DESIGN OF A CONTRACTION FOR A TRANSONIC LUDWIEG TUBE

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ABSTRACT

The Mach 4 Ludwieg tube at Purdue University is being converted to test airfoils at transonic speeds. The modification consists of a new contraction and a test section with contoured walls. The contraction was designed using matched cubics and a superellipse function for the cross-sections. The three dimensional contraction was approximated as axi-symmetric, so that the Rott-Crabtree method can be applied to compute the boundary layer behavior. The new contraction was designed so that there would be no separation of the boundary layer anywhere in the contraction, in order to avoid any unsteady oscillations that may result in the core flow. This in turn may lead to being the world’s first quiet-flow transonic wind tunnel.

INTRODUCTION

It has been almost a century since the first manned aircraft designed and built by the Wright Brothers flew, and several advanced methods for designing airfoils and simulating flowfields using high-powered computers have been developed. However the final word on the aerodynamic performance of a vehicle and/or a vehicle component must still come from wind tunnel testing [8]. Today, with the severe budget cuts in the aerospace industry and research area, it is crucial that wind tunnels are capable of operating at low cost [6]. To further complicate matters, boundary layer transition tests done in noisy facilities with pressure fluctuations and noise in the freestream flow do not properly simulate the quiet flow of actual flight [7]. The answer to the above two problems is the Ludwieg tube, a wind tunnel design first proposed by Ludwieg of Germany [5].

Many Ludwieg tubes have been built around the world due to the low cost of construction and operation [11]. Ludwieg tubes have an advantage over a conventional blowdown type wind tunnel because of the high Reynolds number and high quality flow. According to the author of reference [10], “The authors are unable to conceive of a fundamentally quieter process for providing the nozzle airflow”. This is due to the shockless inlet design of the Ludwieg tube [10]. A more detailed explanation of the Ludwieg tube is available in references [7] and [10].

In the early 90’s, a Mach 4 Ludwieg tube was built at Purdue University to study high-speed boundary layer transition. A schematic of the facility is shown in Figure 1. Detailed information on this facility can be found in [6] and [7]. A new Mach 6 Ludwieg tube with a larger test section and longer test time is currently under construction to replace the Mach 4 Ludwieg tube. A few possibilities for the future use of the older facility have been discussed, and one option would be to modify the tunnel so that

Figure 1 – Purdue University Mach 4 Ludwieg Tube
CONTRACTION DESIGN METHOD

The section of the Ludwieg tube to be modified is the contraction that will fit between the driver tube and the test section. The driver tube is a steel pipe that has a circular cross-section with a radius of 6.033 inches, and the test section has a rectangular cross-section with a width of 3.804 inches and a height of 4.325 inches. The contraction must smoothly match to the driver tube and to the test section. The contraction was designed using a superellipse function, so that the cross-sectional geometry of the contraction can be transitioned from a circle to a rectangle. The general equation for a superellipse is given below.

\[ \left( \frac{x}{x_0} \right)^a + \left( \frac{y}{y_0} \right)^a = 1 \]

In this equation, \(x_0\) and \(y_0\) are the semi-major and semi-minor axes as a function of \(z\), the downstream distance down the contraction. These values were calculated using a cubic function of \(z\), with \(z\) set to 0 at the inlet. The contraction was split into two regions to allow for more input parameters which ultimately controlled the shape of the contraction. The first section started at the inlet to the contraction and ended at some arbitrary point \(z_1\) in the contraction. The second region started at \(z_1\) and ended at \(z_2\), the inlet to the test section. This resulted in 16 coefficients to solve for using known boundary conditions. The cubic equations for the semi-minor axes in shown below.

\[ x_{0,1}(z) = a_1 + a_2 z + a_3 z^2 + a_4 z^3 \]
\[ x_{0,2}(z) = b_1 + b_2 z + b_3 z^2 + b_4 z^3 \]

Here, the top equation represents the first region, and the second equation represents the second region. Similar equations were also formed for \(y_0\).

