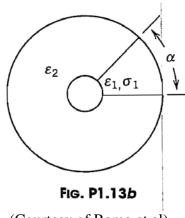
## ECE 604 Electromagnetic Field Theory Fall 2020

Homework No. 4. Due Date: Oct 2, 2020.

## Read lecture notes 1-14, and ECE 350X notes on transmission line theory at: https://engineering.purdue.edu/wcchew/ece350.html

- 1. The line admittance and impedance of a transmission line can be found by solving an electrostatic problem and a magnetostatic problem, respectively.
- (i) The coaxial cable geometry below with inner radius a and outer radius b, find the admittance per unit length that you can substitute into the telegrapher's equations. Assume uniform radial electric field inside the coax.



(Courtesy of Ramo et al)

- (ii) Assume a current I that flows in the inner conductor, using the magnetic field you have found before, find the magnetic energy stored per unit length. Knowing that the energy storage for an inductor is  $\frac{1}{2}LI^2$ , find the inductance per unit length.
- (iii) Use your results to find the characteristic impedance  $Z_0$  of this transmission line, and also the propagation constant  $\gamma$ .

(Note: This problem cannot be solved exactly in a closed form, and we are using a circuit, or quasistatic approximation in finding the transmission line parameters.)

2. Look at Lecture 8 of the notes from ECE350x:

https://engineering.purdue.edu/wcchew/ece350/ee350-08.pdf

(i) Walk yourself through the example, for part (a) change the load to

$$Z_i = 20 + j30$$
 ohms

and find the new answer.

(ii) For part (b), change

$$d_{\min} = 3\lambda/16$$

and find the new answer.

3. The multi-section (or junction) transmission line is as shown in the figure below.

- (i) Use the generalized reflection coefficient derived in class, find  $\tilde{\Gamma}_{23}$  and  $Z_{in3}$ .
- (ii) Then find  $\tilde{\Gamma}_{12}$  and  $Z_{in2}$  .
- (iii) What is the value of  $Z_{02}$  you can choose to have zero reflection at Junction 1? (Note: This problem can also be solved using the graphical calculator, the Smith chart, but the closed form formulas allow one to calculate and program the reflection coefficients and the impedances exactly. Part (iii) of this problem is that of a quarter wave transformer matching which can be found in many textbooks.)

$$l_{1} = \frac{\lambda}{4} \qquad 1 \qquad l_{2} = \frac{\lambda}{4} \qquad 2 \qquad l_{3} = \frac{\lambda}{2}$$

$$Z_{01} = 50 \Omega \qquad Z_{02} = 75 \Omega \qquad Z_{03} = 50 \Omega \qquad Z_{L}$$

$$\beta_{1} \qquad \tilde{\Gamma}_{12} \rightarrow \qquad \beta_{2} \qquad \tilde{\Gamma}_{23} \rightarrow \qquad \beta_{3} \qquad Z_{L} = 60 \Omega$$

$$Z_{in2} \rightarrow \qquad Z_{in3} \rightarrow \qquad Z_{in3} \rightarrow \qquad Z_{L} = 60 \Omega$$

- 4. (i) For Lect 12, derive (12.1.14) from (12.1.13).
- (ii) For Lect 13, derive (13.1.8) from (13.1.7).
- (iii) For Lect 13, show that (13.1.13) and (13.1.14) are the same as each other.