ECE 604, Lecture 15

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Contents

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1 Interesting Physical Phenomena–Contd.

1.1 Brewster Angle

Since most materials at optical frequencies have $\varepsilon_2 \neq \varepsilon_1$, but $\mu_2 \approx \mu_1$, the TM polarization for light behaves differently from TE polarization. For R^{TM}, it is possible that $R^{TM} = 0$ if

$$\varepsilon_2 \beta_{1z} = \varepsilon_1 \beta_{2z} \tag{1.1}$$

Squaring the above, making note that $\beta_{iz} = \sqrt{\beta_i^2 - \beta_x^2}$, one gets

$$\varepsilon_2{}^2({\beta_1}^2 - {\beta_x}^2) = \varepsilon_1{}^2({\beta_2}^2 - {\beta_x}^2) \tag{1.2}$$

Solving the above, assuming $\mu_1 = \mu_2 = \mu$, gives

$$\beta_x = \omega\sqrt{\mu}\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1 + \varepsilon_2}} = \beta_1 \sin\theta_1 = \beta_2 \sin\theta_2 \tag{1.3}$$

The latter two equations come from phase matching at the interface. Therefore,

$$\sin\theta_1 = \sqrt{\frac{\varepsilon_2}{\varepsilon_1 + \varepsilon_2}}, \quad \sin\theta_2 = \sqrt{\frac{\varepsilon_1}{\varepsilon_1 + \varepsilon_2}} \tag{1.4}$$

or that

$$\sin^2\theta_1 + \sin^2\theta_2 = 1, \tag{1.5}$$

Then, assuming that θ_1 and θ_2 are less than $\pi/2$,

$$\sin\theta_2 = \cos\theta_1 \tag{1.6}$$

or that

$$\theta_1 + \theta_2 = \pi/2 \tag{1.7}$$

This is used to explain why at Brewster angle, no light is reflected back to Region 1. Figure 1 shows that the induced polarization dipoles in Region 2 always have their axes aligned in the direction of reflected wave. A dipole does not radiate along its axis, which can be verified heuristically by field sketch and looking at the Poynting vector. Therefore, these induced dipoles in Region 2 do not radiate in the direction of the reflected wave.

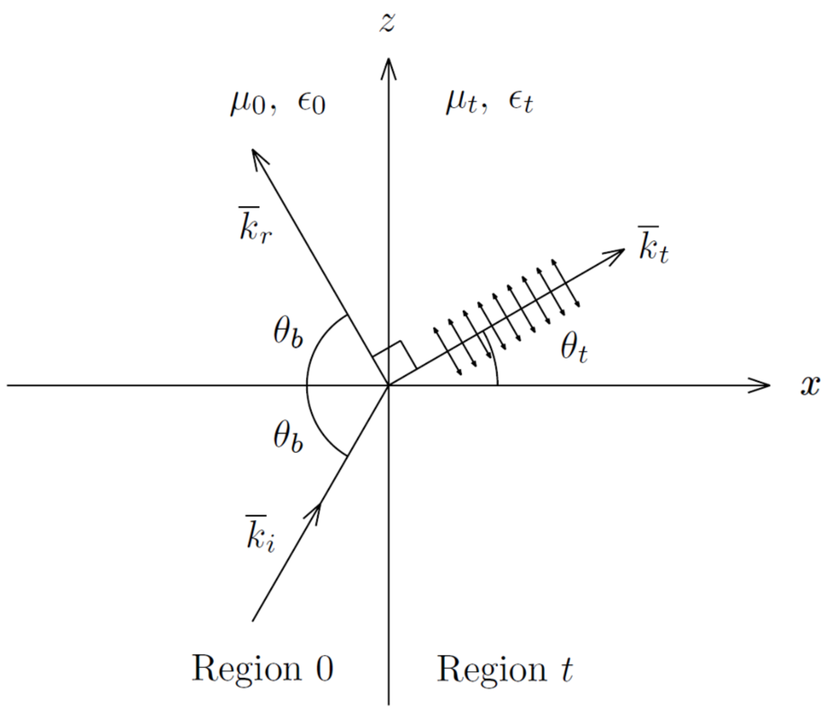

Figure 1: Courtesy of J.A. Kong, EM Wave Theory.

