

ECE 604 Electromagnetic Field Theory

Fall 2019

Homework No. 11. Due Date: Dec 6, 2019.

Read lecture notes 1-37.

1. (i) Show that (36.2.14) expands to (36.2.12) after using (36.2.13). Hence, derive (36.2.18).

(ii) Show that (36.2.32) is indeed true. Give the physical meaning of (36.2.24).

(iii) Derive (36.2.40) and (36.2.41). What is the physical meaning of the extra δI (36.2.40)?

2. (i) Derive (37.1.12).

(ii) Explain why it can be simplified to (37.1.13) and (37.1.14).

(iii) Explain how you can convert these integral equations into matrix equations.

3. (i) Derive eq. (37.1.2) from Maxwell's equations.

(ii) Assuming that the operator $\bar{\mathbf{L}}$ in (37.3.1) is not symmetric now. Derive a functional I such that its optimal point is the solution to eq. (37.3.1) and another auxiliary equation. Show that such a functional can be defined as

$$I = \mathbf{w}^t \cdot \bar{\mathbf{L}} \cdot \mathbf{f} - \mathbf{w}^t \cdot \mathbf{g} - \mathbf{g}_a^t \cdot \mathbf{f}$$

By taking the first variation of the above functional with respect to \mathbf{f} and \mathbf{w} , show that the optimal solution to the above are solutions to the equations

$$\bar{\mathbf{L}} \cdot \mathbf{f} = \mathbf{g}$$

$$\bar{\mathbf{L}}^t \cdot \mathbf{w} = \mathbf{g}_a$$

(iii) Show that the gradient of a functional I in the N dimensional space is given by (37.3.8).