Lecture 32

Shielding, Image Theory

The physics of electromagnetic shielding and electromagnetic image theory (also called image theorem) go hand in hand. They work by the moving of charges around so as to cancel the impinging fields. By understanding simple cases of shielding and image theory, we can gain enough insight to solve some real-world problems. For instance, the art of shielding is very important in the field of electromagnetic compatibility (EMC) and electromagnetic interference (EMI). In the modern age where we have more electronic components working side by side in a very compact environment, EMC/EMI become an increasingly challenging issue. These problems have to be solved using heuristics with a high dosage of physical insight.

32.1 Shielding

We can understand shielding by understanding how electric charges move around in a conductive medium. They move around to shield out the electric field, or cancel the impinging field inside the conductor. There are two cases to consider: the static case and the dynamic case. The physical arguments needed to understand these two cases are quite different. Moreover, since there are no magnetic charges around, the shielding of magnetic field is quite different from the shielding of electric field, as shall be seen below.

32.1.1 A Note on Electrostatic Shielding

We begin with the simple case of electrostatic shielding. For electrostatic problems, a conductive medium suffices to produce surface charges that shield out the electric field from the conductive medium. If the electric field is not zero, then since $\mathbf{J} = \sigma\mathbf{E}$, the electric current inside the conductor will keep flowing. The current will produce charges on the surface of the conductor to cancel the imping field, until inside the conductive medium $\mathbf{E} = 0$. In this case, electric current ceases to flow in the conductor.

In other words, when the field reaches the quiescent state, the charges redistribute themselves so as to shield out the electric field, and that the total internal electric field, $\mathbf{E} = 0$ at equilibrium. And from Faraday's law that tangential $\mathbf{E}$ field is continuous, then $\hat{n} \times \mathbf{E} = 0$ on the conductor surface since $\hat{n} \times \mathbf{E} = 0$ inside the conductor. Figure 32.1 shows the static electric field, in the quiescent state, between two conductors (even though they are not PECs), and the electric field has to be normal to the conductor surfaces. Moreover, since $\mathbf{E} = 0$ inside the conductor, $\nabla\Phi = 0$ implying that the potential is a constant inside a conductor at equilibrium.

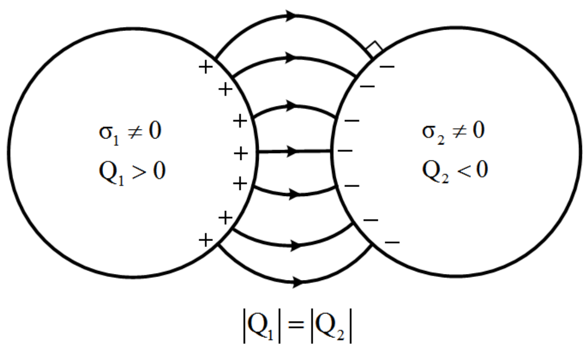

Figure 32.1: The objects can just be conductors, and in the quiescent state (static state), the tangential electric field will be zero on their surfaces. Also, $\mathbf{E} = 0$ inside the conductor, or $\nabla\Phi = 0$, or Φ is a constant.

32.1.2 Relaxation Time

The time it takes for the charges to move around until they reach their quiescent distribution such that $\mathbf{E}(t) = 0$ is called the relaxation time. It is very much similar to the RC time constant of an RC circuit consisting of a resistor in series with a capacitor (see Figure 32.2). It can be proven that this relaxation time is related to ε/σ, but the proof is beyond the scope of this course and it is worthwhile to note that this constant has the same unit as the RC time constant of an RC circuit where a charged capacitor relaxes as $\exp(-t/\tau)$ where the relaxation time $\tau = RC$. Note that when $\sigma \to \infty$, the relaxation time is zero. In other words, in a perfect conductor or a superconductor, the charges reorient themselves instantaneously if the external field is time-varying so that $\mathbf{E}(t) = 0$ always.

Electrostatic shielding or low-frequency shielding is important at low frequencies. The Faraday cage or Faraday shield is an important application of such a shielding (see Figure 32.3) [188].

