ECE 604 Electromagnetic Field Theory

Spring 2020

Homework No. 5. Due Date: Feb 28, 2020.

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Read lecture notes 1-16, and ECE 350X notes on transmission line theory at:
https://engineering.purdue.edu/wcchew/ece350.html
Read H.A. Haus, Section 1.7 (EM Noise and Quantum Optical Measurements).

1. This problem is on energy storage in a dispersive medium.
(i) Assume a cold collisionless plasma medium as we have studied in the Drude-Lorentz-Sommerfeld model, show that the energy storage density in the polarization density part is given by

$$\frac{1}{4}\frac{q^2 N}{m\omega^2}E^2 \qquad (1)$$

(ii) Show that this formula is very different from if we assume that the energy density for the polarization density part is given by

$$\frac{\varepsilon_0}{4}\chi_e E^2 \qquad (2)$$

(iii) If we assume that the time average kinetic energy density is given by

$$\frac{1}{2}\frac{mv^2}{2}N \qquad (3)$$

Show that (1) and (3) are the same.

2. This problem is about magnetic dipole using an electric current loop.
(i) By using the formula for the vector potential given the electric current we have derived in class, derive the formula given by 2.10 (6) of Ramo et al (an ecopy of the this book is available if needed). Furthermore, by learning how to use the curl operator in spherical coordinates, derive the magnetic flux as given by equations (7), (8), and (9) in the same section of Ramo et al (use the handout on Some Useful Formulas).
(ii) Now go to Example 1.8d of the textbook where the scalar potential for an electric dipole has been derived. Find the electric field and the electric flux from this scalar potential. Discuss the similarity and difference between the magnetic and electric fluxes that you have derived. Explain why a current loop is also termed a magnetic dipole.

3. This problem shows that the telegrapher's equations can be derived by field theory in addition to lumped element model used in lecture notes.
In between a coax, the region is source free, and Maxwell's equations can be written as

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} = -\mu\frac{\partial\mathbf{H}}{\partial t}$$

$$\nabla\times\mathbf{H} = \frac{\partial\mathbf{D}}{\partial t} = \varepsilon\frac{\partial\mathbf{E}}{\partial t}$$

(i) In a coax, by symmetry, we can let $\mathbf{E} = \hat{\rho} E_\rho, \mathbf{H} = \hat{\phi} H_\phi$. By learning how to write the curl operator in cylindrical coordinates (see Some Useful Formulas handout), convert and simplify the above time varying Maxwell's equations so that it involves only a single space derivative in *z*. Assume that the fields are independent of ϕ.
(ii) From these equations, relate the electric field to voltage, and the magnetic field to current, and then derive the telegrapher's equations from Maxwell's equations.
(iii) From these equations, find the velocity of the wave in terms of the line capacitance and the line inductance.

4. This problem is about physical interpretation of the generalized reflection coefficient.
(i) By using $T_{ij} = 1 + R_{ij}$ and that $R_{ij} = -R_{ji}$, show that the generalized reflection coefficient can be written as

$$\tilde{R}_{12} = \frac{R_{12} + R_{23} e^{-2j\beta_{2z} t_2}}{1 - R_{23} R_{21} e^{-2j\beta_{2z} t_2}} = R_{12} + \frac{T_{12} R_{23} T_{21} e^{-2j\beta_{2z} t_2}}{1 - R_{23} R_{21} e^{-2j\beta_{2z} t_2}}$$

Where t_2 is the thickness of region 2.
(ii) Use the geometric series expansion that $1/(1-x) = 1 + x + x^2 + x^3 + \cdots$, show that the generalized reflection coefficient can be rewritten as

$$\tilde{R}_{12} = R_{12} + T_{12} R_{23} T_{21} e^{-2j\beta_{2z} t_2} + T_{12} R_{23}^2 R_{21} T_{21} e^{-4j\beta_{2z} t_2} + T_{12} R_{23}^3 R_{21}^2 T_{21} e^{-6j\beta_{2z} t_2} + \cdots$$

(iii) Give the physical meanings of each of the terms above, and the meaning of the phase delay term.