- Scalar Magnetic Potential Φ_m
- Boundary Conditions
- Magnetic Energy
- Inductance

Interested Readers:

- 2.13
- 2.14
- 2.16
- 2.5, 2.17

$\boxed{\text{Scalar Magnetic Potential } \phi_m}$

In source-free region

$$\nabla \times \vec{H} = 0$$

$$\vec{H} = -\nabla \underline{\phi_m}$$

Scalar magnetic potential

$$\because \quad \nabla \cdot \vec{B} = 0$$

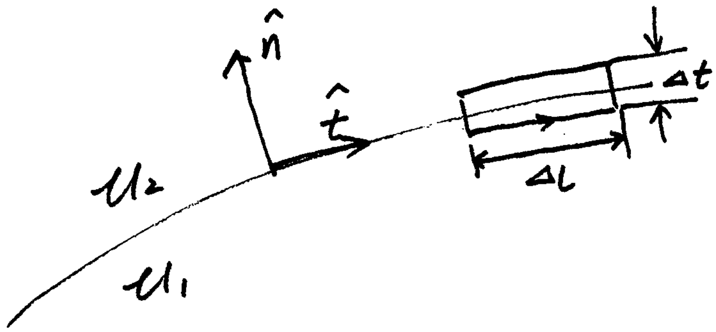
$$\therefore \quad \nabla \cdot (\mu \nabla \phi_m) = 0$$

In homogeneous media where μ does not depend on $\vec{r}$

$$\underline{\nabla^2 \phi_m = 0}$$

Boundary Conditions

For a discontinuous interface



$$\oint \vec{H} \cdot d\vec{l} = I$$

$$\vec{H_1} \cdot \hat{t} \Delta l + \vec{H_1} \cdot \hat{n} \frac{\Delta t}{2} + \vec{H_2} \cdot \hat{n} \frac{\Delta t}{2} - \vec{H_2} \cdot \hat{t} \Delta l$$
$$- \vec{H_2} \cdot \hat{n} \frac{\Delta t}{2} - \vec{H_1} \cdot \hat{n} \frac{\Delta t}{2} = \vec{J} \cdot (\hat{t} \times \hat{n}) \Delta t \, \Delta l$$

Let $\Delta t \to 0$

$$\vec{H_1} \cdot \hat{t} \Delta l - \vec{H_2} \cdot \hat{t} \Delta l = \underline{\vec{J_s}} \cdot (\hat{t} \times \hat{n}) \Delta l$$

$\vec{J} \Delta t$ surface current density
unit A/m

$$\Rightarrow \quad \vec{H_1} \cdot \hat{t} - \vec{H_2} \cdot \hat{t} = \vec{J_s} \cdot (\hat{t} \times \hat{n})$$

Vector identity:

$$\boxed{\vec{a} \times (\vec{b} \times \vec{c}) = (\vec{a} \cdot \vec{c})\vec{b} - (\vec{a} \cdot \vec{b})\vec{c}}$$

$$\therefore \quad \hat{n} \times (\hat{t} \times \hat{n}) = \hat{t} - (\hat{n} \cdot \hat{t})\hat{n}$$

$$= \hat{t}$$

$$\therefore \quad \vec{H}_1 \cdot [\hat{n} \times (\hat{t} \times \hat{n})] - \vec{H}_2 \cdot [\hat{n} \times (\hat{t} \times \hat{n})] = \vec{J}_s \cdot (\hat{t} \times \hat{n})$$

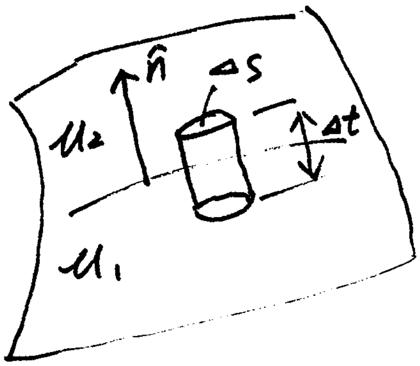
Vector identity:

$$\boxed{\vec{a} \cdot (\vec{b} \times \vec{c}) = \vec{b} \cdot (\vec{c} \times \vec{a}) = \vec{c} \cdot (\vec{a} \times \vec{b})}$$

$$\therefore \quad [\vec{H}_1 \times \hat{n} - \vec{H}_2 \times \hat{n}] \cdot (\hat{t} \times \hat{n}) = \vec{J}_s \cdot (\hat{t} \times \hat{n})$$

$$\therefore \quad \boxed{\hat{n} \times (\vec{H}_2 - \vec{H}_1) = \vec{J}_s}$$

How about $\vec{B}$?