However, if the conductor charges are induced by an external electric field that is time varying, then the charges have to constantly redistribute/re-orient themselves to try to shield out the incident time-varying electric field. Currents have to be constantly flowing around the conductor. Then the electric field cannot be zero inside the conductors as shown in Figure 32.4. In other words, an object with finite conductivity cannot shield out completely a time-varying electric field. It can be shown that the depth of penetration of the field into

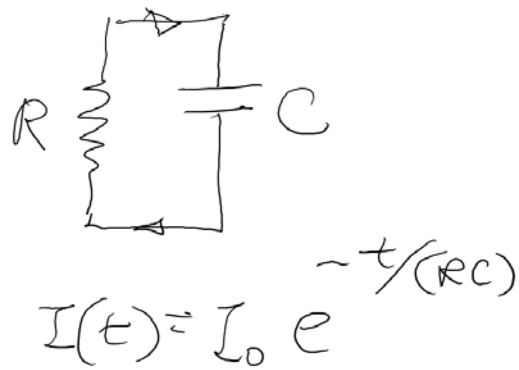

Figure 32.2: The relaxation (or disappearance of accumulated charges) in a conductive object is similar to the relaxation of charges from a charged capacitance in an RC circuit as shown.

the conductive object is about a skin depth $\delta = \sqrt{2/(\omega\mu\sigma)}$. Or the lower the frequency ω or the conductivity σ, the large the penetration depth.

For a perfect electric conductor (PEC) , $\mathbf{E} = 0$ inside with the following argument: Because if $\mathbf{E} \neq 0$, then $\mathbf{J} = \sigma\mathbf{E}$ where $\sigma \to \infty$. Let us assume an infinitesimally small time-varying electric field in the PEC to begin with. It will induce an infinitely large electric current, and hence an infinitely large time-varying magnetic field. An infinite time-varying magnetic field in turn yields an infinite electric field that will drive an electric current, and these fields and current will be infinitely large. This is an unstable chain of events if it is true. Moreover, it will generate infinite energy in the system, which is not physical. Hence, the only possibility for a stable solution is for the time-varying electromagnetic fields to be zero inside a PEC.

Thus, for the PEC, the charges can re-orient themselves instantaneously on the surface when the inducing electric fields from outside are time varying. In other words, the relaxation time ε/σ is zero. As a consequence, the time-varying electric field $\mathbf{E}$ is always zero inside PEC, and therefore, $\hat{n} \times \mathbf{E} = 0$ on the surface of the PEC, even for time-varying fields.

32.2 Image Theory

The image theory here in electromagnetics is quite different from that in optics. As mentioned before, when the frequency of the fields is high, the waves associated with the fields can be described by rays. Therefore ray optics can be used to solve many high-frequency problems. We can use ray optics to understand how an image is generated in a mirror. But the image theory in electromagnetics is quite different from that in ray optics.

Image theory can be used to derived closed form solutions to boundary value problems when the geometry is simple and has a lot of symmetry. These closed form solutions in turn

Figure 32.3: Faraday cage demonstration on volunteers in the Palais de la Découverte in Paris (courtesy of Wikipedia). When the cage is grounded, charges will surge from the ground to the cage surface so as to make the field inside the cage zero, or the potential is constant. Therefore, a grounded Faraday cage effectively shields the external fields from entering the cage.

offer physical insight into the problems. This theory or method is also discussed in many textbooks [1, 43, 54, 65, 79, 181, 189].

32.2.1 Electric Charges and Electric Dipoles

Image theory for a flat conductor surface or a half-space is quite easy to derive. To see that, we can start with electro-static theory of putting a positive charge above a flat plane. As mentioned before, for electrostatics, the plane or half-space does not have to be a perfect conductor, but only a conductor (or a metal). From the previous Section 32.1.1, the tangential static electric field on the surface of the conductor has to be zero.

By the principle of linear superposition, the tangential static electric field can be canceled by putting an image charge of opposite sign at the mirror location of the original charge. This is shown in Figure 32.5. Now we can mentally add the total field due to these two charges. When the total static electric field due to the original charge and image charge is sketched, it will look like that in Figure 32.6. It is seen that the static electric field satisfies the boundary condition that $\hat{n} \times \mathbf{E} = 0$ at the conductor interface due to symmetry.