Because of the Brewster angle effect, and that $\varepsilon_2 \neq \varepsilon_1$, $|R^{TM}| \leq |R^{TE}|$ as shown in Figure 2. Then when a randomly polarized light is incident on a surface, the polarization where the electric field is parallel to the surface (TE polarization) is reflected more than the polarization where the magnetic field is parallel to the surface (TM polarization). This phenomenon is used to design sun glasses to reduce road glare for drivers. For light reflected off a road surface, they are predominantly horizontally polarized with respect to the surface of the road. When sun glasses are made with vertical polarizers, they will filter out and mitigate the reflected rays from the road surface to reduce road glare.

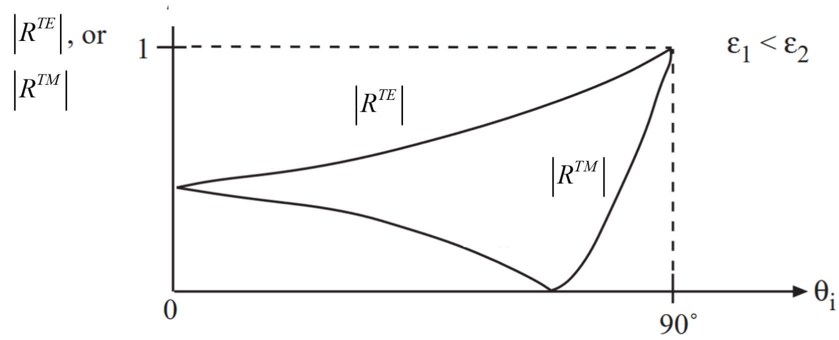

Figure 2:

1.2 Surface Plasmon Polariton

Surface plasmon polariton occurs for the same mathematical reason for the Brewster angle effect but the physical mechanism is quite different. The reflection coefficient R^{TM} can become infinite if $\varepsilon_2 < 0$, as in a plasma medium.

In this case, the criterion is

$$-\varepsilon_2\beta_{1z} = \varepsilon_1\beta_{2z} \tag{1.8}$$

When the above is satisfied, R^{TM} becomes infinite. This implies that a reflected wave exists when there is no incident wave. In other words, a reflected wave exists while there is no incident wave. This is often encountered in a resonance system like an LC tank circuit. Current flows in the tank circuit despite the absence of an exciting voltage. Hence, there is a plasmonic resonance or guided mode existing at the interface without the presence of an incident wave.

Solving (1.8) after squaring it, as in the Brewster angle case, yields

$$\beta_x = \omega\sqrt{\mu}\sqrt{\frac{\varepsilon_1\varepsilon_2}{\varepsilon_1+\varepsilon_2}} \tag{1.9}$$

This is the same equation for the Brewster angle except now that ε_2 is negative. Even if $\varepsilon_2 < 0$, but $\varepsilon_1 + \varepsilon_2 < 0$ is still possible so that the expression under the square root sign (1.9) is positive. Thus, β_x can be pure real. The corresponding β_{1z} and β_{2z} in (1.8) can be pure imaginary, and (1.8) can still be satisfied.

This corresponds to a guided wave propagating in the x direction. When this happens,

$$\beta_{1z} = \sqrt{{\beta_1}^2 - {\beta_x}^2} = \omega\sqrt{\mu}\left[\varepsilon_1\left(1 - \frac{\varepsilon_2}{\varepsilon_1+\varepsilon_2}\right)\right]^{1/2} \tag{1.10}$$

Since $\varepsilon_2 < 0$, $\varepsilon_2/(\varepsilon_1+\varepsilon_2) > 1$, then β_{1z} becomes pure imaginary. Moreover, $\beta_{2z} = \sqrt{{\beta_2}^2 - {\beta_x}^2}$ and ${\beta_2}^2 < 0$ making β_{2z} becomes even a larger imaginary number. This corresponds to a trapped wave at the interface. The wave decays exponentially in both directions away from the interface and they are evanescent waves. This mode is shown in Figure 3, and is the only case in electromagnetics where a single interface can guide a surface wave, while such phenomena abound for elastic waves.

