ECE 604, Lecture 2

August 23, 2018

1 Introduction

In this lecture, we will cover the following topics:

- Gauss's Law - Differential form
- Faraday's Law - Differential form
- Constitutive Relations
- Electric Scalar Potential Φ
- Poisson's and Laplace's Equations

Relevant reading in the textbook can be found in:

- Sections 1.7, 1.8, 1.11, 1.12

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2 Gauss's Law, Differential Form

First, we will need to prove Gauss's divergence theorem, namely, that:

$$\iiint_V dV \nabla \cdot \mathbf{D} = \oiint_S \mathbf{D} \cdot d\mathbf{S} \tag{2.1}$$

In the above, $\nabla \cdot \mathbf{D}$ is defined as

$$\nabla \cdot \mathbf{D} = \lim_{\Delta V \to 0} \frac{\oiint_{\Delta S} \mathbf{D} \cdot d\mathbf{S}}{\Delta V} \tag{2.2}$$

and eventually, we will find an expression for it. We know that if $\Delta V \approx 0$ or small, then the above,

$$\Delta V \nabla \cdot \mathbf{D} \approx \oiint_{\Delta S} \mathbf{D} \cdot d\mathbf{S} \tag{2.3}$$

First, we assume that a volume V has been discretized[1] into a sum of small cuboids, where the i-th cuboid has a volume of ΔV_i as shown in Figure 1. Then

$$V \approx \sum_{i=1}^{N} \Delta V_i \tag{2.4}$$

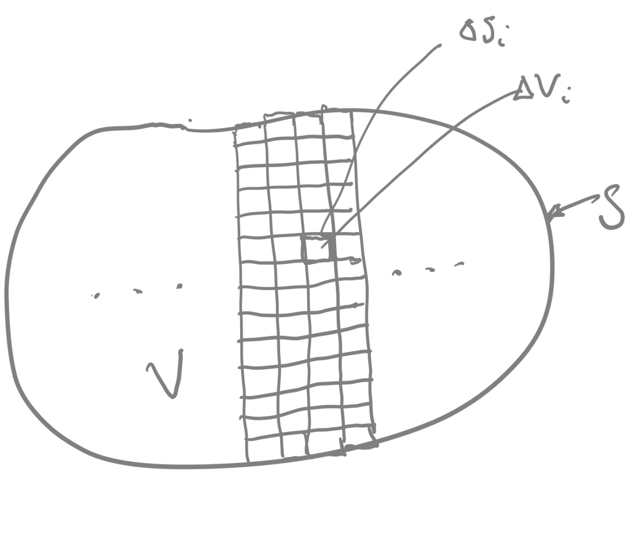


Figure 1: The discretization of a volume V into sum of small volumes ΔV_i each of which is a small cuboid. Stair-casing error occurs near the boundary of the volume V but the error diminishes as $\Delta V_i \to 0$.

[1] Other terms are "tesselated", "meshed", or "gridded".

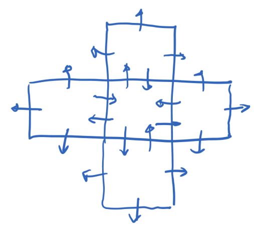


Figure 2: Fluxes from adjacent cuboids cancel each other leaving only the fluxes at the boundary that remain uncancelled. Please imagine that there is a third dimension of the cuboids in this picture where it comes out of the paper.

Then from (2.2),

$$\Delta V_i \nabla \cdot \mathbf{D}_i \approx \oiint_{\Delta S_i} \mathbf{D}_i \cdot d\mathbf{S}_i \tag{2.5}$$

By summing the above over all the cuboids, or over i, one gets

$$\sum_i \Delta V_i \nabla \cdot \mathbf{D}_i \approx \sum_i \oiint_{\Delta S} \mathbf{D}_i \cdot d\mathbf{S}_i \approx \oiint_S \mathbf{D} \cdot d\mathbf{S} \tag{2.6}$$

It is easily seen the the fluxes out of the inner surfaces of the cuboids cancel each other, leaving only the fluxes flowing out of the cuboids at the edge of the volume V as explained in Figure 2. The right-hand side of the above equation (2.6) becomes a surface integral over the surface S except for the stair-casing approximation (see Figure 1). Moreover, this approximation becomes increasingly good as $\Delta V_i \to 0$, or that the left-hand side becomes a volume integral, and we have

$$\iiint_V dV \nabla \cdot \mathbf{D} = \oiint_S \mathbf{D} \cdot d\mathbf{S} \tag{2.7}$$

The above is Gauss's divergence theorem.

