ECE 604 Electromagnetic Field Theory
Fall 2019

Homework No. 3. Due Date: Sept 13, 2019

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Read lecture notes 7 and 8.

1. For Lecture 7:

For uniaxial medium, the permittivity tensor is given by:

$$\overline{\boldsymbol{\varepsilon}} = \begin{bmatrix} \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon_z \end{bmatrix} \tag{1}$$

Assume a plane wave propagating as

$$\mathbf{E}_0 e^{-j\mathbf{k}\bullet\mathbf{r}} \tag{2}$$

(i) From Maxwell's equations, show that the following equation must be satisfied:

$$\mathbf{k}\times\mathbf{k}\times\mathbf{E} = -\omega^2\mu\overline{\boldsymbol{\varepsilon}}\bullet\mathbf{E} \tag{3}$$

(ii) When the electric field $\mathbf{E}$ is polarized in the *xy* plane, ε_z is not felt by the wave. This is called the ordinary wave. Show that the dispersion relation from the above equation simplifies to:

$$k_x^2 + k_z^2 = \omega^2\mu\varepsilon \tag{4}$$

(iii) When the electric field $\mathbf{E}$ is polarized in the *xz* plane, ε_z is now felt by the wave. The wave is now called the extra-ordinary wave. Show that the electric field has to be of the form:

$$\mathbf{E} = \left(\hat{x} - \hat{z}\frac{k_x\varepsilon}{k_z\varepsilon_z}\right)E_0 e^{-j\mathbf{k}\bullet\mathbf{r}} \tag{5}$$

And the corresponding electric flux is:

$$\mathbf{D} = \left(\hat{x} - \hat{z}\frac{k_x}{k_z}\right)\varepsilon E_0 e^{-j\mathbf{k}\bullet\mathbf{r}} \tag{6}$$

Explain your reasoning.

(iv) From (3), for the extra-ordinary wave, show that the dispersion relation can be reduced to:

$$\frac{k_x^2}{\omega^2\mu\varepsilon_z} + \frac{k_z^2}{\omega^2\mu\varepsilon} = 1 \tag{7}$$

(v) The equations (4) and (7) are equations of surfaces known as k-surfaces. Please draw these two surfaces on the same graph (in 2D, it will just be a contour), and explain the physical meanings of the two surfaces.

2. For Lecture 8:

This solution can be used to explain why plasmonic particles, when embedded in glass or lacquer, glitter in light. When a dielectric sphere is immersed in a static electric field as shown in the Figure 1, the electric field does not satisfy the boundary condition. Hence, the sphere responds by producing a dipolar potential in order to satisfy the boundary condition.

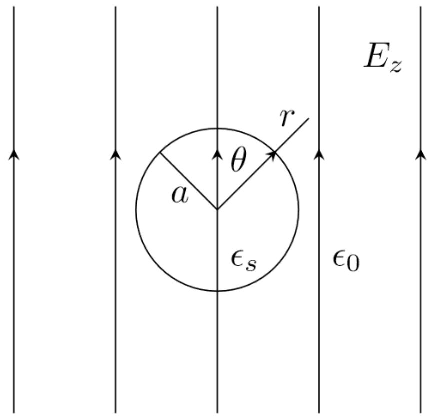

Figure 1

(i) Show that the potential outside the sphere can be written as

$$\Phi_{out} = -E_0 z + \frac{A}{r^2}\cos\theta$$

Explain the physical meaning of the first term on the right-hand side of the above expression.

(ii) The potential inside the sphere can be written as

$$\Phi_{in} = B\, z$$

where *B* is another unknown coefficient here. What kind of electric field corresponds to the above potential?

(iii) Now, assume that the sphere has radius a. Decide on the boundary conditions at the dielectric interface $r = a$.

(iv) From the boundary conditions, derive the expressions for *A* and *B*.

(v) Explain why gold plasmonic nano-particles can glitter in light.

3. Lecture 8:

(i) Estimate the skin depth of the signal in your induction cooker. Assume that it operates around 50 KHz, and that the relative permeability μ_r is 100, and that the conductivity is about 10^7 siemens/m.

(ii) Estimate the electron density of the plasma layer in the ionosphere if it is known that radio frequency below 10 MHz cannot penetrate the ionosphere.

(iii) The conductivity of a conductive medium has been estimated to be

$$\sigma = \varepsilon_0 \frac{\omega_p^2}{\Gamma}$$

using the Drude-Lorentz-Sommerfeld model. Arrive at the same formula using collision frequency argument.