An electric dipole is made from a positive charge placed in close proximity to a negative

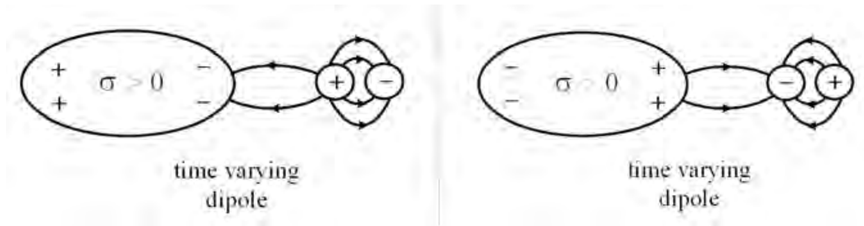

Figure 32.4: If the source that induces the charges on the conductor is time varying, the current in the conductor is always nonzero so that the charges can move around to respond to the external time-varying charges. The two figures above show the orientation of the charges for two snap-shot in time. In other words, a time-varying field can penetrate the conductor to approximately within a skin-depth $\delta = \sqrt{2/(\omega\mu\sigma)}$.

charge. Using that an electric charge reflects to an electric charge of opposite polarity above a conductor, one can easily see that a static horizontal electric dipole reflects to a static horizontal electric dipole of opposite polarity. By the same token, a static vertical electric dipole reflects to static vertical electric dipole of the same polarity as shown in Figure 32.7.

If this electric dipole is a Hertzian dipole whose field is time-varying, then one needs a PEC half-space to shield out the electric field. Also, the image charges will follow the original dipole charges instantaneously. Then the image theory for static electric dipoles over a half-space still holds true if the dipoles now become Hertzian dipoles, but the surface will have to be a PEC surface so that the fields can be shielded out instantaneously.

32.2.2 Magnetic Charges and Magnetic Dipoles

A static magnetic field can penetrate a conductive medium. This is apparent from our experience when we play with a bar magnet over a copper sheet: the magnetic field from the magnet can still be experienced by iron filings put on the other side of the copper sheet.

However, this is not the case for a time-varying magnetic field. Inside a conductive medium, a time-varying magnetic field will produce a time-varying electric field, which in turn produces the conduction current via $\mathbf{J} = \sigma\mathbf{E}$. This is termed eddy current, which by Lenz's law, repels the magnetic field from the conductive medium.[1]

Now, consider a static magnetic field penetrating into a perfect electric conductor, an minute amount of time variation will produce an electric field, which in turn produces an infinitely large eddy current. So the stable state for a static magnetic field inside a PEC is for it to be expelled from the perfect electric conductor. This in fact is what we observe when a magnetic field is brought near a superconductor. Therefore, for the static magnetic field, where $\mathbf{B} = 0$ inside the PEC, then $\hat{n} \cdot \mathbf{B} = 0$ on the PEC surface (see Figure 32.8).

[1]The repulsive force occurs by virtue of energy conservation. Since "work done" is needed to set the eddy current in motion in the conductor, or to impart kinetic energy to the electrons forming the eddy current, a repulsive force is felt in Lenz's law so that work is done in pushing the magnetic field into the conductive medium.

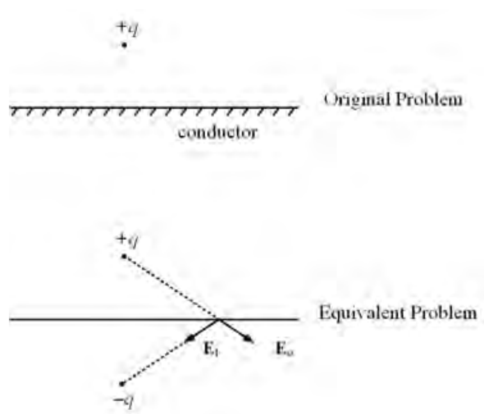

Figure 32.5: The use of image theory to solve the BVP of a point charge on top of a conductor. The boundary condition is that $\hat{n} \times \mathbf{E} = 0$ on the conductor surface. By placing a negative charge with respect to the original charge, by the principle of linear superposition, both of them produce a total field with no tangential component at the interface.