Next, we will derive the details of the definition embodied in (2.2). To this end, we evaluate the numerator of the right-hand side carefully, in accordance to Figure 3.

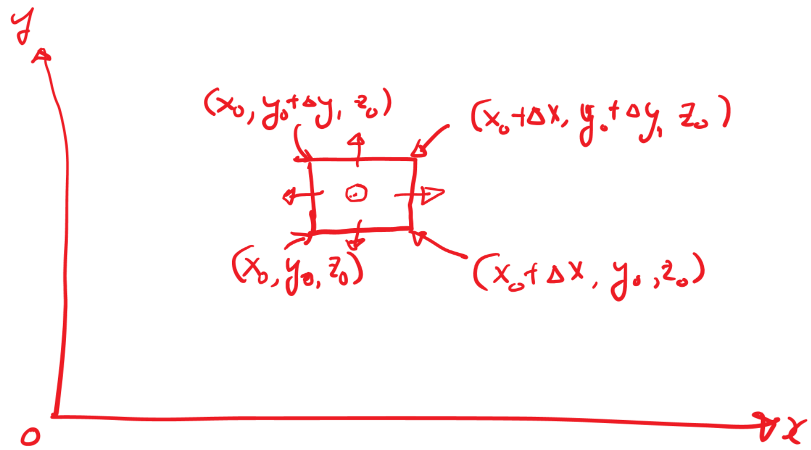


Figure 3: Figure to illustrate the calculation of fluxes from a small cuboid where a corner of the cuboid is located at (x_0, y_0, z_0). There is a third z dimension of the cuboid not shown, and coming out of the paper. Hence, this cuboid, unlike as shown in the figure, has six faces.

Accounting for the fluxes going through all the six faces, assigning the appropriate signs in accordance with the fluxes leaving and entering the cuboid, one arrives at

$$\begin{aligned}\oiint_{\Delta S} \mathbf{D}\cdot d\mathbf{S} \approx &-\mathrm{D}_x(x_0, y_0, z_0)\Delta y\Delta z + \mathrm{D}_x(x_0+\Delta x, y_0, z_0\mathrm{x}_o)\Delta y\Delta z\\ &-\mathrm{D}_y(x_0, y_0, z_0)\Delta x\Delta z + \mathrm{D}_y(x_0, y_0+\Delta y, z_0)\Delta x\Delta z\\ &-\mathrm{D}_z(x_0, y_0, z_0)\Delta x\Delta y + \mathrm{D}_z(x_0, y_0, z_0+\Delta z)\Delta x\Delta y\end{aligned} \tag{2.8}$$

Factoring out the volume of the cuboid $\Delta V = \Delta x\Delta y\Delta z$ in the above, one gets

$$\begin{aligned}\oiint_{\Delta S} \mathbf{D}\cdot d\mathbf{S} \approx \Delta V\,\{&[D_x(x_0+\Delta x, \ldots) - D_x(x_0, \ldots)]/\Delta x\\ &+[D_y(\ldots, y_0+\Delta y, \ldots) - D_y(\ldots, y_0, \ldots)]/\Delta y\\ &+[D_z(\ldots, z_0+\Delta z) - D_z(\ldots, z_0)]/\Delta z\}\end{aligned} \tag{2.9}$$

Or that

$$\frac{\oiint \mathbf{D}\cdot d\mathbf{S}}{\Delta V} \approx \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} \tag{2.10}$$

In the limit when $\Delta V \to 0$, then

$$\lim_{\Delta V\to 0}\frac{\oiint \mathbf{D}\cdot d\mathbf{S}}{\Delta V} = \frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \nabla\cdot\mathbf{D} \tag{2.11}$$

where

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \tag{2.12}$$

$$\mathbf{D} = \hat{x}D_x + \hat{y}D_y + \hat{z}D_z \tag{2.13}$$

The divergence operator $\nabla\cdot$ has its complicated representations in cylindrical and spherical coordinates, a subject that we would not delve into in this course. But they are best looked up at the back of some textbooks on electromagnetics.

