ECE 604, Lecture 19

November 6, 2018

In this lecture, we will cover the following topics:

- Hollow Waveguide:
  - TE Case
  - TM Case
- Rectangular Waveguides:
  - TE Modes
  - TM Modes
- Circular Waveguides
  - TE Modes
  - TM Modes

Additional Reading:

- Sections 6.6, 6.8 of Ramo, Whinnery, and Van Duzer.
- Lecture Notes 11, Prof. Dan Jiao.
- Section 2.5, J.A. Kong, Electromagnetic Wave Theory.
- Lecture 18, ECE 350X.

You should be able to do the homework by reading the lecture notes alone. Additional reading is for references.

Printed on November 13, 2018 at 15:12: W.C. Chew and D. Jiao.

1 Hollow Waveguides

Hollow waveguides are useful for high-power microwaves. Air has a higher breakdown voltage compared to most materials, and hence, could be a good medium for propagating high power microwave. Also, they are sufficiently shielded from the rest of the world so that interference from other sources is minimized. Also, for radio astronomy, they can provide a low-noise system immune to interference. Air generally has less loss than materials, and loss is often the source of thermal noise. A low loss waveguide is also a low noise waveguide.

Many waveguide problems can be solved in closed form. An example is the coaxial waveguide previously discussed. But there are many other waveguide problems that have closed form solutions. Closed form solutions to Laplace and Helmholtz equations are obtained by the separation of variables method. The separation of variables method works only for separable coordinate systems. There are 11 separable coordinates for Helmholtz equations, but 13 for Laplace equation. Some examples of separable coordinate systems are cartesian, cylindrical, and spherical coordinates. But these three coordinates are about all we need to know for solving many engineering problems. More complicated cases are often handled with numerical methods.

When a waveguide has a center conductor or two conductors like a coaxial cable, it can support a TEM wave where both the electric field and the magnetic field are orthogonal to the direction of propagation. The uniform plane wave is a TEM wave, for instance.

However, when the waveguide is hollow or is filled completely with a homogeneous medium, it can only support a TE_z or TM_z wave like the case of a layered medium. For a TE_z wave (or TE wave), $E_z = 0$, $H_z \neq 0$ while for a TM_z wave (or TM wave), $H_z = 0$, $E_z \neq 0$. These classes of problems can be decomposed into two scalar problems like the layerd medium case, by using the pilot potential method.

1.1 TE Case ($E_z = 0$, $H_z \neq 0$)

In this case, the field inside the waveguide is TE to z. We can write the **E** field as

$$\mathbf{E}(\mathbf{r}) = \nabla \times \hat{z}\Psi_h(\mathbf{r}) \tag{1.1}$$

Equation (1.1) will guarantee that $E_z = 0$ due to its construction. Here, $\Psi_h(\mathbf{r})$ is a scalar potential and $\hat{z}$ is the pilot vector.[1]

The waveguide is assumed source free and filled with a lossless, homogeneous material. Eq. (**??**) also satisfies the source-free condition since $\nabla \cdot \mathbf{E} = 0$. And hence, from Maxwell's equations, it follows that

$$(\nabla^2 + \beta^2)\mathbf{E}(\mathbf{r}) = 0 \tag{1.2}$$

[1] It "pilots" the field so that it is transverse to z.

where $\beta^2 = \omega^2 \mu \varepsilon$. Substituting (1.1) into (1.2), we get

$$(\nabla^2 + \beta^2)\nabla \times \hat{z}\Psi_h(\mathbf{r}) = 0 \tag{1.3}$$

In the above, we assume that $\nabla^2 \nabla \times \hat{z}\Psi = \nabla \times \hat{z}(\nabla^2 \Psi)$, or that these operators commute.[2] Then it follows that

$$\nabla \times \hat{z}(\nabla^2 + \beta^2)\Psi_h(\mathbf{r}) = 0 \tag{1.4}$$

Thus, if

$$(\nabla^2 + \beta^2)\Psi_h(\mathbf{r}) = 0 \tag{1.5}$$

then (1.4) is satisfied, and so is (1.2). Hence, the $\mathbf{E}$ field constructed with (1.1), where $\Psi_h(\mathbf{r})$ satisfies (1.5) satisfies Maxwell's equations.

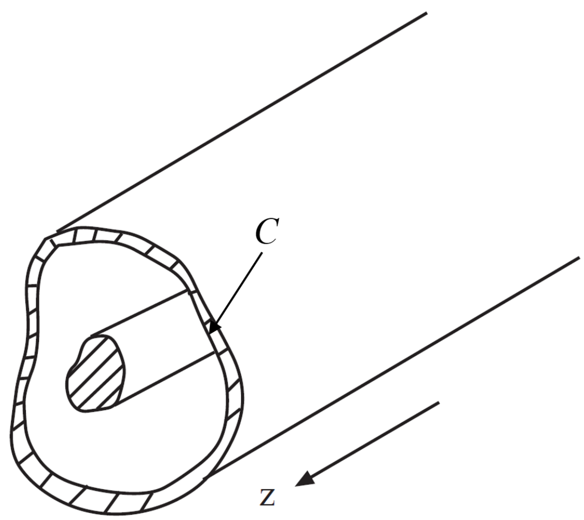

Figure 1:

Next, we look at the boundary condition for $\Psi_h(\mathbf{r})$. The boundary condition for $\mathbf{E}$ is that $\hat{n} \times \mathbf{E} = 0$ on C, the wall of the waveguide. But from (1.1), using the back-of-the-cab (BOTC) formula,

$$\hat{n} \times \mathbf{E} = \hat{n} \times (\nabla \times \hat{z}\Psi_h) = -\hat{n} \cdot \nabla \Psi_h = 0 \tag{1.6}$$

[2]This is a mathematical parlance, and a commutator is defined to be $[A, B] = AB - BA$ for two operators A and B. If these two operators commute, then $[A, B] = 0$.

In applying the BOTC formula, one has to be mindful that ∇ operates on a function, and the function Ψ_h is always placed to the right of the ∇ operator.

In the above $\hat{n}\cdot\nabla = \hat{n}\cdot\nabla_s$ where $\nabla_s = \hat{x}\frac{\partial}{\partial x}+\hat{y}\frac{\partial}{\partial y}$ since $\hat{n}$ has no z component. The boundary condition (1.6) becomes

$$\hat{n}\cdot\nabla_s\Psi_n = \partial_n\Psi_n = 0 \text{ on } C \tag{1.7}$$

which is also known as the homogeneous Neumann boundary condition.

Furthermore, in a waveguide, just as in a transmission line case, we are looking for traveling solutions of the form $\exp(\mp j\beta_z z)$ for (1.5), or that

$$\Psi_h(\mathbf{r}) = \Psi_{hs}(\mathbf{r}_s)e^{\mp j\beta_z{}^z} \tag{1.8}$$

where $\mathbf{r}_s = \hat{x}x + \hat{y}y$, or in short, $\Psi_{hs}(\mathbf{r}_s) = \Psi_{hs}(x, y)$. With this assumption, $\frac{\partial^2}{\partial z^2} \to -\beta_z{}^2$, and (1.5) becomes even simpler, namely,

$$({\nabla_s}^2 + \beta^2 - {\beta_z}^2)\Psi_{ns}(\mathbf{r}_s) = ({\nabla_s}^2 + \beta_s^2)\Psi_{ns}(\mathbf{r}_s) = 0 \ , \quad \partial_n\Psi_{ns}(\mathbf{r}_s) = 0, \text{ on } C \tag{1.9}$$

where $\beta_s^2 = \beta^2 - \beta_z^2$. The above is a boundary value problem for a 2D waveguide problem. The above 2D wave equation is also known as the reduced wave equation.

1.2 TM Case ($E_z \neq 0$, $H_z = 0$)

Repeating similar treatment for TM waves, the TM magnetic field is

$$\mathbf{H} = \nabla\times\hat{z}\Psi_e(\mathbf{r}) \tag{1.10}$$

where

$$(\nabla^2 + \beta^2)\Psi_e(\mathbf{r}) = 0 \tag{1.11}$$

The corresponding $\mathbf{E}$ field is obtained by taking the curl of the magnetic field in (1.10), and thus the $\mathbf{E}$ field is proportional to

$$\mathbf{E} \sim \nabla\times\nabla\times\hat{z}\Psi_e(\mathbf{r}) = \nabla\nabla\cdot(\hat{z}\Psi_e) - \nabla^2\hat{z}\Psi_e = \nabla\frac{\partial}{\partial z}\Psi_e + \hat{z}\beta^2\Psi_e \tag{1.12}$$

Taking the z component of the above, we get

$$E_z \sim \frac{\partial^2}{\partial z^2}\Psi_e + \beta^2\Psi_e \tag{1.13}$$

Assuming that

$$\Psi_e \sim e^{\mp j\beta_z{}^z} \tag{1.14}$$

then in (1.13), $\partial^2/\partial z^2 \to -\beta_z^2$, and

$$E_z \sim (\beta^2 - \beta_z)\Psi_e \tag{1.15}$$

Therefore, if

$$\Psi_e(\mathbf{r}) = 0 \text{ on } C, \tag{1.16}$$

then,

$$E_z(\mathbf{r}) = 0 \text{ on } C \tag{1.17}$$

One can further show from (1.12) that the homogeneous Dirichlet boundary condition also implies that the other components of tangential $\mathbf{E}$ are zero, namely $\hat{n} \times \mathbf{E} = 0$ on the waveguide wall C.

Thus, with some manipulation, the boundary value problem related to equation (1.11) reduces to a simpler 2D problem, i.e.,

$$(\nabla_s{}^2 + \beta_s^2)\Psi_{es}(\mathbf{r}_s) = 0 \tag{1.18}$$

$$\Psi_{es}(\mathbf{r}_s) = 0,\ \mathbf{r}_s \text{ on } C \tag{1.19}$$

the homogenous Dirichlet boundary condition where we have assumed that

$$\Psi_e(\mathbf{r}) = \Psi_{es}(\mathbf{r}_s)e^{\mp j\beta_z{}^z} \tag{1.20}$$

We can solve some simple waveguides as illustrations.

