ECE 604 Electromagnetic Field Theory

Fall 2020

Homework No. 9. Due Date: Nov 13, 2020.

Read lecture notes 1-33.

1. (i) Using reciprocity theorem, show that an impressed current source on a PEC surface cannot radiate any field.
(ii) A dyad is defined in physics as juxtaposed of two 3 vectors, e.g., $\mathbf{ab}$ where $\mathbf{a}$ and $\mathbf{b}$ are three-component vectors called 3 vectors in physics. In matrix algebra, this is written as $\mathbf{a}\bullet\mathbf{b}^t$ called an outer product (this outer product is denoted $\mathbf{a}\otimes\mathbf{b}$ in the math literature as well), where $\mathbf{a}^t\bullet\mathbf{b}$ is called an inner product. In physics, an inner product is just written $\mathbf{a}\bullet\mathbf{b}$. Assuming that $\mathbf{a}$ and $\mathbf{b}$ are independent of each other but not orthogonal, even though the dyad $\mathbf{ab}$ behaves like a 3x3 matrix, it only has one nonzero eigenvector with one nonzero eigenvalue. Find this eigenvector. It can be shown that a dyad has at most one nonzero eigenvector. The number of nonzero eigenvalues of a matrix is the rank of the matrix. A dyad has a rank of at most 1. (This exercise teaches you what a dyad is.)
(iii) On the other hand, a dyadic is a superposition of dyads. The dyadic Green's function is homomorphic to the scalar Green's function, albeit with more complicated vector algebra. By direct back substitution, show that the solution to the following equation

$$\nabla\times\nabla\times\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')-k^2\,\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')=\overline{\mathbf{I}}\,\delta(\mathbf{r}-\mathbf{r}').$$

is given by

$$\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')=\left(\overline{\mathbf{I}}+\frac{\nabla\nabla}{k^2}\right)g(\mathbf{r}-\mathbf{r}')$$

In the above, the dyadic $\overline{\mathbf{I}}$ is similar to an identity matrix because $\overline{\mathbf{I}}\bullet\mathbf{a}=\mathbf{a}$. Discuss how you would write $\overline{\mathbf{I}}$ in a three space. You can write $\overline{\mathbf{I}}$ in cartesian, cylindrical, or spherical coordinates.
(iv) Given the vector wave equation for the electric field has a source term, namely that

$$\nabla\times\nabla\times\mathbf{E}-k^2\mathbf{E}=-j\omega\mu\mathbf{J}$$

By the principle of linear superposition, show that the solution to the above equation can be written as

$$\mathbf{E}(\mathbf{r})=-j\omega\mu\iiint d\mathbf{r}'\,\overline{\mathbf{G}}(\mathbf{r},\mathbf{r}')\bullet\mathbf{J}(\mathbf{r}')$$

2. (i) Explain why perfect electric conductor is not necessary to shield out the electric field at statics.
(ii) We simplify the cavity-backed slot antenna to a geometry shown below. Its radiation behavior can be replaced by an equivalence problem 1 shown below. What should $\mathbf{J}_s$

and $\mathbf{M}_s$ be in the equivalence problem 1 so that the fields to the left are zero, and the equivalence currents radiate to the right to create the same fields?

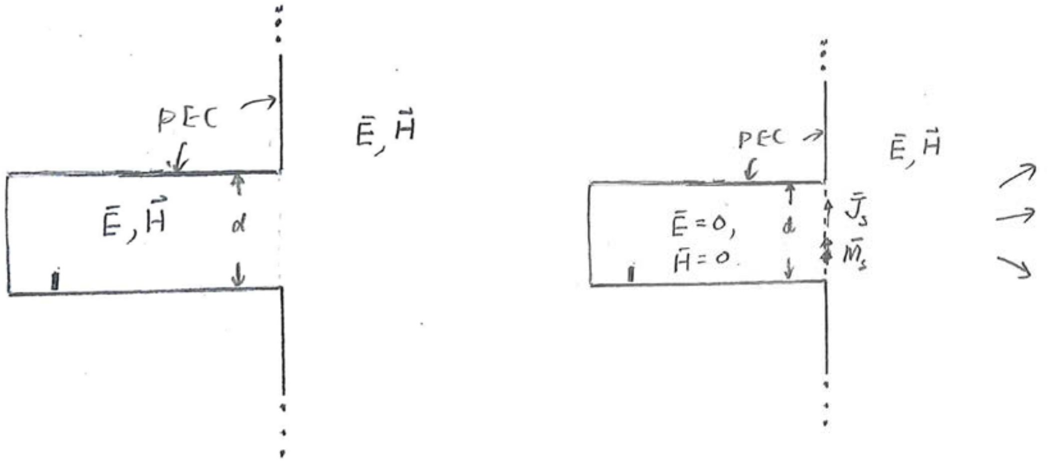

Figure 1. Original problem (left). Equivalence problem 1 (right).

(iii) Explain why the equivalence problem 1 above can be replaced by equivalence problem 2 and equivalence problem 3 shown below. What addition theorem do you need to arrive at equivalence probem 3? Given the values of $\mathbf{M}_{SA}$ and $\mathbf{M}_{SB}$ below in terms of $\mathbf{M}_s$ in Part (iii) above.

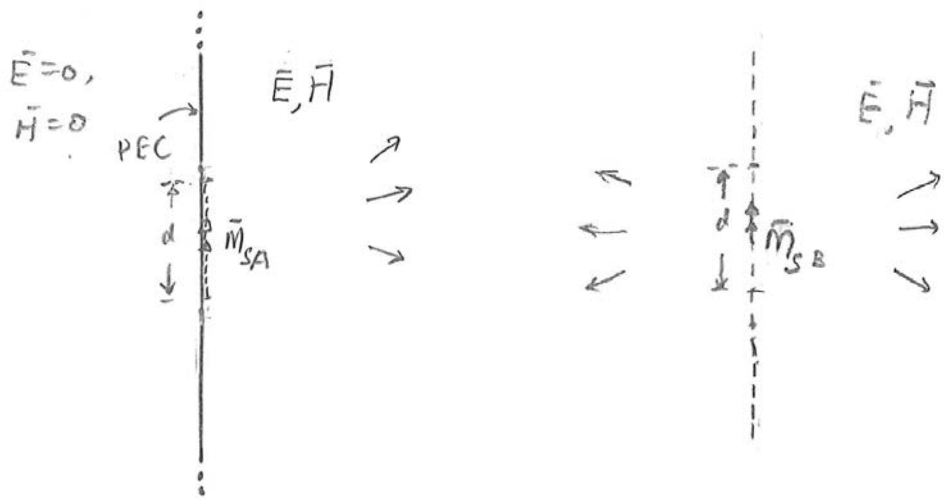

Figure 2. Equivalence problem 2 (left). Equivalence problem 3 (right).

3. (i) Derive the following equations of the notes on Gaussian beam and paraxial wave equation.

$$\Psi(x,y,z) = \frac{A_0}{\sqrt{1+z^2/b^2}} e^{-j\beta \frac{x^2+y^2}{2R}} e^{-\frac{x^2+y^2}{w^2}} e^{j\psi}$$

where

$$w^2 = \frac{2b}{\beta}\left(1+\frac{z^2}{b^2}\right), \qquad R = \frac{z^2+b^2}{z}, \qquad \psi = \tan^{-1}\left(\frac{z}{b}\right)$$

(ii) Explain why R is related to the radius of curvature of the wavefront of the Gaussian beam.

4. Use Fermat's principle, derive the law of reflection for a simple interface, and then a metasurface where a phase shift of $\Phi(x,y)$ can happen at the surface.