Consequently, one gets Gauss's divergence theorem given by

$$\iiint_V dV \nabla \cdot \mathbf{D} = \oiint_S \mathbf{D} \cdot d\mathbf{S} \tag{2.14}$$

By further using Gauss's or Coulomb's law implies that

$$\oiint_S \mathbf{D} \cdot d\mathbf{S} = Q = \iiint dV \varrho \tag{2.15}$$

which is equivalent to

$$\iiint_V dV \nabla \cdot \mathbf{D} = \iiint_V dV \varrho \tag{2.16}$$

When $V \to 0$, we arrive at the pointwise relationship, a relationship at a point in space:

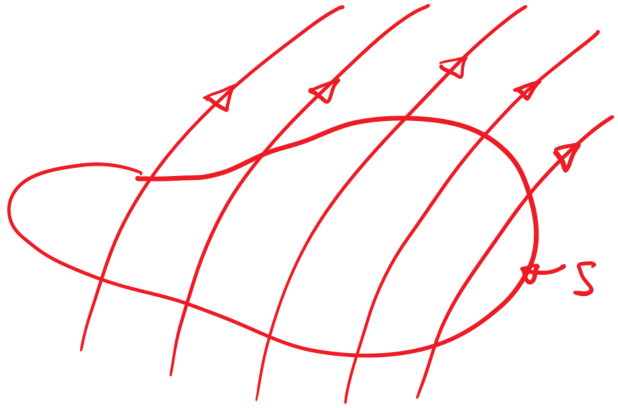
$$\nabla \cdot \mathbf{D} = \varrho \tag{2.17}$$

The physical meaning of divergence is that if $\nabla \cdot \mathbf{D} \neq 0$ at a point in space, it implies that there are fluxes oozing or exuding from that point in space. On the other hand, if $\nabla \cdot \mathbf{D} = 0$, if implies no flux oozing out from that point in space. The flux is termed divergence free. Thus, $\nabla \cdot \mathbf{D}$ is a measure of how much sources or sinks exists for the flux at a point. The sum of these sources or sinks gives the amount of flux leaving or entering the surface that surrounds the sources or sinks.

Moreover, if one were to integrate a divergence-free flux over a volume V, and invoking Gauss's divergence theorem, one gets

$$\oiint_S \mathbf{D} \cdot d\mathbf{S} = 0 \tag{2.18}$$

In such a scenerio, whatever flux that enters the surface S must leave it. In other words, what comes in must go out of the volume V, or that flux is conserved. This is true of incompressible fluid flow, electric flux flow in a source free region, as well as magnetic flux flow, where the flux is conserved.



$$\nabla \cdot \bar{D} = 0 \Rightarrow \oint_S \hat{n} \cdot \bar{D} \, dS = 0$$

Figure 4: In an incompressible flux flow, flux is conserved: whatever flux that enters a volume V must leave the volume V.

2.1 Example

If $\mathbf{D} = (2y^2 + z)\mathbf{a_x} + 4xy\mathbf{a_y} + x\mathbf{a_z}$, find:

1. Volume charge density ρ_v at $(-1, 0, 3)$.
2. Electric flux through the cube defined by

$$0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1.$$

3. Total charge enclosed by the cube.

3 Faraday's Law

3.1 Integral Form

Faraday's law is experimentally motivated. Michael Faraday (1791-1867) was an extraordinary experimentalist who documented this law with meticulous care. It was only decades later that a mathematical description of this law was arrived at. In the static limit, it can be written as

$$\oint \mathbf{E} \cdot d\mathbf{l} = 0 \tag{3.1}$$

The above was the limiting case of the dynamic Faraday's law that states that

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_S \mathbf{B} \cdot d\mathbf{S} \tag{3.2}$$

In the static limit, one assumes that $\frac{\partial}{\partial t} = 0$ and the electrostatic version of the law (3.1) follows. At this juncture, it will be prudent to derive Stokes's theorem.