To solve the coefficients in the cubic equations, eight equations were formed each for \(x_0\) and \(y_0\) using boundary conditions.

1. \( x_0(0) \) and \( y_0(0) = 6.033 \text{ in.} \)
2. \( x_0'(0) \) and \( y_0'(0) = 0 \).
3. \( x_0''(0) \) and \( y_0''(0) = 0 \).
4. \( x_0(z_2) = 1.902 \text{ in.}, \text{ half the width of the test section.} \)
5. \( y_0(z_2) = 2.1625 \text{ in.}, \text{ half the height of the test section.} \)
6. \( x_0(z_i) \) and \( y_0(z_i) \) must be equal for the two regions.
7. \( x_0''(z_i) \) and \( y_0''(z_i) \) must be equal for the two regions.
8. \( x_0''(z_j) \) and \( y_0''(z_j) \) must be equal for the two regions.

The coefficients were then solved using \(z_i\) and \(z_j\) as inputs to vary the length of the two regions.

A similar technique was also used to find the distribution of the exponents along the contraction centerline. In the beginning, the exponents were also split into two regions, with the same match point as that for the semi-minor and semi-major axes. At the end of the contraction where the cross-section must be rectangular, an exponent of infinity would be needed. This was approximated by using a value of 30 to avoid any numerical round-off errors that might result by using a much higher value. During the design process however, the equations to determine the exponents had to be somewhat modified.

The first major change was that the superelliptical region not necessarily started at \(z\) equal to 0, but at \(z_s\), an arbitrary point downstream of the inlet to the contraction, which was made an input. Second, the first derivative at \(z_s\), and the inflection point in the second cubic function for the exponents was made an input. Reasons for these changes will be discussed in detail later, since it is tied with problems in designing the axes and keeping the boundary layer attached. As with before, the boundary conditions to determine the coefficients in the cubic functions for the exponents are listed below.

1. \( a(z_s) = 2 \)
2. \( a'(z_s) = 0 \)
3. \( a''(z_s) = 0 \)
4. \( a(z_i) \) must be equal for the two regions.
5. \( a'(z_i) \) must be equal for the two regions.
6. \( a''(z_i) = 0 \) location of the inflection point.
7. \( a(z_2) = 30 \)
8. \( a'(z_2) = \text{d}a/\text{d}z, \text{ the derivative at end of contraction.} \)
BOUNDARY LAYER ANALYSIS

METHOD

The properties of the boundary layer that grows on the contraction wall were analyzed using the E0D code written by Douglas Aircraft Company, along with the Rott-Crabtree method, which is the axisymmetric analog of the Thwaites method, outlined in [12]. Although the method may not be accurate as some of the other known boundary layer calculation methods, this method offers the advantage that the computing time is very short, on the order of a few minutes. This makes the Rott-Crabtree method an excellent analysis tool for design. However, using a simple method means that the problem has to be simplified by making a few approximations and/or assumptions.

The E0D code is a panel method program for axisymmetric incompressible flow. The approximations made were that the three-dimensional contraction can be approximated as axisymmetric, and that the boundary layer is incompressible. Because the Mach number stays subsonic, and compressibility does not affect the boundary layer significantly, this should not cause major errors. Detailed information on the E0D code can be found in [1] and [2]. The following information on the Rott-Crabtree method is from reference [12]. The Rott-Crabtree method computes the momentum thickness \( \theta \) as

\[
\theta^2 = \frac{0.45v}{r_0^2U^6} \int_0^z r_0^2U^5dz
\]

Here, \( r_0 \) is the local radius, \( U \) is the edge velocity, \( v \) is the kinematic viscosity, and \( z \) is the streamwise coordinate. A pressure gradient parameter \( \lambda \) is given by

\[
\lambda = \frac{\theta^2}{\nu} \frac{dU}{dz}
\]

where \( \lambda \) must be greater than -0.09 to avoid separation.