When one operates close to the resonance of the mode so that the denominator in (1.9) is almost zero, then β_x can be very large. The wavelength becomes very short in this case, and since $\beta_{iz} = \sqrt{\beta_i^2 - \beta_x^2}$, then β_{1z} and β_{2z} become even larger imaginary numbers. Hence, the mode becomes tightly confined to the surface, making the confinement of the mode very tight. It portends use in tightly packed optical components, and has caused some excitement in the optics community.

https://en.wikipedia.org/wiki/Surface_plasmon

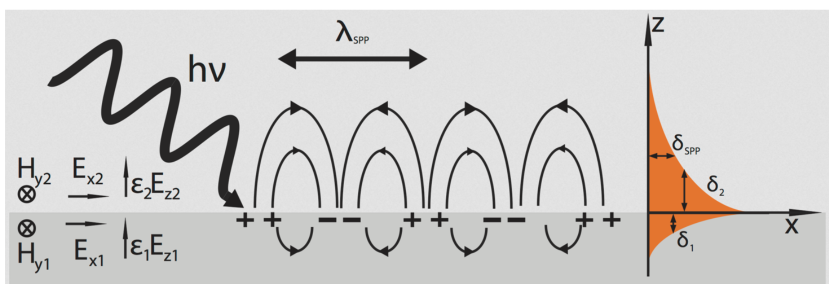

Figure 3: Courtesy of Wikipedia.

2 Homomorphism of Uniform Plane Waves and Transmission Lines Equations

It turns out that the plane waves through layered medium can be mapped into the multi-section transmission line problem due to mathematical homomorphism between the two problems. Hence, we can kill two birds with one stone: apply all the transmission line techniques and equations that we have learnt to solve for the solutions of waves through layered medium problems.

For uniform plane waves, since they are proportional to $\exp(-j\boldsymbol{\beta}\cdot\mathbf{r})$, we know that with $\nabla \to -j\boldsymbol{\beta}$, Maxwell's equations becomes

$$\boldsymbol{\beta} \times \mathbf{E} = \omega\mu\mathbf{H} \tag{2.1}$$

$$\boldsymbol{\beta} \times \mathbf{H} = -\omega\varepsilon\mathbf{E} \tag{2.2}$$

for a general isotropic homogeneous medium. We will specialize these equations for different polarizations.

2.1 TE or TE$_z$ Waves

For this, one assumes a TE wave traveling in z direction with electric field polarized in the y direction, or $\mathbf{E} = \hat{y}E_y$, $\mathbf{H} = \hat{x}H_x + \hat{z}H_z$, then we have from (2.1)

$$\beta_z E_y = -\omega\mu H_x \tag{2.3}$$
$$\beta_x E_y = \omega\mu H_z \tag{2.4}$$

From (2.2), we have

$$\beta_z H_x - \beta_x H_z = -\omega\varepsilon E_y \tag{2.5}$$

Then, expressing H_z in terms of E_y from (2.4), we can show from (2.5) that

$$\begin{aligned}\beta_z H_x &= -\omega\varepsilon E_y + \beta_x H_x = -\omega\varepsilon E_y + \frac{\beta_x^2}{\omega\mu}E_y \\ &= -\omega\varepsilon(1-\beta_x^2/\beta^2)E_y = -\omega\varepsilon\cos^2\theta E_y\end{aligned} \tag{2.6}$$

where $\beta_x = \beta\sin\theta$ has been used.

Eqns. (2.3) and (2.6) can be written to look like the telegrapher's equation by letting $-j\beta_z \to d/dz$ to get

$$\frac{d}{dz}E_y = j\omega\mu H_x \tag{2.7}$$
$$\frac{d}{dz}H_x = j\omega\varepsilon\cos^2\theta E_y \tag{2.8}$$

If we let $E_y \to V$, $H_x \to -I$, $\mu \to L$, $\varepsilon\cos^2\theta \to C$, the above is exactly analogous to the telegrapher's equation. The equivalent characteristic impedance of these equations above is then

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\varepsilon}}\frac{1}{\cos\theta} = \sqrt{\frac{\mu}{\varepsilon}}\frac{\beta}{\beta_z} = \frac{\omega\mu}{\beta_z} \tag{2.9}$$

The above is the wave impedance for a propagating plane wave with propagation direction or the $\boldsymbol{\beta}$ inclined with an angle θ respect to the z axis. When $\theta = 0$, the wave impedance becomes the intrinsic impedance of space.

