)$:

$$y(x) = \sum_{n=1}^{\infty} B_n \sin \left(\frac{n\pi x}{L}\right)$$

Suppose we want the amplitude associated with a particular value of $n$ - say $n_1$. To find it we multiply both sides of the equation by $\sin \left(\frac{n_1\pi x}{L}\right)$ and integrate with respect to $x$ over the range from zero to $L$:

$$\int_{0}^{L} y(x) \sin \left(\frac{n_1\pi x}{L}\right) dx = \sum_{n=1}^{\infty} B_n \int_{0}^{L} \sin \left(\frac{n\pi x}{L}\right) \sin \left(\frac{n_1\pi x}{L}\right) dx$$

On the right we still appear to have an infinite series of terms. But now consider the properties of an integral whose integrand is a product of sines. Given any two angles, $\theta$ and $\psi$, we have

$$\cos(\theta - \psi) = \cos \theta \cos \psi + \sin \theta \sin \psi$$

$$\cos(\theta + \psi) = \cos \theta \cos \psi - \sin \theta \sin \psi$$

Therefore,

$$\sin \theta \sin \psi = \frac{1}{2} \left[ \cos(\theta - \psi) - \cos(\theta + \psi) \right]$$

Hence we can put

$$\sin \left(\frac{n\pi x}{L}\right) \sin \left(\frac{n_1\pi x}{L}\right) = \frac{1}{2} \left\{ \cos \left(\frac{(n-n_1)\pi x}{L}\right) - \cos \left(\frac{(n+n_1)\pi x}{L}\right) \right\}$$

Therefore,
\[ \int \sin \left( \frac{n \pi x}{L} \right) \sin \left( \frac{n_1 \pi x}{L} \right) \, dx = \frac{L}{2\pi(n-n_1)} \sin \left( \frac{(n-n_1)\pi x}{L} \right) - \frac{L}{2\pi(n+n_1)} \sin \left( \frac{(n+n_1)\pi x}{L} \right) \]

If we insert the limits \( x = 0, x = L \), the values of \( \sin(n \pm n_1)\pi x/L \) are all zero. Thus at first sight it would appear that we had got rid of the right-hand side of the equation altogether. But then we notice that the quantity \( (n - n_1) \) appears in the denominator of one of the integrals. Thus if \( n = n_1 \), we have one integral of the form 0/0. And it at once turns out that, although all other terms are zero, this one is not. For if \( n = n_1 \), the integral to be evaluated is the following:

\[ \int_0^L \sin^2 \left( \frac{n_1 \pi x}{L} \right) \, dx = \frac{1}{2} \int_0^L 1 - \cos \left( \frac{2n_1 \pi x}{L} \right) \, dx \]

The cosine term contributes nothing between the given limits, but the other part gives us \( L/2 \). Thus we arrive at the following identity:

\[ \int_0^L y(x) \sin \left( \frac{n \pi x}{L} \right) \, dx = \frac{L}{2} B_n \]

i.e.,

\[ B_n = \frac{2}{L} \int_0^L y(x) \sin \left( \frac{n \pi x}{L} \right) \, dx \]

This equation determines for us the amplitude \( B_n \) associated with any given value of \( n \) in the harmonic analysis of \( y(x) \).

If \( y(x) \) is a purely empirical curve, the evaluation of the Fourier coefficients \( B_n \) is a matter for computers or graphical integration. But if the form of \( y(x) \) can be described by an exact analytic function, we can obtain a general formula for all the \( B_n \)'s. To illustrate the procedure, let us take the profile shown in Figure 4(a).

\[ y = kL \]
This is like the shape of a string that is stretched aside at one extreme end. (Of course, at \( x = L \) exactly we must have \( y = 0 \); we are assuming, however, that our equation for \( y \) holds good for points arbitrarily close to the end.) The evaluation of the \( B_n \)'s then proceeds as follows:

\[
B_n = \frac{2}{L} \left[ kx \sin \left( \frac{n\pi x}{L} \right) \right]_0^L
\]

Integrating by parts, we find

\[
B_n = \frac{2k}{L} \left\{ -\frac{L}{n\pi} \left[ x \cos \left( \frac{n\pi x}{L} \right) \right]_0^L + \frac{L}{n\pi} \left[ \cos \left( \frac{n\pi x}{L} \right) \right]_0^L \right\}
\]
\[ \frac{2k}{n\pi} \left\{ -x \cos \left[ \frac{n\pi x}{L} \right] \bigg|_0^L + \frac{L}{n\pi} \sin \left[ \frac{n\pi x}{L} \right] \bigg|_0^L \right\} \]

\[ = -\frac{2kL}{\pi} \cos n\pi \]

