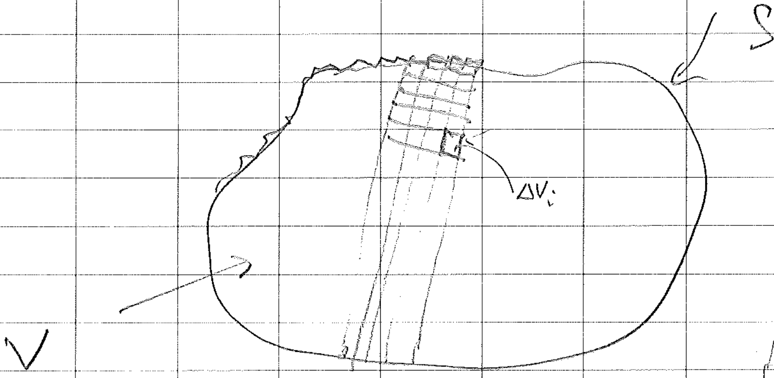
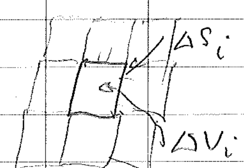
ECE 604, Aug 23, 2018 WC Chew

$$\nabla \cdot \bar{D} = \lim_{\Delta V \to 0} \frac{\oint\!\!\oint_{\Delta S} \bar{D} \cdot d\bar{S}}{\Delta V} = ?$$

if $\Delta V \approx 0$,

$$\Delta V \, \nabla \cdot \bar{D} \approx \oint\!\!\oint_{\Delta S} \bar{D} \cdot d\bar{S}$$





$$V \approx \sum_{i=1}^{N} \Delta V_i$$

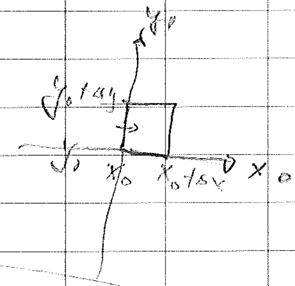
$$\Delta V_i \, \nabla \cdot \bar{D}_i \approx \oint\!\!\oint_{\Delta S_i} \bar{D}_i \cdot d\bar{S}_i$$

$$\sum_i \Delta V_i \, \nabla \cdot \bar{D}_i \approx \sum_i \oint\!\!\oint_{\Delta S_i} \bar{D}_i \cdot d\bar{S}_i \approx \oint\!\!\oint_S \bar{D} \cdot d\bar{S}$$

All inner surface fluxes ~~mutually~~ cancel. mutually

$$\Delta V_i \to 0 \Rightarrow \iiint_V dV \, \nabla \cdot \bar{D} = \oint\!\!\oint_S \bar{D} \cdot d\bar{S}$$ Gauss's Theorem

$$\oint_{\Delta S} \bar{D} \cdot d\bar{S}$$



$$\oint_{\Delta S} \bar{D}\, dS \cong -D_x(x_0)(\overbrace{\cancel{\Delta x}\Delta y \Delta z}^{\Delta V}) + D_x(x_0+\Delta x)\,\cancel{\Delta x}\Delta y \Delta z$$
$$- D_y(y_0)\Delta x \Delta y \cancel{\Delta z} + D_y(y_0+\Delta y)\,\Delta x \Delta y \cancel{\Delta z}$$
$$- D_z(z_0)\Delta x \Delta y \cancel{\Delta z} + D_z(z_0+\Delta z)\,\Delta x \Delta y \cancel{\Delta z}$$

$$= \Delta V \left\{ \{D_x(x_0+\Delta x, \ldots) - D_x(x_0, \,)\}/\Delta x \right.$$
$$+ (D_y(y_0+\Delta y, \ldots) - D_y(\ldots, y_0) \ldots))/\Delta y$$
$$\left. + (D_z(\ldots, z_0+\Delta z) - D_z(\ldots, z_0)))/\Delta z \right\}$$

$$\frac{\oint \bar{D}\, dS}{\Delta V} \cong \frac{\partial}{\partial x} D_x + \frac{\partial}{\partial y} D_y + \frac{\partial}{\partial z} D_z$$

$$\boxed{\lim_{\Delta V \to 0} \frac{\oint \bar{D}\, dS}{\Delta V} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \nabla \cdot \bar{D}}$$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

Gauss' Theorem

$$\iiint_V dV\, \nabla \cdot \bar{D} = \oint_S \bar{D} \cdot d\bar{S}$$ ← Gauss Th

$$\oint_S \bar{D} \cdot d\bar{S} = Q = \iiint_V dV \rho$$

$$\iiint_V dV\, \nabla \cdot \bar{D} = \iiint_V dV \rho$$

make $V \to 0$, $S \to 0$

$$\nabla \cdot \bar{D} = \rho$$, point-wise relationship

physical meaning of divergence

Faraday's Law

$$\oint_C \bar{E} \cdot d\bar{l} = 0$$

$$\oint_C \bar{E} \cdot d\bar{l} = -\frac{\partial}{\partial t} \iint_S \bar{B} \cdot d\bar{s} - K$$

fictitious magnetic current

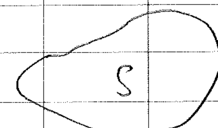
$\frac{\partial}{\partial t} = 0$, static