Now, assuming a magnetic monopole exists, it will reflect to itself on a PEC surface so that $\hat{n} \cdot \mathbf{B} = 0$ as shown in Figure 32.8. Therefore, a magnetic charge reflects to a charge of similar polarity on the PEC surface.

By extrapolating this to magnetic dipoles, they will reflect themselves to the magnetic dipoles as shown in Figure 32.9. A horizontal magnetic dipole reflects to a horizontal magnetic dipole of the same polarity, and a vertical magnetic dipole reflects to a vertical magnetic dipole of opposite polarity. Hence, a dipolar bar magnet can be levitated by a superconductor when this magnet is placed closed to it. This is also known as the Meissner effect [190], which is shown in Figure 32.10.

A time-varying magnetic dipole can be made from a electric current loop. Over a PEC, a time-varying magnetic dipole will reflect the same way as a static magnetic dipole as shown in Figure 32.9.

32.2.3 Perfect Magnetic Conductor (PMC) Surfaces

Magnetic conductor does not come naturally in this world since there are no free-moving magnetic charges around. Magnetic monopoles are yet to be discovered. On a PMC surface, by duality, $\hat{n} \times \mathbf{H} = 0$. At low frequency, it can be mimicked by a high μ material. One can see that for magnetostatics, at the interface of a high μ material and air, the magnetic flux is approximately normal to the surface, resembling a PMC surface.

High μ materials are hard to find at higher frequencies. Since $\hat{n} \times \mathbf{H} = 0$ on such a surface, no electric current can flow on such a surface. Hence, a PMC can be mimicked by a surface where no surface electric current can flow. This has been achieved in microwave engineering with a mushroom surface as shown in Figure 32.11 [192]. The mushroom structure

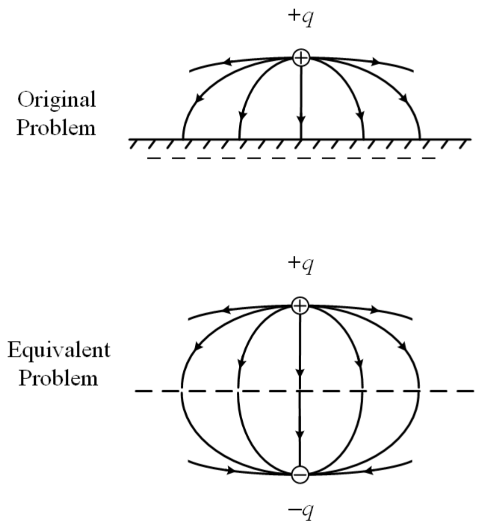

Figure 32.6: By image theory, the total electric of the original problem and the equivalent problem when we add the total electric field due to the original charge and the image charge.

consisting of a wire and an end-cap, can be thought of as forming an LC tank circuit. Close to the resonance frequency of this tank circuit, the surface of mushroom structures essentially becomes open circuits with no or little current flowing on the surface, or $\mathbf{J}_s \cong 0$. In other words, $\hat{n} \times \mathbf{H} \cong 0$. This resembles a PMC, because with no surface electric current on this surface, the tangential magnetic field is small, the hallmark of a good magnetic conductor, by the duality principle.

Mathematically, a surface that is dual to the PEC surface is the perfect magnetic conductor (PMC) surface. The magnetic dipole is also dual to the electric dipole. Thus, over a PMC surface, these electric and magnetic dipoles will reflect differently as shown in Figure 32.12. One can go through Gedanken experiments and verify that the reflection rules are as shown in the figure.