Computing whether the boundary layer will separate can be predicted by solving the above equation. The Mach number in the driver tube was taken as 0.06 which is from the estimated area ratio, the total temperature as 300K, and the contraction walls to be isothermal.

An important computational issue was that \( \lambda \) should be 0 in the driver tube, because the velocity derivative is 0 in a region with a constant cross-sectional area. To simulate this, the simulated portion of the driver tube had to be made long enough so that the growth of the boundary layer could be properly simulated. The simulation was performed at first using a driver tube length of 2 inches, and was gradually increased to 20 inches, at which point a satisfactory result was obtained.

Often in wind tunnel designs, after the boundary layer is calculated the wall contours are re-shaped to account for the boundary layer’s displacement thickness. For this contraction, the displacement thickness was calculated using the polynomial curve fit, shown below, which was given in [10].

\[
H = 2 + 4.14z - 83.5z^2 + 854z^3 - 3337z^4 + 4576z^5
\]

Here, \( H \) is the shape factor, or the ratio between the displacement thickness and the momentum thickness, and \( z \) is given by

\[
z = (0.25 - \lambda)
\]

for \( \lambda \) less than 0.25. When \( \lambda \) increased to above 0.25, \( H \) was taken to be constant at 2.0. The polynomial provided an empirical relationship between the momentum thickness, displacement thickness and the pressure gradient parameter.

FINAL DESIGN

There were no special requirements for the shape of the plot of \( x_0 \) and \( y_0 \), except that the contraction must keep the boundary layer attached to the contraction wall. Generally, the shorter the contraction the better because the cost for the material will be reduced, and the contraction will take less time to manufacture. However, the shorter the contraction, the steeper the curvature will be, which will result in an increased possibility of separation.

The plot for the designed \( x_0 \) and \( y_0 \) can be found on Figure 2. \( z_2 \) was chosen to be 30 inches and \( z_1 \) was set at 25 inches. Separation bubbles are sometimes seen in the concave region of wind tunnels. Often times this is a matter of confusion, because the one dimensional flow accelerates uniformly in a contraction, and so the pressure gradient should be favorable. In reality however, there is a two dimensional effect, which produces a higher pressure near the wall than at the edge of the

\[
\text{Fig. 2 - Semi-major and Semi-minor Axes}
\]
boundary layer, in order to turn the flow. If the contraction is too steep, then this pressure gradient can be large enough to separate the flow.

To avoid this effect, the curvature near the inlet was relaxed, and made steep farther downstream. Originally, the length of the contraction was set at 15 inches, the length of the current contraction in use. With this length however, it was not possible to find a value for the match point and a distribution of the exponents in the superellipse function to avoid separation. The length was increased until a satisfactory contraction was designed. Detailed discussion on the boundary layer of the designed contraction will be given later, and for now, it will simply be mentioned that the boundary layer should not separate anywhere.

Several problems existed in calculating satisfactory values for the exponents. One other design issue that came into issue at this point was that the cross-sectional area must continuously decrease. The rate of decrease does not necessarily have to be constant, but the derivative with respect to \( z \) must be negative everywhere in the contraction. A contraction that has a region where the cross-sectional area increase is essentially a diffuser mounted inside a nozzle, clearly not an acceptable design.

Looking at Figure 2, because the rate at which the axes decrease at the beginning of the contraction is very slow, the exponents have to increase very slowly. Otherwise, the cross-sectional area would increase, due to the mathematics of the superellipse function. This is why the starting location of the superelliptical region was made an input. Keeping the region where the axes decrease very slowly an axi-symmetric region, and starting the superelliptical region where the axes are decreasing fast enough was the only way the above problem could be solved.

To complicate matters furthermore, the axes shown in Figure 2 appears to be very smooth, but mathematically, the plot is separated by two distinct equations, and similar with the exponents. This caused the cross-sectional area distribution to always have a small bump at the match point. In order to smooth out this bump as much as possible, the equations for the exponents and the choices of which parameters to make inputs were modified several times, and perhaps hundreds of combinations of different values for the inputs were tried. Some of the options that were tried included using a single 5th order polynomial for the exponents, making the entire first region axi-symmetric, and controlling the derivatives of the exponents at the inlet instead of at the end of the contraction.

