Note Title 8/20/2018

WW01-02 Outline

- Electrostatics
  - Coulumb's Law
  - Gauss's Law

Interested Readers:

- Textbook 1.2
- Textbook 1.3, 1.4

①

$$\boxed{\text{Maxwell's Equations}}$$

Fundamental postulates:

$$\oint_c \vec{E}\cdot d\vec{l} = -\frac{\partial}{\partial t}\iint_s \vec{B}\cdot d\vec{s} - K \quad \text{Faraday's Law}$$

$$\oint_c \vec{H}\cdot d\vec{l} = \frac{\partial}{\partial t}\iint_s \vec{D}\cdot d\vec{s} + I \quad \text{Ampère's law}$$

$$\oiint_s \vec{D}\cdot d\vec{s} = Q \quad \text{Gauss's Law}$$

$$\oiint_s \vec{B}\cdot d\vec{s} = Q_m \quad \text{Gauss's Law-magnetic}$$

$$\boxed{\text{Maxwell's equations in integral form}}$$

Basic quantities:

$\vec{E}$: V/m $\quad$ $\vec{H}$: A/m

$\vec{D}$: C/m^2 $\quad$ $\vec{B}$: webers/m^2

K: V $\quad$ I: A

Q: coulumbs $\quad$ Q_m: webers

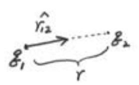
②

$\boxed{\text{Chapter I}}$

Electrostatics

$\boxed{\text{Coulomb's Law}}$

$$\vec{f}_{1\to2} = \frac{q_1 q_2}{4\pi\varepsilon r^2}\hat{r}_{12}$$



$\vec{f}$ (Force): Newton

q (charge): Coulombs

ε (permittivity), farads/meter

r (distance between q_1 and q_2): m

$\hat{r}_{12}$: unit vector pointing ~~away~~ from ~~the~~ charge 1 to charge 2

$$\hat{r}_{12} = \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|} \qquad r = |\vec{r}_2 - \vec{r}_1|$$

$$\therefore\ \vec{f}_{12} = \frac{q_1 q_2 (\vec{r}_2 - \vec{r}_1)}{4\pi\varepsilon|\vec{r}_2 - \vec{r}_1|^3}$$

($\vec{r}_1$, $\vec{r}_2$ are position vectors)

$\hat{r}_{12} \Rightarrow$ unit vector pointing in direction of q_1 to q_2

ϵ. epsilon. ε. ε_0, ϵ_0 free space permittivity

③

$\boxed{\text{Electric Field } \vec{E}}$

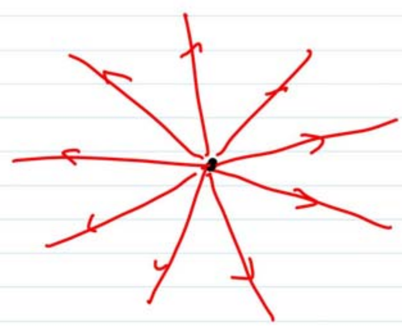
$$\vec{E} = \frac{\vec{f}}{\Delta q} \qquad (V/m)$$

$\vec{E}$ from a point charge q at origin

$$\vec{E} = \frac{q}{4\pi\varepsilon r^2}\hat{r}$$

$\vec{E}(\vec{r})$ from a point charge q at $\vec{r}'$

$$\vec{E}(\vec{r}) = \frac{q(\vec{r} - \vec{r}')}{4\pi\varepsilon|\vec{r} - \vec{r}'|^3}$$



Example

Field of a ring of charge of density ρ_ℓ c/m

Question, what is $\vec{E}$ along z axis?

Remark: If you know $\vec{E}$ due to a point charge, you know $\vec{E}$ due to any charge distribution $\because$ any charge distribution can be decomposed into point charges

④

Gauss's Law

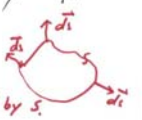
$$\boxed{\oiint_S \vec{D}\cdot d\vec{s} = Q}$$

$\vec{D}$: Electric flux density C/m^2

$\vec{D} = \varepsilon\vec{E}$

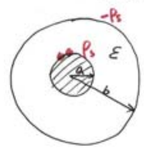
$d\vec{s}$: normal to S pointing outward

Q: Total charge enclosed by S.



Example

Field Between Coaxial Cylinders of unit length



Question: What is $\vec{E}$?

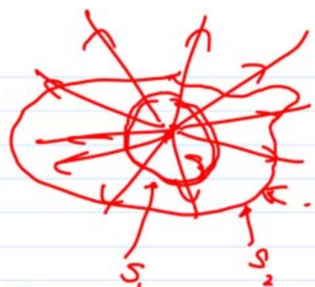
$$\oiint_S \bar{D}\cdot d\bar{S} = Q$$

$\bar{D} = \epsilon_0 \bar{E}$, ϵ_0 = permittivity of vacuum.

$$\bar{D} = \frac{q}{4\pi r^2}$$

Count flux $\underline{\sum_i \bar{D}_i\cdot\Delta\bar{S}_i}$ $\quad \bar{D}_i\cdot\Delta\bar{S}_i$

$\Delta\bar{S}_i = \hat{n}_i \Delta S_i$



$= 4\pi r^2 \bar{D}\cdot\hat{r} = q$

$$\oiint_S \bar{D}\cdot d\bar{S} \stackrel{?}{=} q$$

$S_1 + S_2 = S_T$

$$\oiint_S \bar{D}\cdot d\bar{S} = 0$$

Fluid flow

incompressible

no sink, no sources.

5

Example, Fields of a sphere of uniform charge density

Question: what is $\vec{E}$?