3.2 Stokes's Theorem–Faraday's Law

The mathematical description of fluid flow was well established before the establishment of electromagnetic theory. Hence, much mathematical description of electromagnetic theory uses the language of fluid. In mathematical notations, Stokes's theorem is

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = \iint_S \nabla \times \mathbf{E} \cdot d\mathbf{S} \tag{3.3}$$

In the above, the contour C is a closed contour, whereas the surface S is not closed.[2]

First, applying Stokes's theorem to a small surface ΔS, we define a curl operator $\nabla\times$ at a point to be

$$\nabla \times \mathbf{E} \cdot \hat{n} = \lim_{\Delta S \to 0} \frac{\oint_{\Delta C} \mathbf{E} \cdot d\mathbf{l}}{\Delta S} \tag{3.4}$$

[2]In other words, C has no boundary whereas S has boundary. A closed surface S has no boundary like when we were proving Gauss's divergence theorem previously.

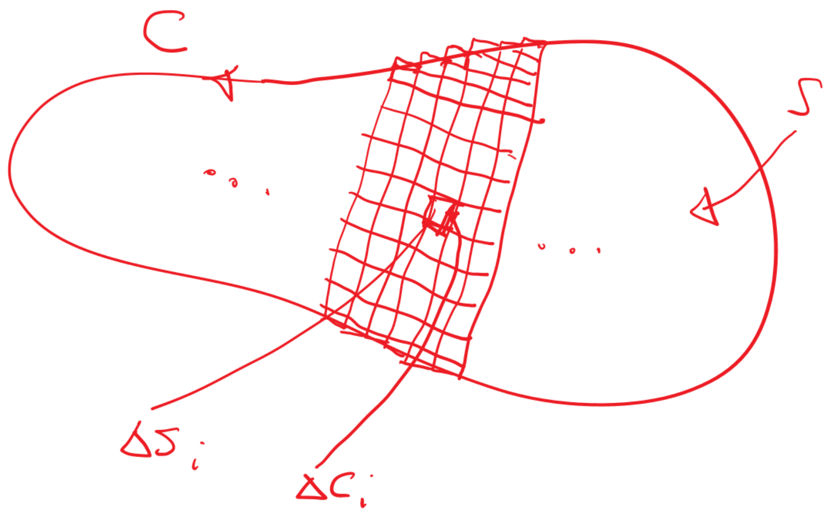


Figure 5: In proving Stokes's theorem, a closed contour C is assumed to enclose an open surface S. Then the surface S is tessellated into sum of small rects as shown. Stair-casing error vanishes in the limit when the rects are made vanishingly small.

First, the surface S enclosed by C is tessellated into sum of small rects (rectangles). Stokes's theorem is then applied to one of these small rects to arrive at

$$\oint_{\Delta C_i} \mathbf{E}_i \cdot d\mathbf{l}_i = (\nabla \times \mathbf{E}_i) \cdot \Delta \mathbf{S}_i \tag{3.5}$$

Next, we sum the above equation over i or over all the small rects to arrive at

$$\sum_i \oint_{\Delta C_i} \mathbf{E}_i \cdot d\mathbf{l}_i = \sum_i \nabla \times \mathbf{E}_i \cdot \Delta \mathbf{S}_i \tag{3.6}$$

Again, on the left-hand side of the above, all the contour integrals over the small rects cancel each other internal to S save for those on the boundary. In the limit when $\Delta S_i \to 0$, the left-hand side becomes a contour integral over the larger contour C, and the right-hand side becomes a surface integral over S. One arrives at Stokes's theorem, which is

$$\oint_C \mathbf{E} \cdot d\mathbf{l} = \iint_S (\nabla \times \mathbf{E}) \cdot d\mathbf{S} \tag{3.7}$$