2 Rectangular Waveguides

Rectangular waveguides are among the simplest waveguides to analyze because closed form solutions exist in cartesian coordinates. One can imagine traveling waves in the xy directions bouncing off the walls of the waveguide causing standing waves to exist inside the waveguide.

It turns out that not all electromagnetic waves can be guided by a hollow waveguide. Only when the wavelength short enough, or the frequency high enough that an electromagnetic wave can be guided by a waveguide.

2.1 TE Modes (H Mode)

The scalar potential $\Psi_{hs}(\mathbf{r}_s)$ satisfies

$$(\nabla_s{}^2 + \beta_s{}^2)\Psi_{hs}(\mathbf{r}_s) = 0, \qquad \frac{\partial}{\partial n}\Psi_{hs}(\mathbf{r}_s) = 0 \quad \text{on } C \tag{2.1}$$

where $\beta_s{}^2 = \beta^2 - \beta_z{}^2$. A solution for $\Psi_{hs}(x, y)$ is then

$$\Psi_{hs}(x, y) = A\cos(\beta_x x)\cos(\beta_y y) \tag{2.2}$$

where $\beta_x{}^2 + \beta_y^2 = \beta_s^2$. One can see that the above is the representation of standing waves in the xy directions. It is quite clear that $\Psi_{hs}(x, y)$ satisfies equation (2.1). Furthermore, cosine functions, rather than sine functions are chosen that

that the above satisfies the homogenous Neumann boundary condition at $x = 0$, and $y = 0$ surfaces.

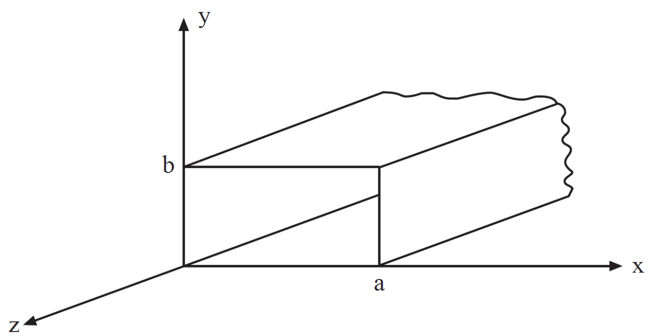

Figure 2:

To further satisfy the boundary condition at $x = a$, and $y = b$ surfaces, it is necessary that the boundary condition for eq. (1.7) is satisfied or that

$$\partial_x \Psi_{hs}(x, y)\big|_{x=a} \sim \sin(\beta_x a) \cos(\beta_y y) = 0, \tag{2.3}$$

$$\partial_y \Psi_{hs}(x, y)\big|_{y=b} \sim \cos(\beta_x x) \sin(\beta_y b) = 0, \tag{2.4}$$

The above put constraints on β_x and β_y, implying that $\beta_x a = m\pi$, $\beta_y b = n\pi$ where m and n are integers. Hence (2.2) becomes

$$\Psi_{hs}(x, y) = A \cos\left(\frac{m\pi}{a} x\right) \cos\left(\frac{n\pi}{b} y\right) \tag{2.5}$$

where

$$\beta_x^2 + \beta_y^2 = \left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 = \beta_s^2 = \beta^2 - {\beta_z}^2 \tag{2.6}$$

The above condition on β_s^2 is the guidance condition for the mode in the waveguide. Furthermore,

$$\beta_z = \sqrt{\beta^2 - \left(\frac{m\pi}{a}\right)^2 - \left(\frac{n\pi}{b}\right)^2} \tag{2.7}$$

Furthermore, from (2.7), when

$$\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2 > \beta^2 = \omega^2 \mu \varepsilon \tag{2.8}$$

β_z becomes pure imaginary and the mode cannot propagate or become evanescent in the z direction.[3] For fixed m and n, the frequency at which the above

[3] We have seen this happening in a plasma medium earlier.

happens is called the cutoff frequency of the TE_{mn} mode of the waveguide. It is given by

$$\omega > \omega_{mn,c} = \frac{1}{\sqrt{\mu\varepsilon}}\sqrt{\left(\frac{m\pi}{a}\right)^2 + \left(\frac{n\pi}{b}\right)^2} \tag{2.9}$$

A corresponding cutoff wavelength is then

$$\lambda < \lambda_{mn,c} = \frac{2}{[\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2]^{1/2}} \tag{2.10}$$

When $m = n = 0$, then $\Psi_h(\mathbf{r})$ is a function independent of x and y. Then $\mathbf{E}(\mathbf{r}) = \nabla \times \hat{z}\Psi_h(\mathbf{r}) = \nabla_s \times \hat{z}\Psi_h(\mathbf{r}) = 0$. It turns out the only way for $H_z \neq 0$ is for $\mathbf{H}(\mathbf{r}) = \hat{z}H_0$ which is a static field in the waveguide. This is not a very interesting mode, and thus TE_{00} propagating mode is assumed not to exist. So the TE_{mn} modes cannot have both $m = n = 0$. Thus, the TE_{10} mode, when $a > b$, is the mode with the lowest cutoff frequency or longest cutoff wavelength.

