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# **Fast and Efficient Algorithms in Computational Electromagnetics**

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eds.**

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*To our wives and our parents*

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# Preface

This book documents recent advances in computational electromagnetics performed under the auspices of the Center for Computational Electromagnetics at the University of Illinois, funded mainly by the Multidisciplinary University Research Initiative (MURI), a program administered by the Air Force Office of Scientific Research. Other funding agencies also contributed to the success of the Center, such as the National Science Foundation, Office of Naval Research, Army Research Office, and Department of Energy.

There is a tremendous need to bring the science of electromagnetic simulation, also known as computational electromagnetics, to the same confidence level as that achieved by circuit simulation. However, computational electromagnetics involves solving Maxwell's equations, which are more complex than circuit equations. It is hoped that one day electromagnetic simulation will master this complexity and enjoy the same pervasiveness in engineering design as does circuit simulation. We are grateful for the foresight of these funding agencies who share our passion for developing this technology.

This book does not pretend to be complete, as it reflects our viewpoint of computational electromagnetics. However, we believe that the knowledge required to support electromagnetic simulation in a sophisticated manner has to come from physicists, engineers, mathematicians, and computer scientists. Since electrical engineering is an offshoot of applied physics, we play the role of applied physicists in the development of this technology: we develop this technology based on our physical insight into the problems, while drawing on knowledge from mathematicians and computer scientists. The presentation style of most of the chapters of this book is in the manner of applied physicists or of traditional electromagneticists—hopefully, we sacrifice mathematical rigor for physical clarity.

This book is not an introduction to computational electromagnetics. It documents recent advances in computational electromagnetics in the manner of a monograph. A seasoned researcher in the area of computational electromagnetics should have little difficulty reading the material. It is also hoped that a graduate student or a professional with some preliminary background in computational electromagnetics or a classicist in electromagnetics who has done some rapid background reading, can easily digest the work reported in this book. For one who intends to perform research in this area, this book will be an excellent starting point. The variety of topics covered is sufficient to nourish many different research directions in this very interesting field.

Even though this book deals only with linear problems associated with Maxwell's equations, it can be gleaned from a cursory reading that such problems are rich; they are amenable to different mathematical analyses, and allow for different and interesting algorithm designs. Because of the linearity of the problems, both differential equation and integral equation solvers can be developed. Moreover, the problems can be solved in the frequency domain as well as the time domain, enhancing the efficiency and enriching the variety of these methods.

Solutions to Maxwell's equations have been sought since the very early days of the equations' discovery. Electromagnetic analysis has always played an important role in understanding many scientific and engineering problems.

Chapter 1 gives an introduction to electromagnetic analysis and explains how the field has evolved into computational electromagnetics in the last few decades. It also introduces, in a very simplified manner, the recent fast algorithms developed to solve Maxwell's equations. The chapter also attempts to give a historical perspective on electromagnetic analysis and to describe how far we have come since the advent of Maxwell's equations.

Chapter 2 presents an introduction to the fast multipole method (FMM) and the multilevel fast multipole algorithm (MLFMA) in two dimensions. Interpolation, truncation, and integration errors are discussed. An attempt is also made to relate FMM to group theory, and to the inherent symmetry of space.

Chapter 3 describes the three-dimensional version of FMM and MLFMA and demonstrates the application of the fast algorithm to real-world problems. The algorithm has also been parallelized on a shared-memory machine, and four-decade computation involving close to 10 million unknowns is the most important achievement of this work.

Chapter 4 outlines the distributed-memory parallelization of MLFMA, encapsulated in a code called ScaleME (Scaleable Multipole Engine). The parallelization of MLFMA on a distributed memory machine is not an easy task, because different parts of the computation may reside on different processors. The increased communication cost with more processors can be an issue here. A 10-million-unknown problem has also been solved with ScaleME.

Chapter 5 reports on the low-frequency solution of Maxwell's equations using fast algorithms. This chapter describes the treatment needed for FMM and MLFMA to prevent their catastrophic breakdown at low frequencies. It also describes a method to apply the LF-MLFMA based on Rao-Willon-Glisson (RWG), wire, and wire-surface bases while the intrinsic expansion bases are still the loop-tree-star bases. These bases are designed for low-frequency problems to make the LF-MLFMA efficient for problems with global loops.

Chapter 6 delves into different error issues involved when solving surface integral equations related to Maxwell's theory. Discretization error due to the use of basis functions, and integration error by replacing integrals with summation are discussed. Errors result from solving the matrix equation, and deconditioning of the matrix equation by MOM and its impact on errors are studied. This chapter also discusses deconditioning due to the near-resonance problem and the low-frequency breakdown problem.

Chapter 7 deals with a recent topic of intense interest in differential equation solvers—the theory of perfectly matched layers (PML). The concept of complex coordinate stretching is discussed. PML is generalized to curvilinear coordinates as well as to complex media. In this chapter, stability issues related to PML are studied, and a unified analysis of various PML formulations using differential forms is included.

