ook like the contour lines of a topographic map and, in fact, measure potential
f a unit charge relative to a selected zero-potential point just as contours meas-
ntial energy relative to some reference altitude, often sea level. It should be
mind that these lines are actually traces in the x–y plane of three-dimensional
cal equipotential surfaces.
ill discuss the boundary conditions on conductors in some detail in Sec. 1.14.
point it is sufficient to say that the electric fields inside of a metallic conductor
considered to be zero in electrostatic systems. Therefore, (1) shows that the
tor is an equipotential region.

Example 1.8a

OTENTIALS AROUND A LINE CHARGE AND BETWEEN COAXIAL CYLINDERS

example of the relations between potential and electric field, consider first the
m of the line charge used as an example in Sec. 1.4, with electric field given by
4(3). By (1) we integrate this from some radius r_0 chosen as the reference of zero
ial to radius r:

$$\Phi = -\int_{r_0}^{r} E_r \, dr = -\int_{r_0}^{r} \frac{q_l \, dr}{2\pi\varepsilon r} = -\frac{q_l}{2\pi\varepsilon}\ln\left(\frac{r}{r_0}\right) \tag{7}$$

is expression for potential about a line charge may be written

$$\Phi = -\frac{q_l}{2\pi\varepsilon}\ln r + C \tag{8}$$

that it is not desirable to select infinity as the reference of zero potential for the
charge, for then by (7) the potential at any finite point would be infinite. As in (6)
onstant is added to shift the position of the zero potential.
a similar manner, the potential difference between the coaxial cylinders of Fig. 1.4b
be found:

$$\Phi_a - \Phi_b = -\int_b^a \frac{q_l \, dr}{2\pi\varepsilon r} = \frac{q_l}{2\pi\varepsilon}\ln\left(\frac{b}{a}\right) \tag{9}$$

Example 1.8b

POTENTIAL OUTSIDE A SPHERICALLY SYMMETRIC CHARGE

saw in Eq. 1.4(4) that the flux density outside a spherically symmetric charge Q is
$= Q/4\pi r^2$. Using $\mathbf{E} = \mathbf{D}/\varepsilon_0$ and taking the reference potential to be zero at in-
ty, we see that the potential outside the charge Q is the negative of the integral of

$\hat{\mathbf{r}}E_r \cdot \hat{\mathbf{r}}\, dr$ from infinity to radius r:

$$\Phi(r) = -\int_\infty^r \frac{Q \, dr_1}{4\pi\varepsilon_0 r_1^2} = \frac{Q}{4\pi\varepsilon_0 r} \tag{10}$$

Example 1.8c

POTENTIAL OF A UNIFORM DISTRIBUTION OF CHARGE HAVING SPHERICAL SYMMETRY

Consider a volume of charge density ρ that extends from $r = 0$ to $r = a$. Taking $\Phi = 0$ at $r = \infty$, the potential outside a is given by (10) with $Q = \frac{4}{3}\pi\varepsilon a^3\rho$, so

$$\Phi(r) = \frac{a^3\rho}{3\varepsilon_0 r} \qquad r \ge a \tag{11}$$

In particular, at $r = a$

$$\Phi(a) = \frac{a^2\rho}{3\varepsilon_0} \tag{12}$$

Then to get the potential at a point where $r \le a$ we must add to (12) the integral of the electric field from a to r. The electric field is given as $E_r = \rho r/3\varepsilon_0$ (Ex. 1.7) and the integral is

$$\Phi(r) - \Phi(a) = -\int_a^r \frac{\rho r_1}{3\varepsilon_0}\, dr_1 = \frac{\rho}{6\varepsilon_0}(a^2 - r^2) \tag{13}$$

So the potential at a radius r inside the charge region is

$$\Phi(r) = \frac{\rho}{6\varepsilon_0}(3a^2 - r^2) \qquad r \le a \tag{14}$$

Example 1.8d

ELECTRIC DIPOLE

A particularly important set of charges is that of two closely spaced point charges of opposite sign, called an *electric dipole*.