One recognizes that the values of $B_n$ fall into two categories, according to whether $n$ is odd or even, because the value of $\cos n\pi$ alternates between the values +1 and -1. We have, in fact,

\[
n\text{odd}: \quad B_n = \frac{2kL}{n\pi} \\
n\text{even}: \quad B_n = -\frac{2kL}{n\pi}
\]

If one wishes, however, one can represent both sets by means of the single formula

\[ B_n = (-1)^{n+1} \frac{2kL}{n\pi} \]

It is now an easy matter to tabulate the various amplitudes (Table 1). Thus our description of the triangular profile becomes:

\[ y(x) = \frac{2kL}{\pi} \left\{ \sin \left[ \frac{\pi x}{L} \right] - \frac{1}{2} \sin \left[ \frac{2\pi x}{L} \right] + \frac{1}{3} \sin \left[ \frac{3\pi x}{L} \right] + \cdots \right\} \]

<table>
<thead>
<tr>
<th>n</th>
<th>( B_n / kL )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \frac{2}{\pi} ) = 0.636</td>
</tr>
<tr>
<td>2</td>
<td>( -\frac{1}{\pi} ) = -0.318</td>
</tr>
<tr>
<td>3</td>
<td>( \frac{2}{3\pi} ) = 0.212</td>
</tr>
<tr>
<td>4</td>
<td>( -\frac{1}{2\pi} ) = -0.159</td>
</tr>
<tr>
<td>5</td>
<td>( \frac{2}{5\pi} ) = 0.127</td>
</tr>
</tbody>
</table>
The result of synthesizing various numbers of terms, using the numerical coefficients of which the first five are listed in Table 1 is shown in Figure 4(b)-(d). By including further terms we can make the fit as good as we have the patience to achieve. And it is quite remarkable that, with so few terms as we have used, one can simulate the general trend of a profile that differs so radically from any sine curve - especially one that departs so far from zero at one end.

The sine curves in terms of which the Fourier analysis is made represent an example of what are called orthogonal functions. The description "orthogonal" belonged originally, of course, to geometry. The orthogonality of two sine functions in Fourier analysis is described by the result

$$\int_0^L \sin\left(\frac{n_1 \pi x}{L}\right) \sin\left(\frac{n_2 \pi x}{L}\right) dx = 0 \hspace{1cm} \text{for } n_1 \neq n_2$$  \hspace{1cm} (1)

This may at first sight appear to have no connection with the geometrical condition, but it is not so far removed as one might think. For if we have two vectors, A and B, the condition that they are orthogonal (perpendicular) to each other is that their scalar product be zero. In terms of their components this can be written:

$$A_xB_x + A_yB_y + A_zB_z = 0$$

Now if we replaced the continuous integral of equation (1) by a summation over a very large number, N, of separate terms (as we might do if we were evaluating the integral by numerical methods), a particular value of x could be written as $x_p$, where

$$x_p = \frac{pL}{N}$$

Thus, equation (1) would be replaced by the following statement:

$$\frac{L}{N} \sum_{p=1}^{N} \sin\left(\frac{n_1 \pi p}{N}\right) \sin\left(\frac{n_2 \pi p}{N}\right) = 0 \hspace{1cm} \text{for } n_1 \neq n_2$$

If we write the condition for orthogonality of two ordinary vectors in the form

$$\sum_{p=1}^{3} A_p B_p = 0 \hspace{1cm} \text{for } A \perp B$$

we see that, in a purely formal sense, the difference between the two statements is merely that one of them involves quantities that are completely described by just three components, whereas the other needs N components (and, in the limit, infinitely many). The possibility of analyzing an arbitrary function of terms of a set of orthogonal functions (not necessarily sines or cosines) is one of the most important and widely used techniques in engineering.
1.1.8 Examples of the Use of the LeCroy Oscilloscope for Spectral Analysis

Figure 5  Sampling of the 1kHz Square Wave

Fig. 5 is an image of the LeCroy screen, sampling the 1kHz square wave from the CAL OUT connector. The upper trace is the signal, the second trace is an expanded version of the signal, and the lower trace is the Fast Fourier Transform (FFT) of the signal. This FFT is similar to the Fourier series decomposition just discussed.
Fig. 6 is a screen-image for a high-pitched whistle into a microphone. The upper trace is the signal, the second trace is an expanded portion, the 3rd trace is the FFT, and the 4th trace is an average of 10 FFT’s (which begins to approach the ‘power spectrum’). Note the peak in the FFT’s at the whistle frequency, and the presence of other peaks.
Fig. 7 is a screen-image for a low-pitched hum. More frequencies are present, for the tone is not as clean.

