Image Theory

Image theory for a flat conductor surface is quite easy to derive. To see that we can start with electro-static theory of putting a positive charge above a flat plane

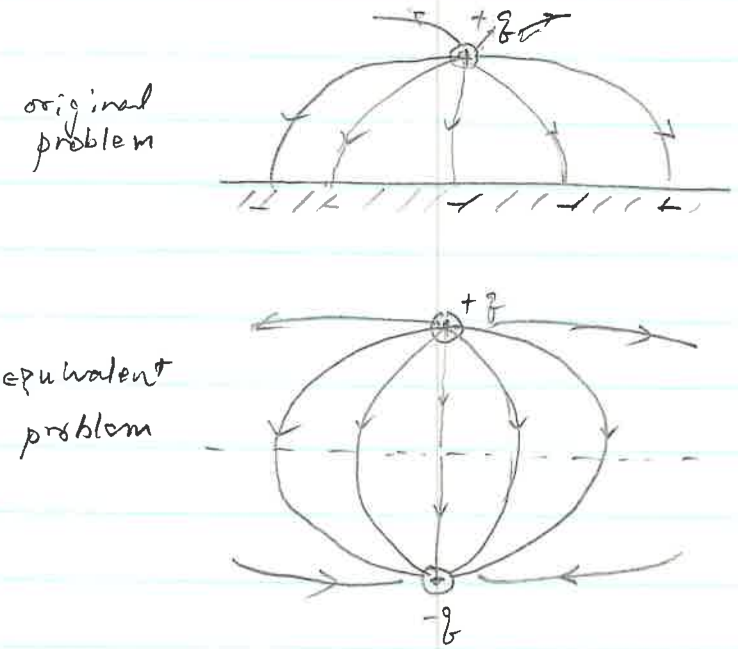


Fig. 1

From the above, one can easily see that a horizontal dipole reflects to one of opposite polarity while a vertical dipole reflects to one of equal polarity

Fig. 2

For electrostatic problems, a conductive medium suffices to produce charges that shield out the electric field from the bottom half space so that $\hat{n}\times\bar{E}=0$ on the conductor surface, and no current flows in the conductor.

For a perfect conductor, the charges can re-orient themselves fast enough so that $\hat{n}\times\bar{E}$ is always zero on the PEC surface even when the dipoles are time varying. The electric field $\bar{E}$ is always zero inside a PEC, and hence $\hat{n}\times\bar{E}=0$ on the surface by Faraday's law. Hence, the reflected images in Fig. 2 are valid even for a time-varying dipole over a PEC half space.

Assuming a magnetic monopole exists, it will reflect to itself on a PEC surface so that $\hat{n}\cdot\bar{B}=0$

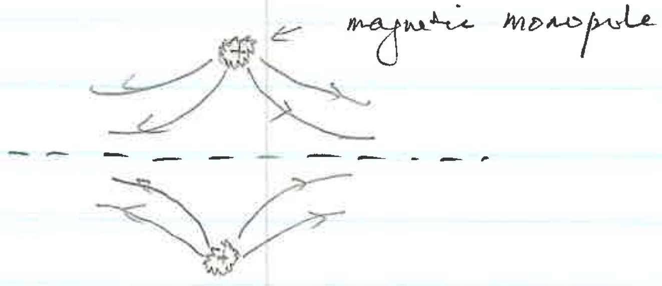


Fig. 3

Fig. 4

On a PMC surface, these electric and magnetic dipoles will reflect differently.

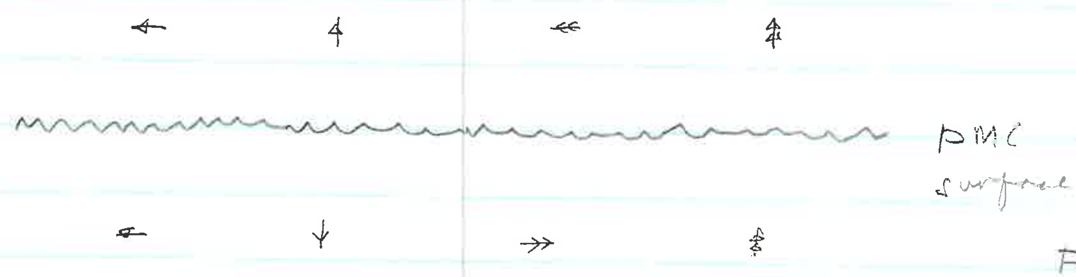


Fig. 5

One can go through Gedanken experiments and verify the above reflection rules.

