ECE 604 Electromagnetic Field Theory

Fall 2020

Homework No. 8. Due Date: Nov 6, 2020.
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Read lecture notes 1-28.

1. (i) Following the method, where the Green's function has been derived for Poisson's equation, show by back substitution that the Green's function for Helmholtz equation is in fact (**Note: you have derived this in your THE1. Just show this by back substitution now**).

$$g(\mathbf{r}-\mathbf{r}')=\frac{e^{-j\beta|\mathbf{r}-\mathbf{r}'|}}{4\pi|\mathbf{r}-\mathbf{r}'|}$$

(ii) With a simplified version of the above, letting $g(r)=\frac{e^{-j\beta r}}{4\pi r}$, and comparing with the static version where $g_s(r)=\frac{1}{4\pi r}$, show that the higher order derivatives of $g(r)$ is always much larger that of $g_s(r)$ when r is large. Can you give a physical meaning to this? (The answer can be found in Chew, W. C., *Philo. Trans. Royal Soc. London. Series A: Math., Phys. and Engg. Sci.* 362.1816 (2004): 579-602.)
(iii) Show that in the far field, the ratio of $|\mathbf{E}|/|\mathbf{H}|=\eta$. Give the physical reason for this.
(iv) In Lecture 26, it was shown that

$$\mathbf{A}(\mathbf{r})\cong\frac{\mu e^{-j\beta r}}{4\pi r}\mathbf{F}(\boldsymbol{\beta})$$

Explain why when we are in the far field such that $\beta r\gg 1$, in the neighborhood of an observation around $\mathbf{r}$, the variation of the field is dominated by $e^{-j\beta r}$, and hence, why we can make a local plane wave approximation. Is it necessary that the radius of curvature of the wavefront be much larger than the wavelength for this approximation? Explain why and why not?

2. (i) Go through the Lecture 27 and derive the expression (27.1.8).
(ii) Repeat Case II of the same lecture notes, but with $N=5$, and plot the far field pattern of this new array.
(iii) Find the leading order approximation, up to the quadratic term, of the expression $|\mathbf{r}-\mathbf{r}'|$ when $r'<<r$. In other words, rederive equation (27.2.7) and reconfirm the definition of Rayleigh distance in (27.2.10). Also, derive the Rayleigh distance defined in (27.2.12).

3. (i) Explain why a folded dipole has a radiation resistance four times that of an unfolded dipole (**Hint: the answer can be found in wiki. But please understand the logic before you regurgitate it**).
(ii) Explain how a cavity backed slot antenna work.

(iii) Explain why the corrugated horn antenna produce an axially symmetric radiation pattern.
(iv) Explain how a lens antenna work.
(v) Explain why PIFA is smaller than a half-wave dipole.
(vi) Explain how you would make the current uniform on a large loop antenna.