A two region, single-interface reflection problem can then be mathematically mapped to a single-junction two-transmission-line problem. The equivalent characteristic impedances of these two regions are then

$$Z_{01} = \frac{\omega\mu_1}{\beta_{1z}}, \quad Z_{02} = \frac{\omega\mu_2}{\beta_{2z}} \tag{2.10}$$

We can use the above to find Γ_{12} as given by

$$\Gamma_{12} = \frac{Z_{02} - Z_{01}}{Z_{02} + Z_{01}} = \frac{(\mu_2/\beta_{2z}) - (\mu_1/\beta_{1z})}{(\mu_2/\beta_{2z}) + (\mu_1/\beta_{1z})} \tag{2.11}$$

The above is the same as the Fresnel reflection coefficient found earlier for TE waves or R^{TE} after some simple re-arrangement.

Assuming that we have a single junction transmission line, one can define a transmission coefficient given by

$$T_{12} = 1 + \Gamma_{12} = \frac{2Z_{02}}{Z_{02} + Z_{01}} = \frac{2(\mu_2/\beta_{2z})}{(\mu_2/\beta_{2z}) + (\mu_1/\beta_{1z})} \tag{2.12}$$

The above is similar to the continuity of the voltage across the junction, which is the same as the continuity of the tangential electric field across the interface. It is also the same as the Fresnel transmission coefficient T^{TE}.

2.2 TM or TM$_z$ Waves

For the TM polarization, by invoking duality principle, the corresponding equations are, from (2.7) and (2.8),

$$\frac{d}{dz} H_y = -j\omega\varepsilon E_x \tag{2.13}$$

$$\frac{d}{dz} E_x = -j\omega\mu \cos^2\theta H_y \tag{2.14}$$

Just for consistency of units, since electric field is in V m^{-1}, and magnetic field is in A m^{-1} we may chose the following map to convert the above into the telegrapher's equations, viz;

$$E_y \to V, \quad H_y \to I, \quad \mu\cos^2\theta \to L, \quad \varepsilon \to C \tag{2.15}$$

Then, the equivalent characteristic impedance is now

$$Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{\mu}{\varepsilon}}\cos\theta = \sqrt{\frac{\mu}{\varepsilon}}\frac{\beta_z}{\beta} = \frac{\beta_z}{\omega\varepsilon} \tag{2.16}$$

The above is also termed the wave impedance of a TM propagating wave making an inclined angle θ with respect to the z axis. Notice again that this wave impedance becomes the intrinsic impedance of space when $\theta = 0$.

Now, using the reflection coefficient for a single-junction transmission line, and the appropriate characteristic impedances for the two lines as given in (2.16), we arrive at

$$\Gamma_{12} = \frac{(\beta_{2z}/\varepsilon_2) - (\beta_{1z}/\varepsilon_1)}{(\beta_{2z}/\varepsilon_2) + (\beta_{1z}/\varepsilon_1)} \tag{2.17}$$

Notice that (2.17) has a sign difference from the definition of R^{TM} derived earlier in the last lecture. The reason is that R^{TM} is for the reflection coefficient of magnetic field while Γ_{12} above is for the reflection coefficient of the voltage or the electric field. This difference is also seen in the definition for transmission coefficients. A voltage transmission coefficient can be defined to be

$$T_{12} = 1 + \Gamma_{12} = \frac{2(\beta_{2z}/\varepsilon_2)}{(\beta_{2z}/\varepsilon_2) + (\beta_{1z}/\varepsilon_1)} \tag{2.18}$$

But this will be the transmission coefficient for the voltage, which is not the same as T^{TM} which is the transmission coefficient for the magnetic field or the current. Different textbooks may define different transmission coefficients for this polarization.