ECE 604 Electromagnetic Field Theory
Fall 2020

Homework No. 2. Due Date: Sep 11, 2020
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Read lecture notes 4, 5, and 6.

1. For Lecture 4:
The Earnshaw's theorem says that a minimum or maximum point cannot appear in the potential of the solution to Laplace's equation. The detail of the proof is quite long, but we can motivate this theorem in the following way:
(i) Laplace's equation can be solved by the separation of variables, namely that in 2D, its solution can be written as
$\Phi(x, y) = A\cos(ax)\exp(-ay)$. Show that this is a solution to Laplace's equation, and that this function does not have a maximum or a minimum point except at the boundary.
(ii) However, if we write $\Phi(x, y) = A\cos(ax)\cos(by)$, show that this function does have a maximum or a minimum, but it is not a solution of Laplace equation.

This theorem means that if Φ is a solution to Laplace's equation in a region *V*, Φ can only have maximum or minimum value at the boundary of *V*.
(iii) Use this to explain that if a region *V* is bounded by a surface *S*, and if Φ is constant on *S*, then it is the same constant everywhere in *V*.
(iv) Use this fact to explain how the Faraday's cage works and why that Coulomb's gauge can be used to guarantee a unique vector potential $\mathbf{A}$.

2. For Lecture 4:
(i) By back substitution, show that eq. (4.1.11) in fact satisfies (4.1.10).
(ii) Coulomb's law give the scalar potential for a monopole charge to be

$$\Phi = \frac{q}{4\pi\varepsilon r}$$

Show that by differentiating this expression with respect to *z*, one can get the scalar potential for a dipole to be

$$\Phi_d = \frac{\ell q\cos(\theta)}{4\pi\varepsilon r^2}$$

where ℓ is the length of the dipole.
(iii) Give the physical meaning of this mathematical procedure.
(iv) When the above Φ_d is back substituted into Poission's equation, what do you expect the charge density to be on the right-hand side of Poisson's equation?
(v) Derive the jump condition or the boundary condition induced by Ampere's law when there is a current sheet at the interface between two media.

3. For Lecture 5:
(i) Derive eqs. (5.1.5) and (5.1.6).
(ii) Explain why for electrostatics, perfect conductor is not needed to shield out the electric field.

(iii) Explain the Meissner effect in a superconductor, and why a small piece of superconductor can levitate on a pole of a permanent magnet.
(iv) Give physical interpretation to equation (5.3.19) and the meaning of each of the terms in the equation.