32.2.4 Multiple Images

For the geometry shown in Figure 32.13, one can start with electrostatic theory, and convince oneself that $\hat{n} \times \mathbf{E} = 0$ on the metal surface with the placement of charges as shown. For conducting media, the charges will relax to the quiescent distribution after the relaxation time. For PEC surfaces, one can extend these cases to time-varying dipoles because the charges in the PEC medium can re-orient instantaneously (i.e. with zero relaxation time) to shield out or expel the $\mathbf{E}$ and $\mathbf{H}$ fields. Again, one can repeat the above exercise for magnetic charges, magnetic dipoles, and PMC surfaces.

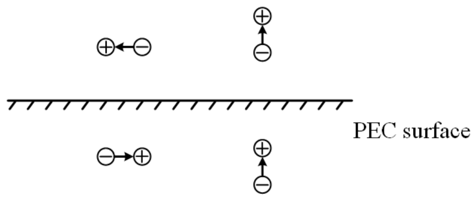

Figure 32.7: By image theory, on a conductor surface, a horizontal static dipole reflects to one of opposite polarity, while a static vertical dipole reflects to one of the same polarity. If the dipoles are time-varying, then a PEC will have a same reflection rule.

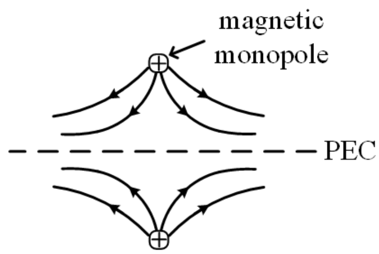

Figure 32.8: On a PEC surface, the requisite boundary condition is $\hat{n} \cdot \mathbf{B} = 0$. Hence, a magnetic monopole on top of a PEC surface will have magnetic field distributed as shown. By image theory, such a field distribution can be obtained by adding a magnetic monopole of the same polarity at its image point.

32.2.5 Some Special Cases—Spheres, Cylinders, and Dielectric Interfaces

One curious case is for a static charge placed near a conductive sphere (or cylinder) as shown in Figure 32.14.[2] A charge of $+Q$ reflects to a charge of $-Q_I$ inside the sphere. For electrostatics, the sphere (or cylinder) need only be a conductor. However, this cannot be generalized to electrodynamics or a time-varying problem, because of the retardation effect: A time-varying dipole or charge will be felt at different points asymmetrically on the surface of the sphere from the original and image charges. Exact cancelation of the tangential electric field on the surface of the sphere or cylinder cannot occur for time-varying field.

When a static charge is placed over a dielectric interface, image theory can be used to find the closed form solution. This solution can be derived using Fourier transform technique which we shall learn later [35]. It can also be extended to multiple interfaces. But image theory cannot be used for the electrodynamic case due to the different speed of light in different media, giving rise to different retardation effects.

[2]This is worked out in p. 48 and p. 49, Ramo et al [31].

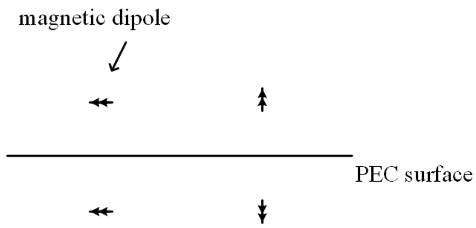

Figure 32.9: Using the rule of how magnetic monopole reflects itself on a PEC surface, the reflection rules for magnetic dipoles can be ascertained. Magnetic dipoles are often denoted by double arrows.

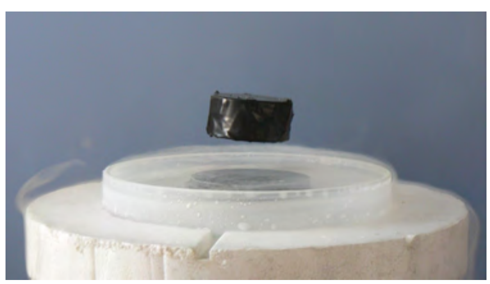

Figure 32.10: On a PEC (superconducting) surface, a vertical magnetic dipole (formed by a small permanent bar magnet here) reflects to one of opposite polarity. Hence, the magnetic dipoles repel each other displaying the Meissner effect. The magnet, because of the repulsive force from its image, levitates above the superconductor (courtesy of Wikipedia [191]).

