ECE 604 Electromagnetic Field Theory

Fall 2020

Homework No. 10. Due Date: Dec 4, 2020.

Read lecture notes 1-37.

1. (i) Explain how the Vikings could have used the physical results of Rayleigh scattering to navigate themselves across the North Atlantic Ocean to arrive at Iceland.
(ii) By using the separation of variables, explain how you would solve the Helmholtz wave equation in 3D in spherical coordinates.

$$(\nabla^2 + \beta^2)\Psi(\mathbf{r}) = 0$$

2. This exercise refers to Lecture 35 of the lecture notes.
(i) By taking the residue of a contour integral around a pole, show how you can go from eq. (35.1.8) to eq. (35.1.9) and eq. (35.1.10).
(ii) Show that eq. (35.2.3) can be derived from (35.2.1), and then show that (35.2.5) can be derived assuming that $z \neq 0$. Similarly, from (35.2.1), derive (35.2.8) and (35.2.9), and then show that (35.2.10) and (35.2.11) can be derived.

(iii) First, for the section on stationary phase, derive (35.3.2) which is the reflected field term in Cartesian coordinates. Assume that $d_1 \neq 0$, derive the equivalence of (35.3.7) with $r \to r_I = \sqrt{x^2 + y^2 + (z + 2d_1)^2}$, using stationary phase argument, and elucidate the physics expressed in the math in accordance to item (ii) below (35.3.7). Explain why this is related to Fermat's principle.

3. This exercise refers to Lecture 36 of the lecture notes.
(i) Derive eq. (36.2.2) from Maxwell's equations.
(ii) In the lecture notes, we assume that $\overline{\mathbf{L}}$ is a symmetric matrix. But now, assume that the operator $\overline{\mathbf{L}}$ in (36.7.1) is not symmetric. The optimal solution is obtained by solving two equations. Derive a functional I such that is optimal point is the solution to eq. (36.7.1) and another auxiliary equation. Show that such a functional can be defined as

$$I = \mathbf{w}^{\mathbf{t}} \bullet \overline{\mathbf{L}} \bullet \mathbf{f} - \mathbf{w}^t \bullet \mathbf{g} - \mathbf{g}_a^t \bullet \mathbf{f}$$

By taking the first variation of the above functional with respect to **f** and **w**, show that the optimal solutions that minimize the above functional are solutions to the equations

$$\overline{\mathbf{L}} \bullet \mathbf{f} = \mathbf{g}$$

$$\overline{\mathbf{L}}^{\mathbf{t}} \bullet \mathbf{w} = \mathbf{g}_{\mathbf{a}}$$

(iii) Using indicial notation, for a symmetric system discussed in the lecture notes, show that the gradient of a functional I in the N dimensional space is given by (36.7.8).

4. This problem refers to Lecture 37.
(i) Show that (37.1.15) expands to (37.1.12) after using (37.1.13) and its equivalence in x and y directions. Hence, derive (37.1.18).
(ii) Show that (37.1.32) is indeed true. Give the physical meaning of (37.1.34).

(iii) Derive (37.1.40) and (37.1.41). What is the physical meaning of the extra δ in (37.1.40)?