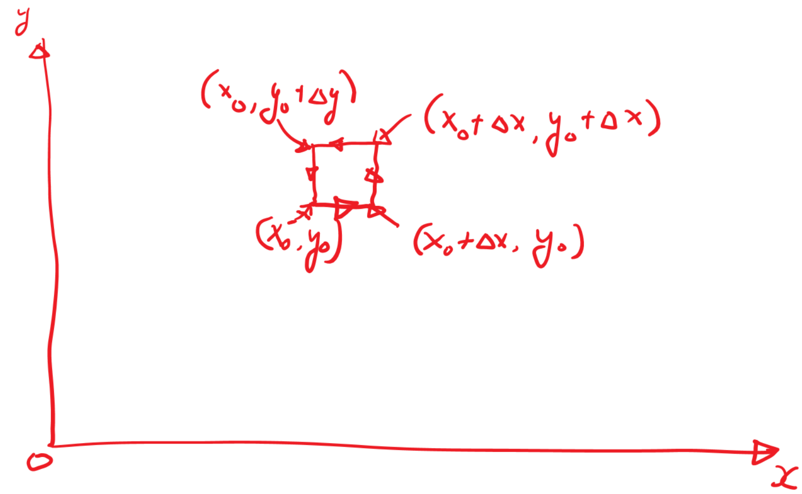


Figure 6: We approximate the integration over a small rect using this figure. There are four edges to this small rect.

Next, we need to prove the details of definition (3.4). Performing the integral over the small rect, one gets

$$\begin{aligned}\oint_{\Delta C} \mathbf{E}\cdot d\mathbf{l} &= E_x(x_0, y_0, z_0)\Delta x + E_y(x_0+\Delta x, y_0, z_0)\Delta y \\ &\quad - E_x(x_0, y_0+\Delta y, z_0)\Delta x - E_y(x_0, y_0, z_0)\Delta y \\ &= \Delta x \Delta y \left(\frac{E_x(x_0, y_0, z_0)}{\Delta y} - \frac{E_x(x_0, y_0+\Delta y, z_0)}{\Delta y} \right. \\ &\quad \left. - \frac{E_y(x_0, y_0, z_0)}{\Delta x} + \frac{E_y(x_0, y_0+\Delta y, z_0)}{\Delta x} \right)\end{aligned} \tag{3.8}$$

We have picked the normal to the incremental surface ΔS to be $\hat{z}$ in the above example, and hence, the above gives rise to the identity that

$$\lim_{\Delta S\to 0} \frac{\oint_{\Delta S} \mathbf{E}\cdot d\mathbf{l}}{\Delta S} = \frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x = \hat{z}\cdot\nabla\times\mathbf{E} \tag{3.9}$$

Picking different $\mathbf{\Delta S}$ with different orientations and normals $\hat{n}$, one gets

$$\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y = \hat{x}\cdot\nabla\times\mathbf{E} \tag{3.10}$$

$$\frac{\partial}{\partial z}E_x - \frac{\partial}{\partial x}E_z = \hat{y}\cdot\nabla\times\mathbf{E} \tag{3.11}$$

Consequently, one gets

$$\nabla \times \mathbf{E} = \hat{x}\left(\frac{\partial}{\partial y}E_z - \frac{\partial}{\partial z}E_y\right) + \hat{y}\left(\frac{\partial}{\partial z}E_x - \frac{\partial}{\partial x}E_z\right) + \hat{z}\left(\frac{\partial}{\partial x}E_y - \frac{\partial}{\partial y}E_x\right) \tag{3.12}$$

where

$$\nabla = \hat{x}\frac{\partial}{\partial x} + \hat{y}\frac{\partial}{\partial y} + \hat{z}\frac{\partial}{\partial z} \tag{3.13}$$

3.3 Faraday's Law, Differential Form

Hence, the differential form of Faraday's law in the static limit is

$$\nabla \times \mathbf{E} = 0 \tag{3.14}$$

The curl operator $\nabla\times$ is a measure of the rotation or the circulation of a field at a point in space. On the other hand, $\oint_{\Delta C} \mathbf{E}\cdot d\mathbf{l}$ is a measure of the circulation of the field $\mathbf{E}$ around the loop formed by C. Again, the curl operator has its complicated representations in other coordinate systems, a subject that will not be discussed in detail here.

