

**ECE 604 Electromagnetic Field Theory  
Fall 2020**

**Homework No. 3. Due Date: Sep 18, 2020**

**Read lecture notes 6, 7 and 8.**

1. For Lecture 6:

- (i) Explain why equation (6.1.17) and the statement after it is true.
- (ii) Explain why equation (6.2.7) and the statement after it is true.
- (iii) Is there a difference in the field quantities obtained from phasor technique and the field quantities obtained from Fourier transform technique?
- (iv) Explain the physical meaning of the imaginary part of complex power.
- (v) Explain the physical meaning of a spatially dispersive medium.

2. For Lecture 7:

For uniaxial medium, the permittivity tensor is given by:



$$\bar{\epsilon} = \begin{bmatrix} \epsilon & 0 & 0 \\ 0 & \epsilon & 0 \\ 0 & 0 & \epsilon_z \end{bmatrix} \quad (1)$$

Assume a plane wave propagating as

$$\mathbf{E}_0 e^{-j\mathbf{k}\cdot\mathbf{r}} \quad (2)$$

(i) From Maxwell's equations, show that the following equation must be satisfied:

$$\mathbf{k} \times \mathbf{k} \times \mathbf{E} = -\omega^2 \mu \bar{\epsilon} \cdot \mathbf{E} \quad (3)$$

(ii) When the electric field  $\mathbf{E}$  is polarized in the  $xy$  plane,  $\epsilon_z$  is not felt by the wave. This is called the ordinary wave. Show that the dispersion relation from the above equation simplifies to:

$$k_x^2 + k_z^2 = \omega^2 \mu \epsilon \quad (4)$$

(iii) When the electric field  $\mathbf{E}$  is polarized in the  $xz$  plane,  $\epsilon_z$  is now felt by the wave. The wave is now called the extra-ordinary wave. Show that the electric field has to be of the form:

$$\mathbf{E} = \left( \hat{x} - \hat{z} \frac{k_x \epsilon}{k_z \epsilon_z} \right) E_0 e^{-j\mathbf{k}\cdot\mathbf{r}} \quad (5)$$

And the corresponding electric flux is:

$$\mathbf{D} = \left( \hat{x} - \hat{z} \frac{k_x}{k_z} \right) \epsilon E_0 e^{-j\mathbf{k}\cdot\mathbf{r}} \quad (6)$$

Explain your reasoning.

(iv) From (3), for the extra-ordinary wave, show that the dispersion relation can be reduced to:

$$\frac{k_x^2}{\omega^2 \mu \epsilon_z} + \frac{k_z^2}{\omega^2 \mu \epsilon} = 1 \quad (7)$$

(v) The equations (4) and (7) are equations of surfaces known as k-surfaces. Please draw these two surfaces on the same graph (in 2D, it will just be a contour), and explain the physical meanings of the two surfaces.

3. For Lecture 8:

This solution here, to a boundary value problem for Laplace equation, can be used to explain why plasmonic particles, when embedded in glass or lacquer, glitter in light. When a dielectric sphere is immersed in a static electric field as shown in the Figure 1, the electric field does not satisfy the boundary condition. Hence, the sphere responds by producing a dipolar potential in order to satisfy the boundary condition.

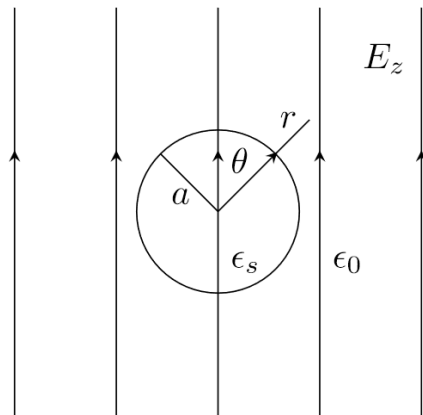


Figure 1

(i) Show that the potential outside the sphere can be written as

$$\Phi_{out} = -E_0 z + \frac{A}{r^2} \cos \theta$$

Explain the physical meanings of the first and second terms on the right-hand side of the above expression.

(ii) The potential inside the sphere can be written as

$$\Phi_{in} = B z$$

where  $B$  is another unknown coefficient here. What kind of electric field corresponds to the above potential?

(iii) Now, assume that the sphere has radius  $a$ . Decide on the boundary conditions at the dielectric interface  $r = a$ .

(iv) From the boundary conditions, derive the expressions for  $A$  and  $B$ .

(v) Explain why gold plasmonic nano-particles can glitter in light.

4. Lecture 8:

(i) Estimate the skin depth of the signal in your induction cooker pan. Assume that it operates around 50 KHz, and that the relative permeability  $\mu_r$  is 100, and that the conductivity is about  $10^7$  siemens/m.

(ii) Estimate the electron density of the plasma layer in the ionosphere if it is known that radio frequency below 10 MHz cannot penetrate the ionosphere.

(iii) The conductivity of a conductive medium has been estimated to be

$$\sigma = \varepsilon_0 \frac{\omega_p^2}{\Gamma}$$

using the Drude-Lorentz-Sommerfeld model. Arrive at the same formula using collision frequency argument. Hint: You may find the answer in some textbooks like Streetman on Semiconductor Devices.