Assume two charges, having opposite signs to be spaced by a distance 2δ as shown in Fig. 1.8d. The potential at some point a distance r from the origin displaced by an angle θ from the line passing from the negative to positive charge can be written as the sum of the potentials of the individual charges:

$$\Phi = \frac{q}{4\pi\varepsilon}\left(\frac{1}{r_+} - \frac{1}{r_-}\right) \tag{15}$$

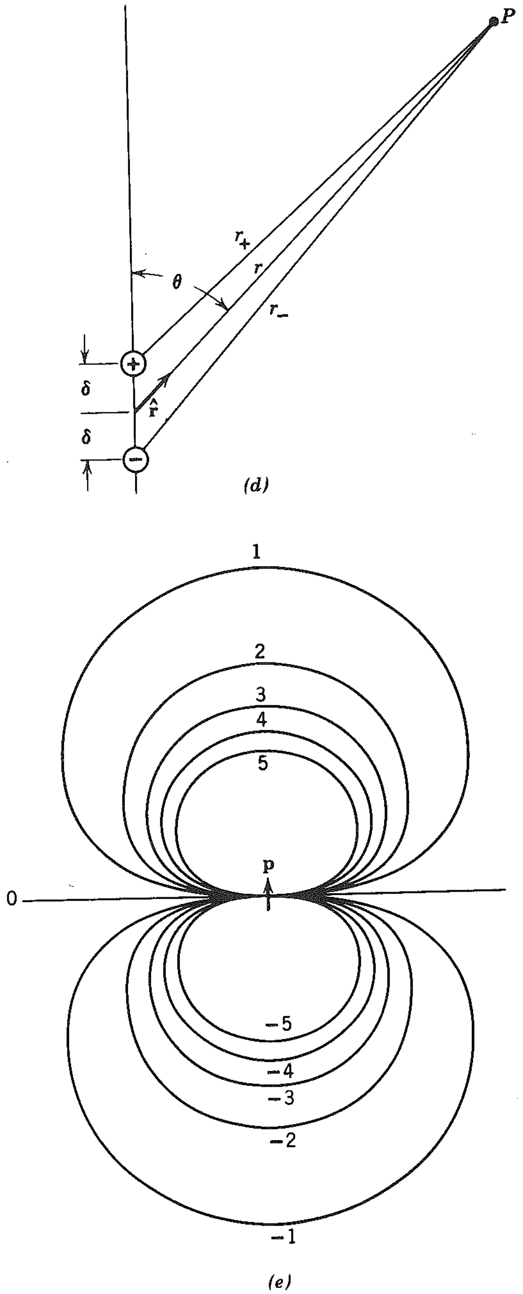

FIG. 1.8 (*d*) Electric dipole. (*e*) Equipotentials of electric dipole.

Using the law of cosines, we have

$$r_+^2 = r^2 + \delta^2 - 2r\delta \cos \theta$$

and similarly for r_- with opposite sign of $\cos \theta$. For $\delta << r$,

$$r_+ \approx r - \delta \cos \theta$$

$$r_- \approx r + \delta \cos \theta$$

Substituting in (15) and again using the restriction $\delta << r$, one obtains

$$\Phi \approx \frac{2\delta q \cos \theta}{4\pi\varepsilon r^2} \tag{16}$$

We define an *electric dipole moment* **p** of a pair of equal charges as the product of the charge and the separation. The direction of vector **p** is from the negative charge to the positive one. Thus (16) may be written as

$$\Phi \approx \frac{\mathbf{p} \cdot \hat{\mathbf{r}}}{4\pi\varepsilon r^2} \tag{17}$$

where $\hat{\mathbf{r}}$ is a unit vector directed outward toward the point of observation. It is seen that the dipole potential decreases as $1/r^2$ with increasing distance from the origin, whereas the potential of the single charge decreases only as the first power. The increased rate of decay of the potential is to be expected as a result of the partial cancellation of the potentials of opposite sign. Equipotential lines are shown plotted on a plane passing through the dipole in Fig. 1.8*e*. The values shown are relative. The equipotentials are surfaces of revolution generated by rotating the lines in Fig. 1.8*e* about the dipole axis.

1.9 Capacitance

The capacitance between two electrodes, widely used in circuit calculations, is a measure of the charge Q on each electrode per volt of potential difference $\Phi_a - \Phi_b$ between them:

$$C = \frac{Q}{\Phi_a - \Phi_b} \tag{1}$$

For capacitance systems having two electrodes, the excess negative charge on one equals the deficiency of negative charge on the other.

Consider first the parallel plates in Fig. 1.9*a*. The separation is small compared with the width. The result is that charges accumulate mainly on the most closely separated surfaces. We shall idealize the structure as a portion of infinitely wide plates (Fig. 1.9*b*) and thereby neglect the *fringing fields* that do not pass straight from one plate to the other. Such idealizations of real situations are extremely useful, but their limitations must always be remembered. The flux density **D** is found from Gauss's law to equal