1.1.9 Summary

In this handout instructions on the use of the oscilloscope were given. The instructions describe and demonstrate how to use the various functions on the oscilloscope.

Lab Procedure

1.1.10 Prelab Preparation

1. Study the scope handout provided in class, 'Getting to Know the LeCroy 9310 Dual-Channel Digital Oscilloscope'.

2. Any periodic function $y(t)$ can be expressed in terms of a sum of sines and cosines of various frequencies, through a process known as Fourier analysis (see Background).
This process is often used in experimental analysis to discuss the so-called 'frequency content' of a signal. In this problem, you can compute theoretical results that you may later compare to experiment. It is adapted from French, *Vibrations and Waves*, MIT Press, 1971, p. 191. Consider a periodic function \( y(t) \) that consists of a square wave, which repeats with period 2\( T \). Between 0 and \( T \), \( y = V \). Between \( T \) and 2\( T \), \( y = -V \). Because of the symmetry of this function, it can be shown that \( y \) can be represented in terms of an infinite series of sines:

\[
y(t) = \sum_{n=1}^{\infty} B_n \sin\left(\frac{n \pi t}{T}\right) \tag{1}
\]

Using the properties of sines and cosines, it is easy to show that

\[
B_n = \frac{2}{T} \int_{0}^{T} y(t) \sin\left(\frac{n \pi t}{T}\right) dt. \tag{2}
\]

a) Sketch the square wave, labeling important points on both axes.

b) By evaluating equation 2 for the given function \( y(t) \), find the formula for the coefficients \( B_n \) of the Fourier series representation of the square wave. Evaluate \( B_n / V \) for the first five coefficients.

c) Let \( T \) be 0.5 millisecond, so that the basic frequency of the square-wave function is 1 kHz. \( B_1 \) is then the amplitude of the 1kHz component of the wave. Find the amplitude of the Fourier component that has frequency 3kHz, as a fraction of the amplitude at 1kHz. Do the same thing for the Fourier components with frequency 5, 7, 9, and 11kHz.

1.1.11 Introduction

After this experiment the student should be able to operate the LeCroy 9304AM digital oscilloscope using continuous signals. The square-wave calibration output on the LeCroy is first examined. The real-time Fourier analysis capability of the scope is used to look at power spectra.

1.1.12 Introduction to the LeCroy 9314AM Oscilloscope

1. All equipment in this experiment should be turned on immediately to allow adequate warm-up time.

2. Disconnect all cables from the front of the oscilloscope.

3. Follow the procedure in the handout “Getting to Know the LeCroy 9310 Dual-Channel Digital Oscilloscope”. This handout applies also to the 9314AM (with some minor differences), and will introduce you to the scope.

4. The FFT of the square wave can be computed. The peaks in the FFT are the Fourier components of the square wave. Using the cursors, record the amplitude and frequency of the first 11 Fourier components of the square
wave. You may compare these amplitudes to the values computed in your prelab. Make two or three screendumps of your FFT to document your results. The scope provides the amplitudes in units of decibels (dBm). These can be converted to volts using a formula given in the 9304 manual, \( \text{dBm} = 10 \log_{10}(V_{mag}^2/V_{ref}^2) \), where \( V_{ref} = 0.316 \) volts.

5. You can dump an image of the screen to a TIF file on a floppy disk, if you’ve brought one, for later inclusion into your report. To do this, access HARDCOPY SETUP through the UTILITIES menu, and set the destination and format accordingly. Then push the SCREENDUMP button, which will save a file of format Dnnnn.TIF. You’ll need to keep track of the conditions under which the file numbers were saved. If you want a paper copy, you’ll have to reset the HARDCOPY SETUP to the Centronics (or printer) port, and to the type of printer being used (HP Deskjet B/W, usually).