$$\oint\!\!\oint_S \vec{B} \cdot d\vec{s} = 0$$

$$\Rightarrow \quad \vec{B_2} \cdot \hat{n} \Delta s + \vec{B_1} \cdot (-\hat{n}) \Delta s + \vec{B} \cdot \hat{\rho}\, 2\pi \rho \Delta t = 0$$

Let $\Delta t \to 0$

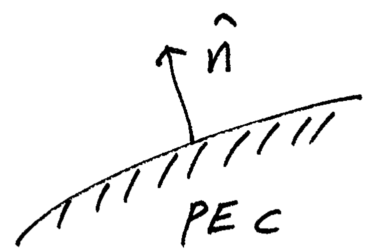
$$\boxed{\hat{n} \cdot (\vec{B_2} - \vec{B_1}) = 0}$$

Boundary conditions

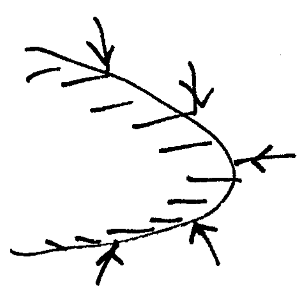
$$\boxed{\begin{aligned} \hat{n} \times (\vec{H_2} - \vec{H_1}) &= \vec{J_s} \\ \hat{n} \cdot (\vec{B_2} - \vec{B_1}) &= 0 \end{aligned}}$$

If medium 1 is a perfect conductor then

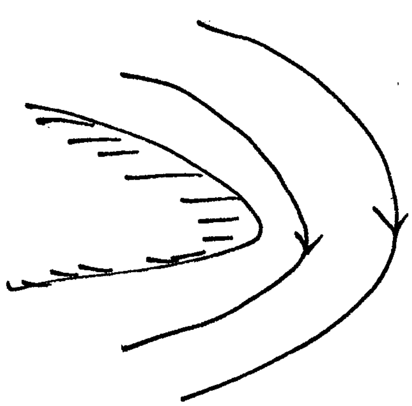
$$\boxed{\begin{aligned} \hat{n} \times \vec{H} &= \vec{J_s} \\ \hat{n} \cdot \vec{B} &= 0 \end{aligned}}$$



Magnetic field line

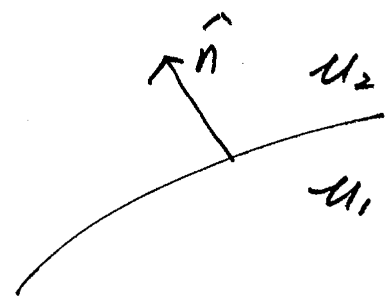


?



Correct !

Boundary Conditions in terms of ϕ_m



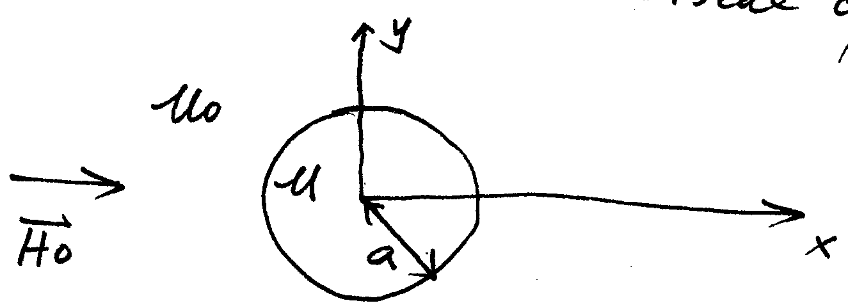
$$\phi_{m1} = \phi_{m2}$$

$$\mu_1 \frac{\partial \phi_{m1}}{\partial n} = \mu_2 \frac{\partial \phi_{m2}}{\partial n}$$

Question: why?

Example

An infinitely long circular cylinder of radius a, permeability μ is placed in a uniform magnetic field $\vec{H}_0$ as shown in the figure below, calculate $\vec{H}$ inside and outside of the cylinder.



Magnetic Energy

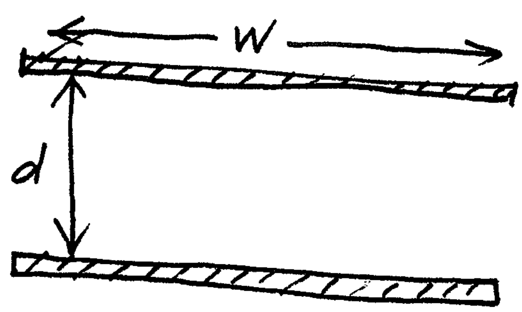
$$W_m = \iiint_V \frac{1}{2}\mu |\vec{H}|^2 \, dV$$

Inductance

$$L = \frac{\psi}{I} = \frac{1}{I}\iint \vec{B}\cdot d\vec{s}$$

Example

A parallel-plate structure of width w, spacing d, with $d \ll w$



Question: What is L per unit length?

What is W_m .. .. .. ?