For the TE_{10} mode, (2.10) reduces to

$$\lambda < \lambda_{10,c} = 2a \tag{2.11}$$

The above has the nice physical meaning that the wavelength has to be smaller than $2a$ in order for the mode to fit into the waveguide. As a mnemonic, we can think that photons have "sizes", corresponding to its wavelength. Only when its wavelength is small enough can the photons go into (or be guided by) the waveguide. The TE_{10} mode, when $a > b$, is also the mode with the lowest cutoff frequency or longest cutoff wavelength.

It is seen with the above analysis, when the wavelength is short enough, or frequency is high enough, many modes can be guided. Each of these modes has a different group and phase velocity. But for most applications, a single guided mode only is desirable. Hence, the knowledge of the cutoff frequencies of the fundamental mode (the mode with the lowest cutoff frequency) and the next higher mode is important. This allows one to pick a frequency window within which only a single mode can propagate in the waveguide.

2.2 TM Modes (E Modes)

The above exercise can be repeated for the TM mode. The scalar wave function for the TM modes is

$$\Psi_{es}(x, y) = A \sin\left(\frac{m\pi}{a}x\right) \sin\left(\frac{n\pi}{b}y\right) \tag{2.12}$$

Here, sine functions are chosen for the standing waves, and the chosen values of β_x and β_y ensure that the homogeneous Dirichlet boundary condition is satisfied on the waveguide wall. Neither of the m and n can be zero, lest the field is zero. In this case, both $m > 0$, and $n > 0$ are needed. Thus, the lowest TM mode is the TM_{11} mode. The corresponding cutoff frequencies and cutoff wavelengths are the same as the TE_{mn} mode. Also, TE_{11} and TM_{11} modes have the same cutoff frequency. These modes are degenerate in this case.

Plots of the fields of different rectangular waveguide modes are shown in Figure 3. Notice that for higher m's and n's, the transverse wavelengths are getting shorter, implying that β_x and β_y are getting larger. Hence, only high frequency fields can generate such modes. Notice also how the electric field and magnetic field curl around each other. Since $\nabla \times \mathbf{H} = j\omega\varepsilon\mathbf{E}$ and $\nabla \times \mathbf{E} = -j\omega\mu\mathbf{H}$, they do not curl around each other "immediately" but with a phase delay.

Therefore, the $\mathbf{E}$ and $\mathbf{H}$ fields do not curl around each other at one location, but at a displaced location due to the $\pi/2$ phase difference. This is shown in Figure 4.