In the end, the method described in the previous section seemed to give the best results, with the superelliptical region starting at 12 inches, the inflection point at 27.5 inches, and the first derivative of the exponents at the end of the contraction at 1.0. A plot of the exponents using the above input parameters are given in Figure 3. Here, only the superelliptical region is shown. Even though the superelliptical region starts well downstream into the contraction, the exponents stays rather small until the match point, at which point the axes start decreasing relatively rapidly, and the exponents are able to increase rapidly also.

Figure 4 shows the plot of the equivalent radius. These values were computed by taking the cross-sectional area distribution, which was computed numerically using the superellipse equation, and finding the radius that would give a circle of the same area. The plot shows that the radius is always decreasing, and has a similar profile with the axes. Also, the plot is smooth enough that to the naked eye, there are no noticeable bumps. This plot provided the necessary inputs for the E0D code.

As a final check of the geometry of the cross-sections, Figure 5 shows the cross-sections in 1 inch intervals. The plot starts at the start of the superelliptical region, since the axi-symmetric region would only show circles of varying radius. The plot also shows only a quarter of the geometry, since the full geometry is just a mirror image of Figure 5 on the \( x \) and \( y \) axis.

It can be seen that the cross-sections smoothly transitions from a circle to a rectangular geometry. Because the exponent at the inlet to the test section is
not infinite as should be for a true rectangle, the corners are slightly rounded, but a rectangular shape can clearly be seen. Also, the downstream cross-sections never cross with the upstream cross-sections, showing again that the area decreases everywhere in the contraction.

BOUNDARY LAYER ANALYSIS

The resulting $\lambda$’s for this contraction are plotted in Figures 6-8. Like any computational problems, the grid resolution showed to have impact on the results. To investigate the grid dependency, three resolutions were tried, 0.025, 0.05, and 0.1 inches. Each plot shows a slight, but significant difference in the results. According to Figure 6, with the finest resolution, the minimum $\lambda$ is about $-0.22$ at about 3 inches into the contraction, and then increases back above $-0.09$ somewhere downstream. This is the two dimensional separation bubble effect which was discussed earlier. The problem with this plot is that the resolution is so fine, that the resulting plot is extremely noisy, even though some of the variables were run through a smoothing routine before taking any derivatives.

Figure 7 shows the $\lambda$’s when using a resolution of 0.05 inches, a factor of two larger than before. There is a significant difference from before, in that there is now a definite pattern to the plot, instead of several points scattered all over the plot. This plot predicts that the minimum $\lambda$ is about $-0.04$, meaning that the boundary layer will not separate. And finally, Figure 8 shows the $\lambda$’s at a resolution of 0.1 inches, another factor of two larger than the previous. The noise in the plot has been significantly reduced, and clearly $\lambda$ is a continuous function of $z$. The minimum $\lambda$ occurring at the inlet has been increased even more, to about $-0.01$.

Deciding on which resolution is the best is always a problem with computational problems. Figure 6 may show a detailed plot of the $\lambda$’s in some respect, but had so much noise, that it was too hard to tell whether the plot is accurate. Figure 8 is very smooth, but was thought that the resolution is so coarse, that some of the important details may be missing. Figure 7 seemed to be right in between these two plots, and was chosen as the final answer – there is no separation anywhere in the contraction. However, it will be stressed again that this result is from the assumption that the contraction is axisymmetric, and the actual superelliptic shape of the contraction might have a greater influence on the boundary layer than expected. The actual flow will be three dimensional which was not simulated, but is
of major concern. However, the existing contraction for the Mach 4 Ludwieg Tube also has a superelliptical section and has worked satisfactorily. Because the transition from circular to rectangular cross-section will be much smoother and gradual for the new contraction, it was reasoned that 3-D flow will not be a problem for the new contraction either.