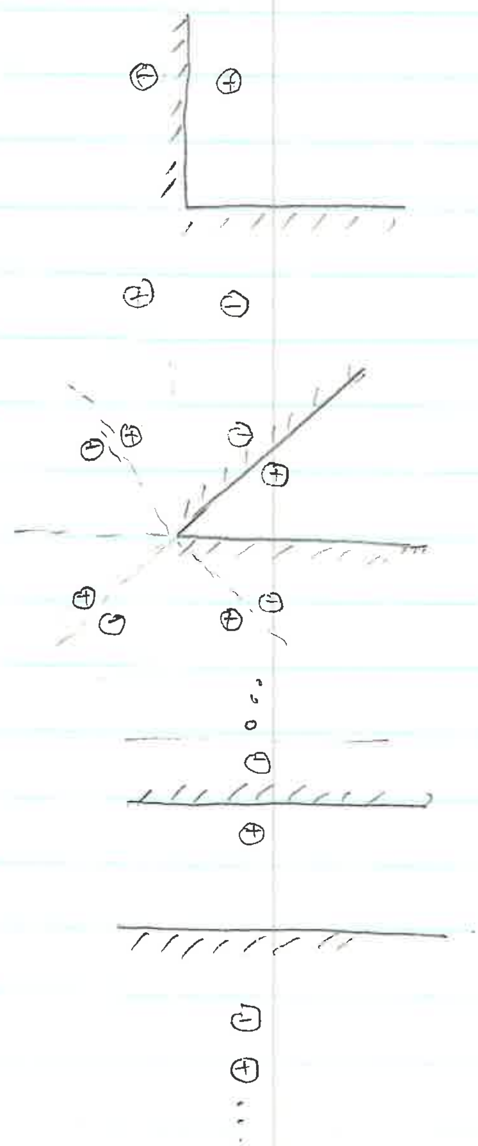


Fig. 6

For the geometry shown in Fig. 6, one can start with electrostatic theory, and convince oneself that $\hat{n} \times \bar{E} = 0$ on the metal surface with the placement of charges as shown. For PEC surfaces, one can extend them to time-varying dipoles. Again, one can repeat the above exercise for magnetic charges, magnetic dipoles, and PMC surfaces.

One curious note is for a static charge placed near a conductive surface.

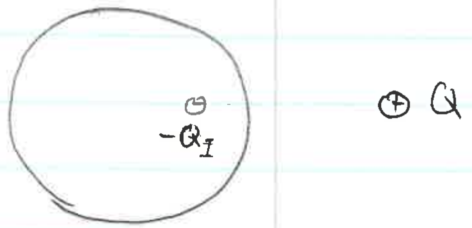
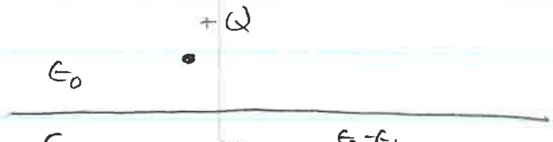


Fig. 7

As worked out in the textbook, a charge of +Q reflects to a charge of $-Q_I$ inside the sphere. However, this cannot be generalized to electrodynamics, because a time-harmonic, charge conserving dipole will reflect to a non-charge conserving dipole.



$$Q_I = \frac{\epsilon_0 - \epsilon_1}{\epsilon_0 + \epsilon_1} Q$$

Fig. 8

When a static charge is placed over a dielectric interface, image theory can be used to find the closed form solution. It can also be extended to multiple interfaces. But image theory cannot be used for the electrodynamic case.