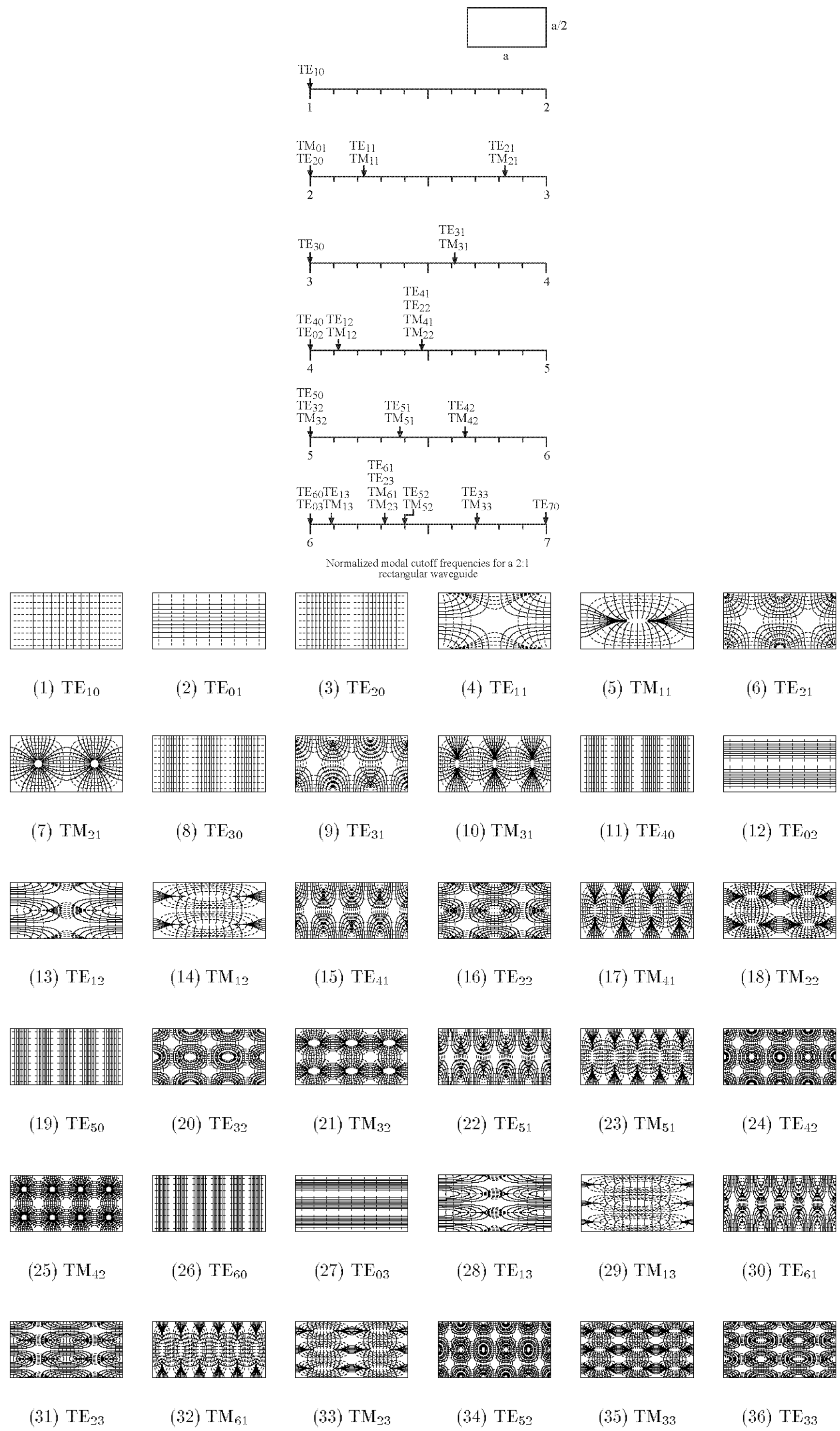

Figure 3: Courtesy of Andy Greenwood. Original plots published in Lee, Lee, and Chuang.

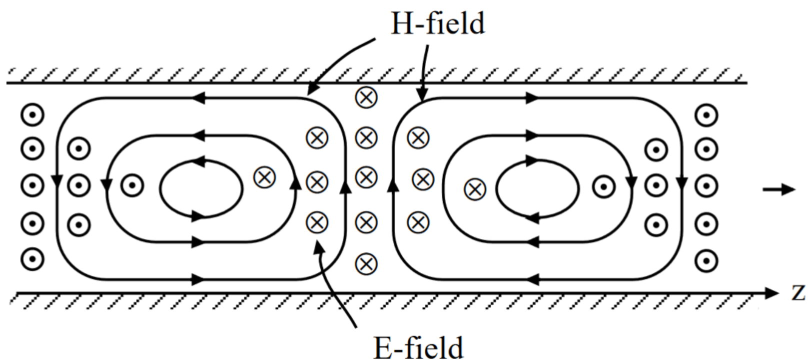

Figure 4:

3 Circular Waveguides

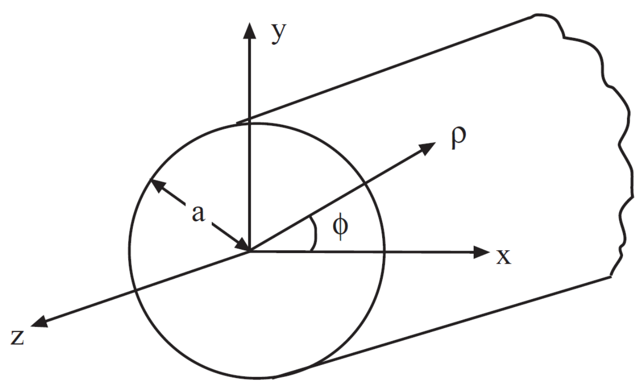

Figure 5:

3.1 TE Case

For a circular waveguide, it is best to express the Laplacian operator, ${\nabla_s}^2 = \nabla_s \cdot \nabla_s$, in cylindrical coordinates. Doing a table lookup, $\nabla_s \Psi = \hat{\rho} \frac{\partial}{\partial \rho} \Psi + \hat{\phi} \frac{1}{\rho} \frac{\partial}{\partial \phi}$, $\nabla_s \cdot \mathbf{A} = \frac{1}{\rho} \frac{\partial}{\partial \rho} \rho A_\rho + \frac{1}{\rho} \frac{\partial}{\partial \phi} A_\phi$. Then

$$({\nabla_s}^2 + {\beta_s}^2)\Psi_{hs} = \left(\frac{1}{\rho} \frac{\partial}{\partial \rho} \rho \frac{\partial}{\partial \rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial \phi^2} + {\beta_s}^2 \right) \Psi_{hs}(\rho, \phi) = 0 \tag{3.1}$$

