ECE 604 Electromagnetic Field Theory

Fall 2019

Homework No. 10. Due Date: Nov 22, 2019.

,

Read lecture notes 1-35.

1. (i) Derive the following equations of the notes on Gaussian beam and paraxial wave equation.

$$\Psi(x,y,z) = \frac{A_0}{\sqrt{1+z^2/b^2}} e^{-j\beta\frac{x^2+y^2}{2R}} e^{-\frac{x^2+y^2}{w^2}} e^{j\psi}$$

where

$$w^2 = \frac{2b}{\beta}\left(1+\frac{z^2}{b^2}\right), \qquad R = \frac{z^2+b^2}{z}, \qquad \psi = \tan^{-1}\left(\frac{z}{b}\right)$$

(ii) Explain why R is related to the radius of curvature of the wavefront of the Gaussian beam.

2. (i) Explain how the Vikings could have used the physical results of Rayleigh scattering to navigate themselves across the North Atlantic Ocean to arrive at Iceland.

(ii) By using the separation of variables, explain how you would solve the Helmholtz wave equation in 3D.

$$(\nabla^2 + \beta^2)\Psi(\mathbf{r}) = 0$$

3. This exercise refers to Lect 35 of the lecture notes.

(i) By taking the residue of a contour integral around a pole, show how you can go from eq. (1.8) to eq. (1.9) and eq. (1.10).

(ii) From eq. (2.3), show that (2.5) can be derived assuming that $z \neq 0$. Similarly, from (2.8) and (2.9), show that (2.10) and (2.11) can be derived.

(iii) For eq. (2.6), explain how the phase term in the reflected wave term can be derived.