It is to be noted that our proof of the Stokes's theorem is for a flat open surface S, and not for a general curved open surface. Since all curved surfaces can be tessellated into a union of flat triangular surfaces, the generalization of the above proof to curved surface is straightforward. An example of such a triangulation of a curved surface into a union of triangular surfaces is shown in Figure 7.

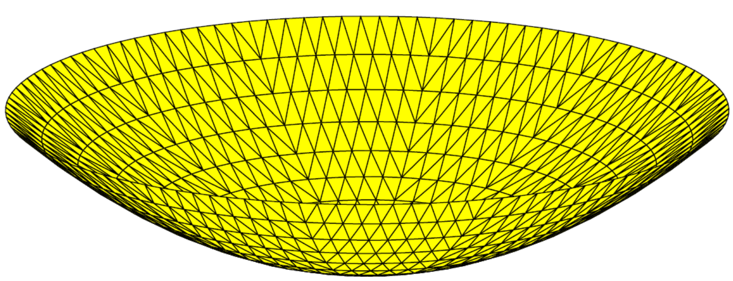


Figure 7: An arbitrary curved surface can be triangulated with flat triangular patches. The triangulation can be made arbitrarily accurate by making the patches arbitrarily small.

3.4 Example

Suppose $\mathbf{E} = \hat{\mathbf{x}}3y + \hat{\mathbf{y}}x$, calculate $\int \mathbf{E} \cdot d\boldsymbol{l}$ along a straight ine in the $x - y$ plane joining (0,0) to (3,1).

4 Constitutive Relations

The constitution relation between **D** and **E** in free space is

$$\mathbf{D} = \varepsilon_0 \mathbf{E} \tag{4.1}$$

When material medium is present, one has to add the contribution to **D** by the polarization density **P** which is a dipole density.[3] Then

$$\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P} \tag{4.2}$$

The second term above is the contribution to the electric flux due to the polarization density of the medium. It is due to the little dipole contribution due to the polar nature of the atoms or molecules that make up a medium.

By the same token, the first term $\varepsilon_0 \mathbf{E}$ can be thought of as the polarization density contribution of vacuum. Vacuum, though represents nothingness, has electrons and positrons, or electron-positron pairs lurking in it. Electron is matter, whereas positron is anti-matter. In the quiescent state, they represent nothingness, but they can be polarized by an electric field **E**. That also explains why electromagnetic wave can propagate through vacuum.

For linear media, $\mathbf{P} = \varepsilon_0 \chi_0 \mathbf{E}$

$$\begin{aligned} \mathbf{D} &= \varepsilon_0 \mathbf{E} + \varepsilon_0 \chi_0 \mathbf{E} \\ &= \varepsilon_0 (1 + \chi_0) \mathbf{E} = \varepsilon \mathbf{E} \end{aligned} \tag{4.3}$$

In other words, for linear material media, one can replace the vacuum permittivity ε_0 with an effective permittivity ε.

In free space:

$$\varepsilon = \varepsilon_0 = 8.854 \times 10^{-12} \approx \frac{10^{-8}}{36\pi} \text{ F/m} \tag{4.4}$$

4.1 Anisotropic Media

For anisotropic media,

$$\begin{aligned} \mathbf{D} &= \varepsilon_0 \mathbf{E} + \varepsilon_0 \overline{\boldsymbol{\chi}}_0 \cdot \mathbf{E} \\ &= \varepsilon_0 (\overline{\mathbf{I}} + \overline{\boldsymbol{\chi}}_0) \cdot \mathbf{E} = \overline{\boldsymbol{\varepsilon}} \cdot \mathbf{E} \end{aligned} \tag{4.5}$$

[3]Note that a dipole moment is given by $Q\ell$ where Q is its charge in coulomb and ℓ is its length in m. Hence, dipole density, or polarization density as dimension of coulomb/m^2, which is the same as that of electric flux **D**.