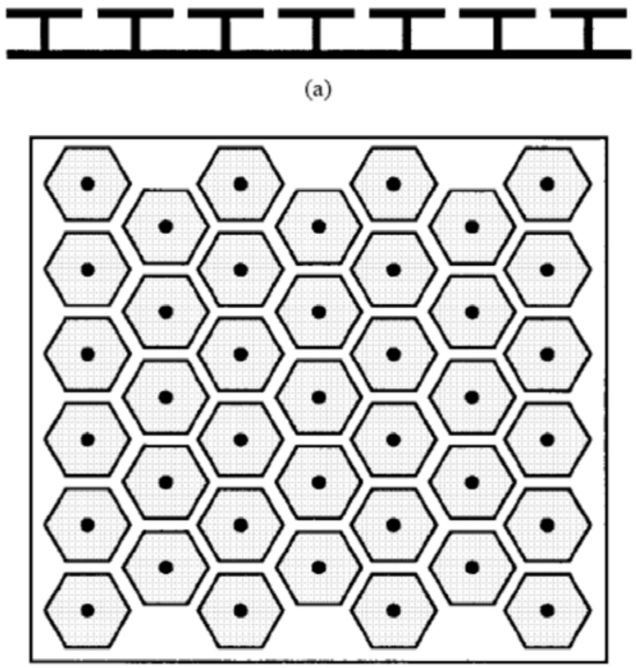

Figure 32.11: A mushroom structure operates like an LC tank circuit. At the right resonant frequency, the surface resembles an open-circuit surface where no current can flow. Hence, tangential magnetic field is zero resembling perfect magnetic conductor (courtesy of Sievenpiper [192]).

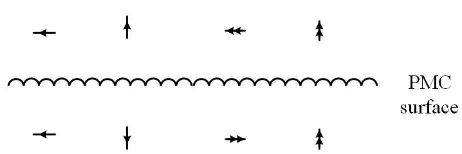

Figure 32.12: Reflection rules for electric and magnetic dipoles over a PMC surface. Magnetic dipoles are denoted by double arrows.

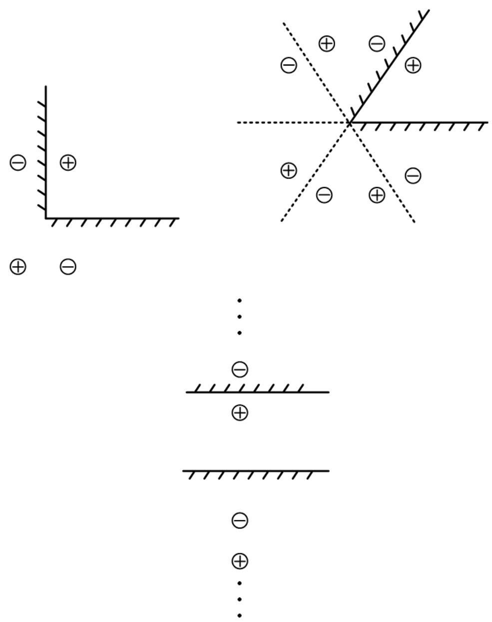

Figure 32.13: Image theory for multiple images [31].

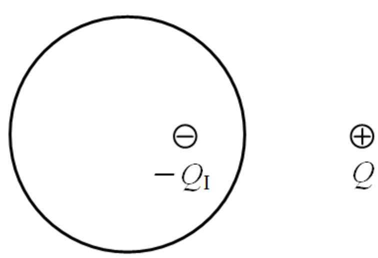

Figure 32.14: Image theory for a point charge near a cylinder or a sphere can be found in closed form [31].

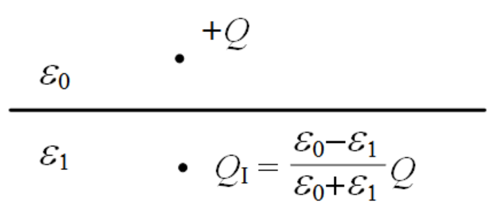

Figure 32.15: A static charge over a dielectric interface can be found in closed form using Fourier transform technique. The solution is beyond the scope of this course.