Chapter 8 addresses the issue of efficiently solving the forward and inverse problems for buried objects using FFT-based methods. The detection of buried objects usually involves loop antennas, and the forward problem involving the solution of loop antennas over a buried object is discussed in great detail. Moreover, recent advances in different inversion algorithms are also described.

Chapter 9 touches upon solving the penetrable problem at very low frequencies. The low-frequency problem encountered in Chapter 5 for metallic objects also occurs for dielectric and lossy material objects. This chapter describes a way to solve this problem so that the solution of integral equations remains stable all the way from zero frequency to microwave frequencies.

Chapter 10 describes an algorithm to solve three-dimensional waveguide structures using numerical mode matching, but using the finite difference method. The spectral Lanczos decomposition method is used to find the modes. An algorithm with  $O(N)$  memory complexity and  $O(N^{1.5})$  computational complexity is achieved.

Chapter 11 addresses the problem of solving the volume integral equation concurrently with the surface integral equation. This is particularly important when dealing with structures having metals as well as dielectric materials. The solutions are also accelerated with MLFMA as demonstrated in the chapter. Many practical illustrations of the use of this solution technique are given in this chapter.

Chapter 12 deals with solving axially symmetric, body-of-revolution (BOR) geometry using the finite element method (FEM). This reduces a three-dimensional problem to two dimensions, greatly enhancing the efficiency of the solution. Both

material-coated and metallic objects are considered. The chapter also shows the practical use of cylindrical PML for truncating the FEM mesh. Treatment of BOR geometry with appendages is also considered.

Chapter 13 reports on the hybridization in computational electromagnetics. Hybridization between FEM and the absorbing boundary condition (ABC) is discussed alongside the boundary integral equation (BIE), MLFMA, adaptive absorbing boundary condition (AABC), and shooting and bouncing ray (SBR). Hybridization between MOM and SBR is also considered. AABC is a promising method of hybridizing FEM with fast solvers in the future.

Chapter 14 presents different higher-order methods in computational electromagnetics. Higher-order methods for the surface integral equation as well as for FEM are considered. Also, the efficient coupling of higher-order methods to fast solvers such as MLFMA is discussed. In particular, the use of point-based MLFMA is illustrated. Moreover, a higher-order grid-robust method is also studied in this chapter.

Chapter 15 touches on the topic of asymptotic waveform evaluation (AWE) for broadband calculation in electromagnetics. Illustrations of this acceleration technique for broadband calculation are given for metallic antennas, wire antennas, dielectric scatterers, and microstrip antennas.

Chapter 16 details the analysis of microstrip structure on top of a layered medium. The derivation of the layered medium Green's function together with its numerical approximation by the complex images is discussed. The use of the fast frequency sweep method, adaptive integral method, and MLFMA to accelerate solution speed is studied. A higher-order method to improve solution accuracy is also demonstrated.

Chapter 17 reviews the steepest-descent FMM (SDFMM) to accelerate the solution speed of quasi-planar structures. For this class of structures, this method reduces both the computational and memory complexity of MLFMA from  $O(N \log N)$  to  $O(N)$ . Applications to scattering from random rough surfaces, quantum-well gratings, and microstrip antennas are demonstrated with this analysis method.

Chapter 18 elaborates on the plane-wave time-domain (PWTD) algorithm, which is an ingenious way of arriving at the time-domain equivalent of FMM and MLFMA. The integral equation is solved using the marching-on-in-time (MOT) method. Stability and accuracy issues are carefully analyzed in this chapter. Both the two-level and multilevel algorithms are presented and demonstrated with examples.

Chapter 19 further develops PWTD for large-scale and real-world applications. The use of PWTD with the magnetic field integral equation (MFIE), electric field integral equation (EFIE), and combined field integral equation (CFIE) is illustrated. Furthermore, scattering and error analysis from complex targets such as aircraft, almond shapes, and cone-spheres are considered.

Even though a large variety of topics is covered here, we do feel that there is still a myriad of problems in computational electromagnetics begging to be solved. Due to the complex nature of computational electromagnetics compared to circuit simulation, the robustness and stability of these algorithms are still issues to be addressed.

Another issue is the computational labor associated with these algorithms—more research needs to be done to enhance their speed. We hope, however, that the work at our Center marks a new beginning in the era of fast algorithms in computational electromagnetics.

During the MURI support, we have demonstrated our ability to solve problems involving 10 million unknowns using the supercomputing facilities of the University of Illinois. With continued support in this field, together with improvements in computer technology, we predict that a decade from now, solving a problem of this size will be routine for many applications.

*If only electromagnetic fields can talk, they will speak volumes!*

WENG CHO CHEW

*Urbana-Champaign, Illinois, June 2001*



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