In the above, $\overline{\varepsilon}$ is a 3×3 matrix also known as a tensor in electromagnetics. The above implies that $\mathbf{D}$ and $\mathbf{E}$ do not necessary point in the same direction, the meaning of anisotropy. A tensor is often associated with a physical notion, whereas a matrix is not.

Similarly for conductive media,

$$\mathbf{J} = \sigma \mathbf{E}, \tag{4.6}$$

For anisotropic conductive media, one can have

$$\mathbf{J} = \overline{\boldsymbol{\sigma}} \cdot \mathbf{E}, \tag{4.7}$$

Here, again, due to the tensorial nature of the conductivity $\overline{\boldsymbol{\sigma}}$, the electric current $\mathbf{J}$ and electric field $\mathbf{E}$ do not necessary point in the same direction.

4.2 Bi-anisotropic Media

In the above, the electric flux $\mathbf{D}$ depends on the electric field $\mathbf{E}$. The concept of constitutive relation can be extended to where $\mathbf{D}$ depends on both $\mathbf{E}$ and $\mathbf{H}$. In general, one can write

$$\mathbf{D} = \overline{\varepsilon} \cdot \mathbf{E} + \overline{\boldsymbol{\xi}} \cdot \mathbf{H} \tag{4.8}$$

A medium where the electric flux is dependent on both $\mathbf{E}$ and $\mathbf{H}$ is known as a bi-anisotropic medium.

4.3 Inhomogeneous Media

If any of the $\overline{\varepsilon}$ or $\overline{\boldsymbol{\xi}}$ is a function of position $\mathbf{r}$, the medium is known as an inhomogeneous medium or a heterogeneous medium. There are usually no simple solution to such media.

5 Electric Scalar Potential Φ

We have shown that Faraday's law in the static limit is

$$\nabla \times \mathbf{E} = 0 \tag{5.1}$$

One way to satisfy the above is to let $\mathbf{E} = -\nabla\Phi$ because of the identity $\nabla \times \nabla = 0$.[4] Alternatively, one can assume that $\mathbf{E}$ is a constant. But we usually are interested in solutions that vanish at infinity, and hence, the latter is not a viable solution. Therefore, we let

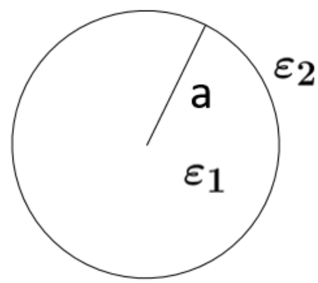
$$\mathbf{E} = -\nabla\Phi \tag{5.2}$$

5.1 Example

Fields of a sphere of uniform charge density ρ:

Assuming that $\Phi|_{r=\infty} = 0$, what is Φ at $r \leq a$? And Φ at $r > a$?

[4]One an easily go through the algebra to convince oneself of this.



6 Poisson's and Laplace's Equations

As a consequence of the above,

$$\nabla \cdot \mathbf{D} = \varrho \Rightarrow \nabla \cdot \varepsilon \mathbf{E} = \varrho \Rightarrow -\nabla \cdot \varepsilon \nabla \Phi = \varrho \tag{6.1}$$

In the last equation above, if ε is a constant of space, or independent of $\mathbf{r}$, then one arrives at the simple Poisson's equation, which is a partial differential equation

$$\nabla^2 \Phi = -\frac{\varrho}{\varepsilon} \tag{6.2}$$

where

$$\nabla^2 = \nabla \cdot \nabla = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$$

If $\varrho = 0$, or if we are in a source-free region,

$$\nabla^2 \Phi = 0 \tag{6.3}$$

which is the Laplace's equation.

