ECE 604 Electromagnetic Field Theory

Fall 2020

Homework No. 6. Due Date: Oct 16, 2020.
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Read lecture notes 1-19, and ECE 350X notes on dielectric waveguide at:
https://engineering.purdue.edu/wcchew/ece350.html

For those interested in further reading on optical fiber, it is available at: Lectures on Theory of Microwave and Optical Waveguides, Sect. 6.2.

1. Show that if we have a wave packet formed by a carrier modulated by a slowly varying function, go through the understanding of the lecture notes, show that the carrier will move at the phase velocity, while the envelope will move at the group velocity. (This homework is the regurgitation of the lecture notes, to ensure that you really understand the very important idea that has been discussed in class.)

2. By going through the lecture notes on dielectric waveguides in ECE350X, explain why when *n* in Lecture 17 (lecture book), eqns. (17.2.8) and (17.2.9) is even or odd, they correspond to mode profiles that are even or odd.

3. Find the group and phase velocity of a guided mode in a hollow waveguide.
(i) Show that their product is always a constant.
(ii) Explain why the group velocities are zero and the phase velocities are infinite right at cut-off of a mode.
(iii) What do the group and phase velocities become when the frequency is very high? Explain why?

4. (i) For a 2 cm by 1 cm rectangular waveguide, and a 1 cm radius circular waveguide, find the first three propagating modes starting with the lowest cutoff frequencies.
(ii) For the TM modes, show that the homogeneous Dirichlet boundary condition that $\psi_e = 0$ on *C*, the waveguide wall, all components of the tangential electric field will be zero also.
(iii) For the TM mode, starting with that

$$\mathbf{H}(\mathbf{r}) = \nabla \times \hat{z}\psi_e(\mathbf{r})$$

Find all the components of electromagnetic fields in a rectangular waveguide.
(iv) Starting with a TM mode in a waveguide where $H_z = 0,\ E_z \neq 0$ initially, show that if this mode becomes a TEM mode so that $H_z = 0,\ E_z = 0$, then $\beta_z = \beta$. (This can happen in a coaxial waveguide, for instance.) What happens to β_s for the TEM mode, and what happens to the Helmholtz equations that $\psi_e(\mathbf{r})$ and $\psi_h(\mathbf{r})$ originally satisfy? Explain why the fields of the TEM mode of a waveguide, if it exists, is electrostatic and magnetostatic in nature.