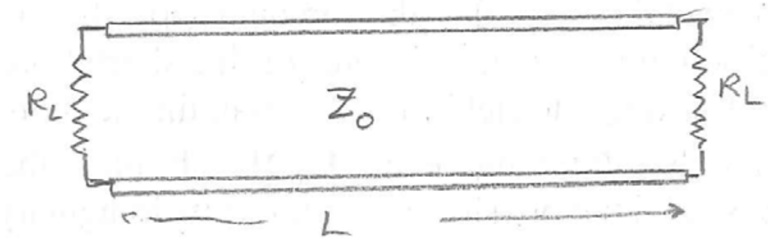
ECE 604 Electromagnetic Field Theory

Fall 2020

Homework No. 7. Due Date: Oct 30, 2020.

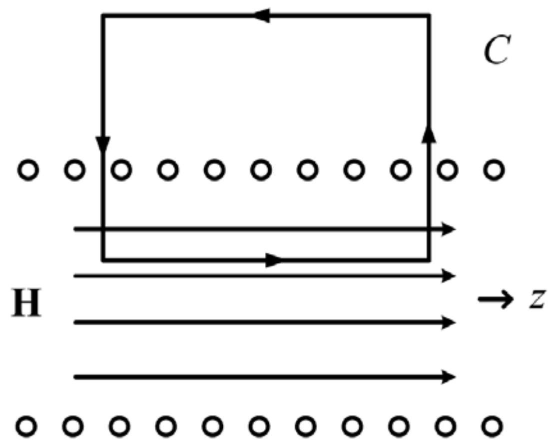
Read lecture notes 1-25.

1. The fundamental mode of a circular waveguide cavity is the TM_{010} mode. (Hint: Discussion on this mode can be found in Kong's book.)
(i) Find the formula for the Q of this mode.
(ii) Assume a transmission line resonator is loaded with two resistors at its two ends. Assume that these resistors are much smaller than the characteristic impedance of the line. Find an approximate expression of the Q of this transmission line resonator with the loads at the two ends. **Hint: Use a perturbation concept that the current at the ends are about the same as that when it is short circuited.**
(iii) Now, assume that you can model the wall loss of the waveguide by replacing $j\omega L$ with $j\omega L + R$ where R is small. Find the contribution to the Q from this wall loss alone.
(iv) If you know the Q due to the end walls and the side wall loss, how would you find the total Q of the system. **Hint: Use a perturbation concept to solve this problem.**

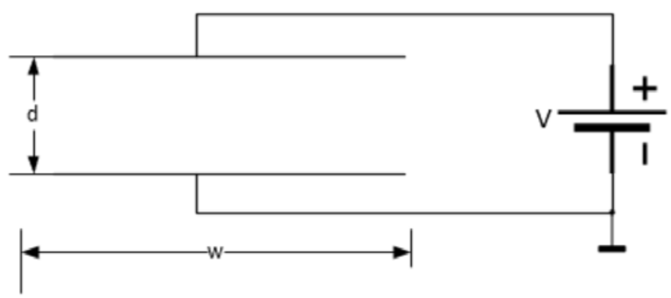


2. The equations for vector and scalar potentials have been found using the Lorenz gauge in the lecture notes.
(i) Now, find these equations using Coulomb's gauge. **Hint: This is described in J.D. Jackson's book, and many other physics texts.**
(ii) Discuss the pros and cons of Coulomb's gauge versus Lorenz gauge.

3. Use the energy storage method:
(i) Find the inductance of the solenoid and show that it is the same as that obtained by the flux linkage method. Hint: Ramo et al has a discussion of this.



(ii) Use the energy storage method, find the capacitance of a parallel plate and show that it is the same as that obtained by solving the boundary value problem between the parallel plates, then followed by calculating the total charge stored in the plates.



4. (i) Assume a sinusoidal current distribution on a short dipole like a half-wave dipole (but retardation effect on the dipole is ignored or the current on the dipole has equal phase), find the radiation resistance of this short dipole. **Hint: This is discussed in the lecture notes.**

(ii) Since we know the complete field of a Hertzian dipole, both in the near field and the far field, find the complex Poynting vector $\mathbf{E}\times\mathbf{H}^*$ as a function of r. Explain the physical meanings of the real part of this vector, and its imaginary part. (You will get good practice of your vector algebra in spherical coordinates here.)

(iii) Find the decay rates with respect to r for the real part and imaginary part.

(iv) Explain the decay rates. Which part decays faster, the real part or the imaginary part? Explain why.