For a point source, we know that

$$\mathbf{E} = \frac{q}{4\pi\varepsilon r^2}\hat{r} = -\nabla\Phi \tag{6.4}$$

From the above, we deduce that[5]

$$\Phi = \frac{q}{4\pi\varepsilon r} \tag{6.5}$$

Therefore, we know the solution to Poisson's equation (6.2) when the source ϱ represents a point source. Since this is a linear equation, we can use the principle of linear superposition to find the solution when ϱ is arbitrary.

A point source located at $\mathbf{r}'$ is described by a charge density as

$$\varrho(\mathbf{r}) = q\delta(\mathbf{r} - \mathbf{r}') \tag{6.6}$$

[5]One can always take the gradient or ∇ of Φ to verify this.

where $\delta(\mathbf{r}-\mathbf{r}')$ is a short-hand notation for $\delta(x-x')\delta(y-y')\delta(z-z')$. Therefore, from (6.2), the corresponding partial differential equation for a point source is

$$\nabla^2\Phi(\mathbf{r}) = -\frac{q\delta(\mathbf{r}-\mathbf{r}')}{\varepsilon} \tag{6.7}$$

The solution to the above equation has to be

$$\Phi(\mathbf{r}) = \frac{q}{4\pi\varepsilon|\mathbf{r}-\mathbf{r}'|} \tag{6.8}$$

where (6.5) is for a point source at the origin, but (6.8) is for a point source located and translated to $\mathbf{r}'$.

6.1 Green's Function

Let us define a partial differential equation given by

$$\nabla^2 g(\mathbf{r}-\mathbf{r}') = -\delta(\mathbf{r}-\mathbf{r}') \tag{6.9}$$

The above is similar to Poisson's equation with a point source on the right-hand side as in (6.7). But such a solution, a response to a point source, is called the Green's function.[6] By comparing equations (6.7) and (6.9), then making use of (6.8), it is deduced that the Green's function is

$$g(\mathbf{r}-\mathbf{r}') = \frac{1}{4\pi|\mathbf{r}-\mathbf{r}'|} \tag{6.10}$$

An arbitrary source can be expressed as

$$\varrho(\mathbf{r}) = \iiint_v dV'\varrho(\mathbf{r}')\delta(\mathbf{r}-\mathbf{r}') \tag{6.11}$$

The above is just the statement that an arbitrary charge distribution $\varrho(\mathbf{r})$ can be expressed as a linear superposition of point sources $\delta(\mathbf{r}-\mathbf{r}')$. Using the above in (6.2), we have

$$\nabla^2\Phi(\mathbf{r}) = -\frac{1}{\varepsilon}\iiint_V dV'\varrho(r')\delta(\mathbf{r}-\mathbf{r}') \tag{6.12}$$

We can let

$$\Phi(\mathbf{r}) = \frac{1}{\varepsilon}\iiint_V dV' g(\mathbf{r}-\mathbf{r}')\varrho(\mathbf{r}') \tag{6.13}$$

By substituting the above into the left-hand side of (6.12), exchanging order of integration and differentiation, and then making use of equation (6.2), it can be shown that (6.13) indeed satisfies (6.5). The above is just a convolutional

[6] George Green (1793-1841), the son of a Nottingham miller, was self-taught, but his work has a profound impact in our world.

integral. Hence, the potential $\Phi(\mathbf{r})$ due to an arbitrary source distribution $\varrho(\mathbf{r})$ can be found by using convolution, namely,

$$\Phi(\mathbf{r}) = \frac{1}{4\pi\varepsilon} \iiint_V \frac{\varrho(\mathbf{r}')}{|\mathbf{r}-\mathbf{r}'|} dV' \tag{6.14}$$

In a nutshell, the solution of Poisson's equation when it is driven by an arbitrary source ϱ, is the convolution of the source with the Green's function, a point source response.

6.2 Example

A capacitor has two parallel plates attached to a battery, what is **E** field inside the capacitor?