In addition to whether the boundary layer will separate, Figure 7 shows a few other interesting properties about the boundary layer. First, there is a bump at the very beginning of the plot. This is strictly a computational issue, in that in reality, there is no bump. As mentioned before, the driver tube had to be made long enough to smooth out this effect from the inlet boundary condition. Another reason for using a long inlet is because the boundary layer at the inlet to the contraction had to be accurately modeled. A boundary layer does not just magically appear at the inlet, but the boundary layer that is growing in the driver tube flows into the contraction.

After the boundary layer enters the contraction, it passes through the region where it is most likely to separate, which was discussed in detail already. The flow then accelerates, and the λ’s increase, in this highly favorable region. The plot reaches a peak at about 0.4 at z equal to about 12 inches, and then starts to decrease. The reason for this is unknown, but the pressure gradient is still highly favorable, that it should not really matter. The plot then increases once again, and a second peak can be seen. The plot then decreases down to about –0.02, in the highly convex region of the contraction.

The growth of the boundary layer can be seen from the displacement thickness, which are plotted in Figure 9. In order to increase the visibility of the individual points, only some of the data are plotted. Also, the displacement thickness was calculated twice, using a total pressure of 15 and 30 psi, which are the target total pressures to be used in the new facility. Although the current hardware is rated up to 120 psi, high pressures will be avoided for safety reasons.

The plot shows a steady increase in the thickness in the driver tube, and then a maximum thickness where the boundary is most likely to separate, an intuitively predicted result. In the accelerating region where the λ’s increase, the thickness decreases, and becomes very thin before thickening again in the highly convex region. The plot shows some noise near the region where the lowest λ’s occur. This is expected, because the displacement thickness was calculated using the already noisy λ’s. Finally, the displacement thickness is at most 0.0033 inches thick, even at the lowest pressure setting of 15 psi. Compared to the axes, this value is so small, that a boundary layer thickness correction was not performed.

Figure 10 shows the Reynolds number based on the momentum thickness. The plot has a similar profile with the displacement thickness, it grows in the driver tube, and then decreases in the favorable region. One interesting characteristic of this plot is that there seems to be a discontinuity around z equal to about 25 inches. This kink in the plot is due to the match point of the axes. Although it was hard to tell by just looking at plots of the geometry, the match point does indeed become a point where the flow pattern changes.

**SUMMARY**

A new contraction for the Mach 4 Ludwieg tube at Purdue University has been designed. The contraction was designed so that there would be no separation of the boundary layer anywhere in the contraction, in order to avoid any unsteady fluctuations. It should be possible to achieve quiet flow even with a transitioning or turbulent boundary layer, as long as the boundary layer does not separate somewhere in the contraction. The design methodology is outlined in a cookbook form below:

1. Decide on the function(s) to use for the semi-minor, semi-major, and the exponents as a function of the downstream coordinate.
2. Decide on what parameters to make as inputs to vary the geometry.
3. Design a contraction using the inputs.
4. Check the cross-sectional area distribution and make sure that it is decreasing everywhere.
5. Check the cross-sections, and make sure that the proceeding cross-section never crosses the preceding cross-section.
6. Check the pressure gradient parameter, and make sure that it does not go below –0.09. To keep a margin of safety, the author recommends a higher value of $\lambda$ to predict separation, of about –0.05.
7. If any of 4-6 are not satisfied, go back to 3.
8. If further problems exist, then go back to 1 or 2.

CURRENT STATUS AND FUTURE PLANS

The aerodynamic contour of the new contraction has been designed, and now the mechanical issues in replacing the contraction will be planned out. The contraction is planned to be fabricated at the machine shop of the Aerospace Sciences Laboratory of Purdue University. The contraction will be fabricated by using a CNC mill to mill out the contraction from a block of aluminum. Fabrication time is expected to be about 3-4 weeks.

Some work has started in designing contoured test section walls. More information can be found in [3]. While the actual hardware is being designed, plans are to do a full Navier-Stokes computation of the contraction as a final check of the performance.

REFERENCES