Using separation of variables , we let

$$\Psi_{hs}(\rho, \phi) = B_n(\beta_s \rho) e^{\pm jn\phi} \tag{3.2}$$

Then $\frac{\partial^2}{\partial \rho^2} \to -n^2$, and (3.1) becomes an ordinary differential equation which is

$$\left(\frac{1}{\rho} \frac{d}{d\rho} \rho \frac{d}{d\rho} - \frac{n^2}{\rho^2} + {\beta_s}^2 \right) B_n(\beta_z \rho) \tag{3.3}$$

The above is known as the Bessel equation whose solutions are special functions. These special functions are $J_n(x)$, $N_n(x)$, $H_n^{(1)}(x)$, and $H_n^{(2)}(x)$ where n is the order, and x is the argument.[4] Since this is a second order ordinary differential equation, only two of the four commonly encountered solutions of Bessel equation are independent. Therefore, they can be expressed then in term of each other. Their relationships are shown below:

Bessel, $$J_n(\beta_s \rho) = \frac{1}{2}[{H_n}^{(1)}(\beta_s \rho) + {H_n}^{(2)}(\beta_s \rho)] \tag{3.4}$$

Neumann, $$N_n(\beta_s \rho) = \frac{1}{2j}[{H_n}^{(1)}(\beta_s \rho) - {H_n}^{(2)}(\beta_s \rho)] \tag{3.5}$$

Hankel–First kind, $${H_n}^{(1)}(\beta_s \rho) = J_n(\beta_s \rho) + jN_n(\beta_s \rho) \tag{3.6}$$

Hankel–second kind, $${H_n}^{(2)}(\beta_s \rho) = J_n(\beta_s \rho) - jN_n(\beta_s \rho) \tag{3.7}$$

It can be shown that

$${H_n}^{(1)}(x) \sim \sqrt{\frac{2}{\pi x}} e^{jx - j(n+\frac{1}{2})\frac{\pi}{2}}, \quad x \to \infty \tag{3.8}$$

$${H_n}^{(2)}(x) \sim \sqrt{\frac{2}{\pi x}} e^{-jx + j(n+\frac{1}{2})\frac{\pi}{2}}, \quad x \to \infty \tag{3.9}$$

They correspond to traveling wave solutions when $\rho \to \infty$. Since $J_n(x)$ and $N_n(x)$ are linear superpositions of these traveling wave solutions, they correspond to standing wave solutions. Moreover, $N_n(x)$, ${H_n}^{(1)}(x)$, and ${H_n}^{(2)}(x) \to \infty$ when $x \to 0$. Since the field has to be regular when $\rho \to 0$ at the center of the waveguide, the only viable solution for the waveguide is that $B_n(\beta_s \rho) = AJ_n(\beta_s \rho)$. Thus

$$\Psi_{hs}(\rho, \phi) = AJ_n(\beta_s \rho) e^{\pm jn\phi} \tag{3.10}$$

The homogeneous Neumann boundary condition on the waveguide wall then translates to

$$\frac{d}{d\rho} J_n(\beta_s \rho) = 0, \quad \rho = a \tag{3.11}$$

[4] Some textbooks use $Y_n(x)$ for Neumann function.

Defining $J_n{}'(x) = \frac{d}{dx} J_n(x)$, the above is the same as

$$J_n{}'(\beta_s a) = 0 \tag{3.12}$$

Plots of Bessel functions and their derivatives are shown in FIgure 7. The above are the zeros of the derivative Bessel function and they are tabulated in many textbooks. The m-th zero of $J_n{}^{`}(x)$ is denoted to be β_{nm} in many books,[5] and some of them are also shown in Figure 8, and hence, the guidance condition for β_s is

$$\beta_s = \beta_{nm}/a \tag{3.13}$$

for the TE_{nm} mode. Using the fact that $\beta_z^2 + \beta_s^2 = \beta^2$, the corresponding cutoff frequency of the TE_{nm} mode is

$$\omega_{nm,c} = \frac{1}{\sqrt{\mu\varepsilon}} \frac{\beta_{nm}}{a} \tag{3.14}$$

with the corresponding cutoff wavelength to be

$$\lambda_{nm,c} = \frac{2\pi}{\beta_{nm}} a \tag{3.15}$$

3.2 TM Case

The corresponding boundary value problem for this case is

$$\left(\frac{1}{\rho} \frac{\partial}{\partial\rho} \rho \frac{\partial}{\partial\rho} + \frac{1}{\rho^2} \frac{\partial^2}{\partial\phi^2} + \beta_s{}^2 \right) \Psi_{es}(\rho, \phi) = 0 \tag{3.16}$$

The solution is

$$\Psi_{es}(\rho, \phi) = A J_n(\beta_n \rho) e^{\pm jn\phi} \tag{3.17}$$

with the boundary condition that $J_n(\beta_s a) = 0$. The zeros of $J_n(x)$ are labeled in α_{nm} is many textbooks, as well as in Figure 8; and hence, the guidance condition is that for the TM_{nm} mode

$$\beta_s = \frac{\alpha_{nm}}{a} \tag{3.18}$$

With $\beta_z = \sqrt{\beta^2 - \beta_s^2}$, the corresponding cutoff frequency is

$$\omega_{nm,c} = \frac{1}{\sqrt{\mu\varepsilon}} \frac{\alpha_{nm}}{a} \tag{3.19}$$

and the cutoff wavelength to be

$$\lambda_{nm,c} = \frac{2\pi}{\alpha_{nm}} a \tag{3.20}$$

It turns out that the lowest mode in a circular waveguide is the TE_{11} mode. It is actually a close cousin of the TE_{10} mode of a rectangular waveguide.

