Chapter II $\boxed{\text{Magnetostatics}}$

- Magnetic Force
- Bio-Savart Law
- Ampere's Law
- Gauss's Law - magnetic
- Magnetic Vector Potential $\vec{A}$
- Vector Poisson's Equation

Interested Readers

- Textbook 2.2, 2.3, 2.4, 2.6-2.9, 2.11-2.12

Chapter II Magnetostatics

Magnetic Force

$$\vec{F} = q\vec{v} \times \vec{B}$$

$$d\vec{f} = I d\vec{l} \times \vec{B}$$

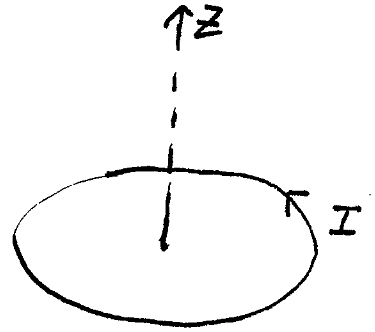
Biot-Savart Law

$$\vec{H}(\vec{r}) = \int \frac{I(\vec{r}')\, d\vec{l}' \times \hat{R}}{4\pi R^2}$$

$$R = |\vec{r} - \vec{r}'|$$

Example

A circular loop of wire carrying dc current I



Question: what's $\vec{H}$ on the z axis?

Ampere's Law

$$\boxed{\oint_c \vec{H} \cdot d\vec{l} = I}$$

Integral Form

For continuous media

$$\because \oint_c \vec{H} \cdot d\vec{l} = \iint_s (\nabla \times \vec{H}) \cdot d\vec{s}$$

Stokes's theorem

$$\therefore \iint_s (\nabla \times \vec{H}) \cdot d\vec{s} = \iint_s \vec{J} \cdot d\vec{s}$$

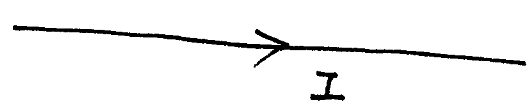
$\because$ S is arbitrary

$$\therefore \boxed{\nabla \times \vec{H} = \vec{J}}$$

Differential Form

Example

A line current I



Question: what's $\vec{H}$ generated by I?

Example

QE Jan. 2006, CC-3, Question 2.

$\boxed{\text{Gass's Law – Magnetic}}$

$$\boxed{\oiint \vec{B} \cdot d\vec{s} = 0}$$ Integral form

$\because$ $\oiint \vec{B} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{B} \, dV$ Divergence (Gauss's) theorem

$\therefore$ $\iiint_V \nabla \cdot \vec{B} \, dV = 0$

$\because$ V is arbitrary

$\therefore$ $\boxed{\nabla \cdot \vec{B} = 0}$ Differential form

Example

Someone has calculated $\vec{B}$ to be $\hat{z}\dfrac{1}{z^2+a^2}$.

Can this be correct? Explain.

$\boxed{\text{Constitutive Relation}}$

$$\vec{B} = \mu \vec{H}$$

μ: permeability H/m

In Free ~~Sap~~ Space

$$\mu = \mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

In Other material

$$\mu = \mu_0 \underline{\mu_r}$$

relative permeability

Magnetic Vector Potential $\vec{A}$

$\because \nabla \cdot \vec{B} = 0$

$\therefore \vec{B}$ can be written as

$$\vec{B} = \nabla \times \vec{A}$$

Vector Poisson's Equation

$\because \nabla \times \vec{H} = \vec{J}$

$\therefore \nabla \times \left(\frac{\vec{B}}{u}\right) = \vec{J}$

$\therefore \nabla \times \left(\frac{1}{u} \nabla \times \vec{A}\right) = \vec{J}$

In homogeneous media

$$\nabla \times (\nabla \times \vec{A}) = u\vec{J}$$

Vector identity

$$\boxed{\nabla \times (\nabla \times \vec{A}) = \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A}}$$

$\therefore \nabla(\nabla \cdot \vec{A}) - \nabla^2 \vec{A} = u\vec{J}$

To uniquely determine $\vec{A}$, we choose

$$\nabla \cdot \vec{A} = 0 \qquad \text{gauge condition}$$

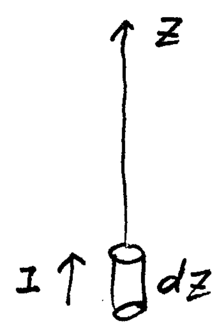
$$\therefore \quad \nabla^2 \vec{A} = -\mu \vec{J} \qquad \boxed{\text{Vector Poisson's equation}}$$

In free space:

$$\vec{A}(\vec{r}) = \frac{\mu}{4\pi} \iiint_V \frac{\vec{J}(\vec{r}')}{R} dV' \qquad R = |\vec{r} - \vec{r}'|$$

<u>Example</u>

A current element $I dz$



<u>Question</u>: what's $\vec{A}$ and $\vec{B}$ generated by this current element?