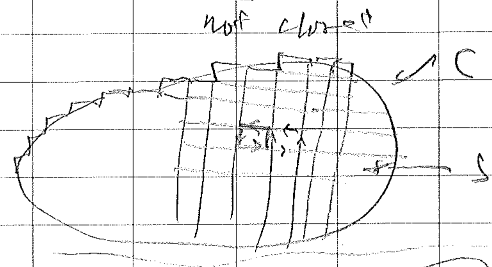
Stokes's Theorem

counting flux

$$\oint_C \bar{E} \cdot d\bar{l} = \iint_S \nabla \times \bar{E} \cdot d\bar{S}$$

not closed





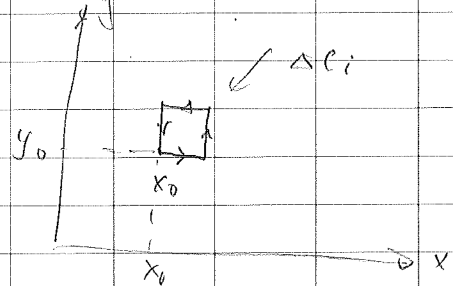
$$\oint_{\Delta C_i} \bar{E}_i \cdot d\bar{l}_i \stackrel{?}{=} \cancel{\iint_{\Delta S_i} (\nabla \times \bar{E}_i) \cdot d\bar{S}_i} \qquad (\nabla \times \bar{E}_i) \cdot \Delta \bar{S}_i$$

$$\sum_i \oint_{\Delta C_i} \bar{E}_i \cdot d\bar{l}_i = \sum_i \nabla \times \bar{E}_i \cdot \Delta \bar{S}_i$$

big C $$\oint_C \bar{E} \cdot d\bar{l} = \iint_S (\nabla \times \bar{E}) \cdot d\bar{S}$$

Stokes' Theorem

$$\nabla \times \bar{E}_i \cdot \hat{n}_i = \frac{\oint_{\Delta C_i} \bar{E}_i \cdot d\bar{l}_i}{\Delta S_i} \qquad \Delta \bar{S}_i = \hat{n}_i \, \Delta S_i$$



$$\oint_{\Delta C} \bar{E} \cdot d\bar{l} = E_x(y_0)\,\Delta x + E_y(x_0+\Delta x)\,\Delta y - E_x(y_0+\Delta y)\,\Delta x - E_y(x_0)\,\Delta y$$

$$= \Delta x \Delta y \left\{ \frac{E_x(y_0)}{\Delta y} - \frac{E_x(y_0+\Delta y)}{\Delta y} - \frac{E_y(x_0)}{\Delta x} + \frac{E_y(x_0+\Delta x)}{\Delta x} \right\}$$

$$\approx \Delta S \left\{ -\frac{\partial E_x}{\partial y} + \frac{\partial E_y}{\partial x} \right\}$$

$$\frac{\oint_{\Delta C} \bar{E} \cdot d\bar{l}}{\Delta S} = \frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x = \hat{z} \cdot \nabla \times \bar{E}$$

$$\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y = \hat{x} \cdot \nabla \times \bar{E}$$

$$\frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_y = \hat{y} \cdot \nabla \times \bar{E}$$

$$\nabla \times \bar{E} = \hat{x}\left(\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y\right) + \hat{y}\left(\frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z\right) + \hat{z}\left(\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x\right)$$

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z}$$

$\nabla \times \bar{E} = 0$, if $\frac{\partial}{\partial t} = 0$, statics

Faraday's law.

Constitutive Relations

$\bar{D} = \epsilon_0 \bar{E}$ in free space or vacuum

$\bar{D} = \underbrace{\epsilon_0 \bar{E}}_{\text{vacuum polarization density}} + \underbrace{\bar{P}}_{\text{Polarization density}}$ in material medium

For linear media $\bar{P} = \epsilon_0 \chi_0 \bar{E}$

$\bar{D} = \epsilon_0 \bar{E} + \epsilon_0 \chi_0 \bar{E}$

$= \epsilon_0 (1 + \chi_0) \bar{E} = \epsilon \bar{E}$

$\epsilon_0 \to \epsilon$

Anisotropic media

$\bar{D} = \epsilon_0 \bar{E} + \epsilon_0 \bar{\bar{\chi}}_0 \cdot \bar{E}$

$= \epsilon_0 (\bar{\bar{I}} + \bar{\bar{\chi}}_0) \cdot \bar{E} = \bar{\bar{\epsilon}} \cdot \bar{E}$

↑ 3×3 tensor or matrix

$\bar{J} = \sigma \bar{E}$, $\bar{J} = \bar{\bar{\sigma}} \cdot \bar{E}$

↑ anisotropic conductor

Electro Static Potential ϕ

$$\nabla \times \bar{E} = 0$$

$\bar{E} = -\nabla \phi$, or $\bar{E}$ is a constant

but $\bar{E} \to 0$, $r \to \infty$

$$\nabla \cdot \bar{D} = \rho$$

$$\nabla \cdot \epsilon \bar{E} = \rho$$

$$-\nabla \cdot \epsilon \nabla \phi = \rho$$

if ϵ is a constant of space, or independent of $\bar{r}$

$\nabla^2 \phi = -\frac{\rho}{\epsilon}$, Poisson's equation

if $\rho = 0$, or source free,

$\nabla^2 \phi = 0$. Laplace's Equation

For a point source

$$\bar{E} = \frac{q}{4\pi \epsilon r^2} \hat{r} = -\nabla \phi$$

$$\phi = \frac{q}{4\pi \epsilon r}$$

$= dV'$ short hand notation

$$\rho(\bar{r}) = \iiint_V d\bar{r}' \, \rho(\bar{r}') \, \delta(\bar{r} - \bar{r}')$$

dV'

$$\phi(\bar{r}) = \frac{1}{4\pi\epsilon} \iiint_V \frac{\rho(\bar{r}')}{|\bar{r} - \bar{r}'|} d\bar{r}'$$