Table in Figure 7 shows the plot of Bessel function $J_n(x)$ and its derivative $J_n'(x)$.

[5] Notably, Abramowitz and Stegun, Handbook of Mathematical Functions.

3.3 An Application of Circular Waveguide

When a real-world waveguide is made, the wall of the metal waveguide is not made of perfect electric conductor, but with some metal with finite conductivity. Hence, tangential **E** is not zero on the wall, and energy can dissipate into the waveguide wall. It turns out that due to symmetry, the TE_{01} of a circular waveguide has the lowest loss of the waveguide modes. Hence, this waveguide mode is of interest to astronomers who are interested in building loss-loss and low-noise systems. Figure 6 shows two ways of engineering a circular waveguide so that the TE_{01} mode is enhanced: by using a mode filter that discourages the guidance of other modes, and second, by designing ridged waveguide wall to discourage the flow of axial current and hence, the propagation of the non-TE_{01} mode.

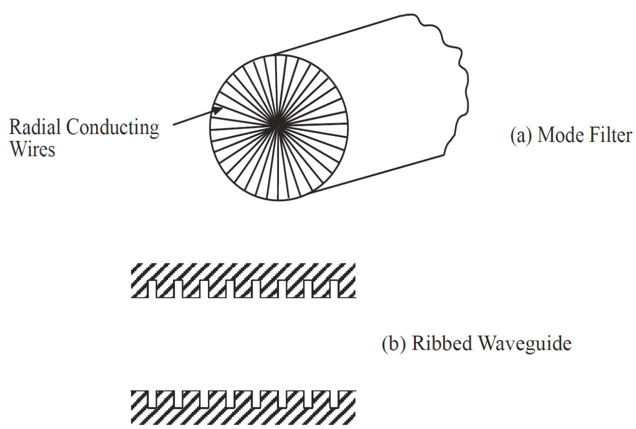

Figure 6:

4 Concluding Remarks

We have analyzed some simple structures where closed form solutions are available. These solutions offer us physical insight into how waves are guided, and how they are cutoff from guidance. For some simple waveguides, the modes can be divided into TEM, TE, and TM modes. However, most waveguides are not simple.

For example, two pieces of metal make a transmission line, and in the case of a circular coax, a TEM mode can propagate in the waveguide. However, most two-metal transmission lines do not support a pure TEM mode but a quasi-TEM mode. When a wave is TEM, it is necessary that the wave propagates with the phase velocity of the medium. But when a uniform waveguide has inhomogeneity in between, this is not possible anymore, and only a quasi-TEM mode can propagate. The lumped element model of the transmission line is still a very good model for such a waveguide.

For most inhomogeneously filled waveguides, the modes inside are not cleanly classed into TE and TM, but with some modes that are the hybrid of TE and TM modes. Sometimes, the hybrid modes are called EH or HE modes, as in an optical fiber. Nevertheless, the guidance is via a bouncing wave picture, where the bouncing waves are reflected off the boundaries of the waveguides. In the case of an optical fiber or a dielectric waveguide, the reflection is due to total internal reflection. But in the case of metalic waveguides, the reflection is due to the metal walls.

But in the transmission line, the guidance is by the exchange of electric and magnetic stored energy via the capacitance and the inductance of the line. In the case of many waveguides, the exchange of energy stored is via the space that stores these energy, like that of a plane wave.

The surface plasmonic waveguide is an exception in that the exchange is between the electric field stored energy with the kinetic energy stored in the moving electrons in the plasma instead of magnetic energy stored. Hence, the dimension of the waveguide can be very small compared to wavelength, and yet it still works.

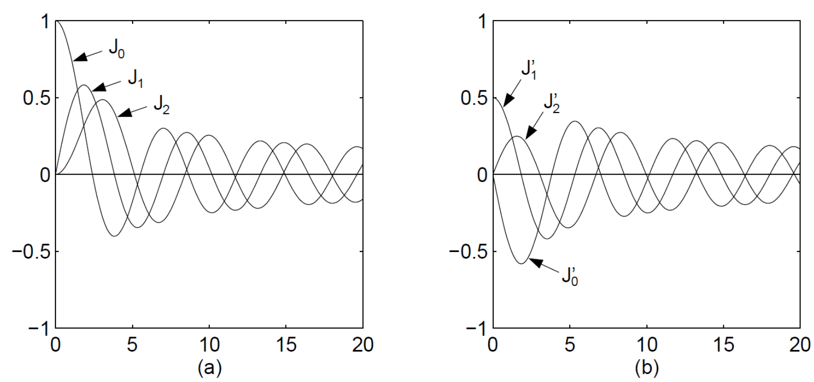

Figure 7:

Table 2.3.1. Roots of $J_n'(x) = 0$.

n	β_{n1}	β_{n2}	β_{n3}	β_{n4}
0	3.832	7.016	10.174	13.324
1	1.841	5.331	8.536	11.706
2	3.054	6.706	9.970	13.170
3	4.201	8.015	11.346	14.586
4	5.318	9.282	12.682	15.964
5	6.416	10.520	13.987	17.313

Table 2.3.2. Roots of $J_n(x) = 0$.

n	α_{n1}	α_{n2}	α_{n3}	α_{n4}
0	2.405	5.520	8.654	11.792
1	3.832	7.016	10.174	13.324
2	5.135	8.417	11.620	14.796
3	6.380	9.761	13.015	16.223
4	7.588	11.065	14.373	17.616
5	8.771	12.339	15.700	18.980

Figure 8:

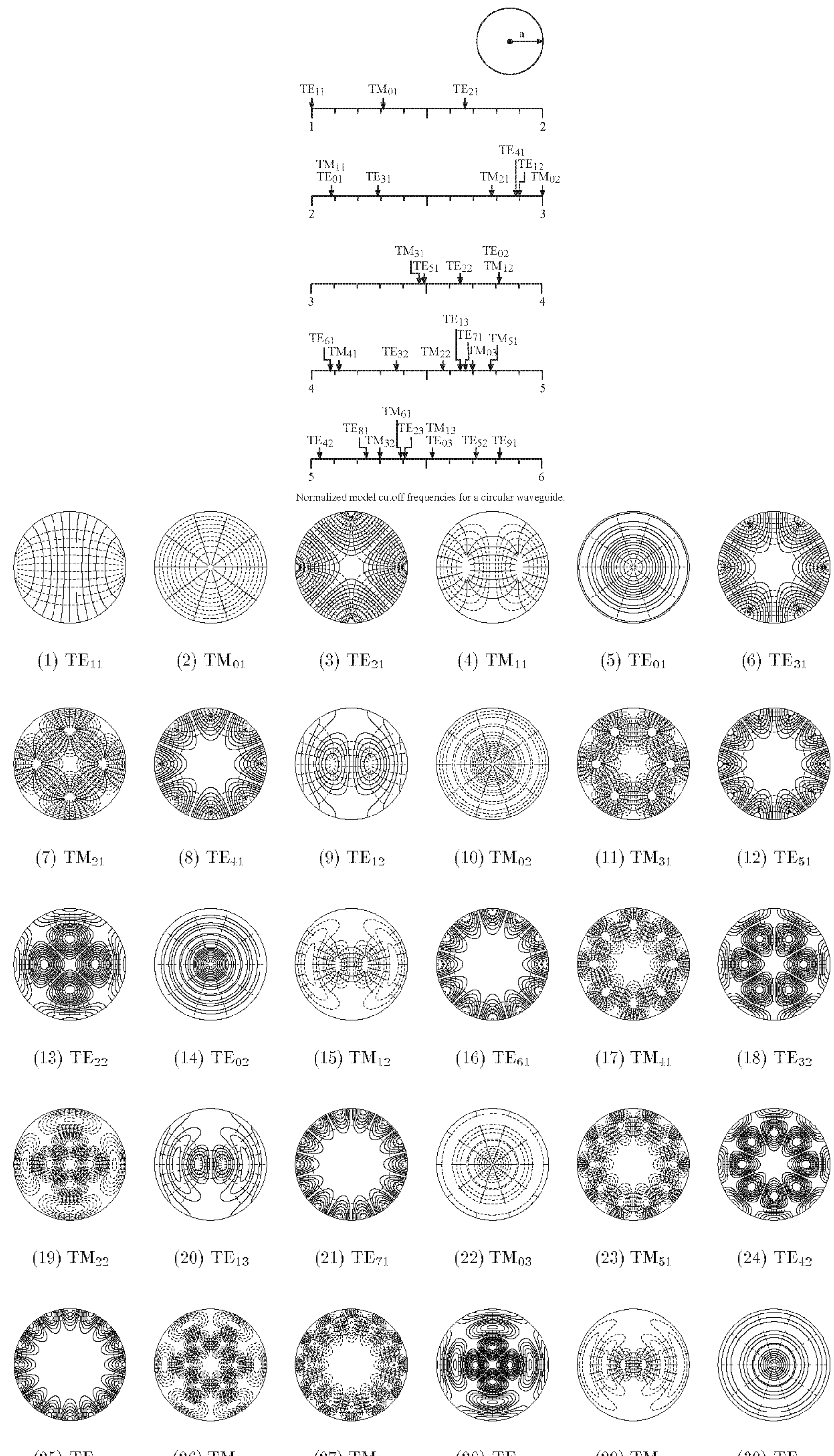

Figure 9: Courtesy of Andy Greenwood. Original plots published in Lee, Lee, and Chuang.

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