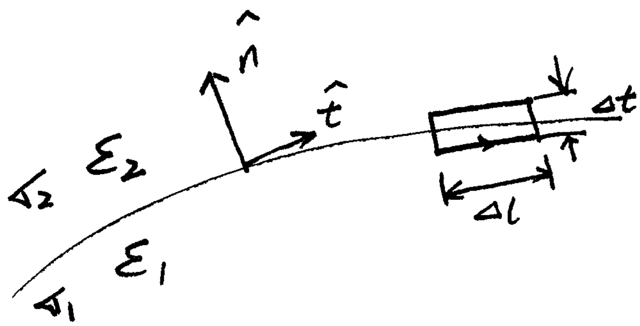
- Boundary Conditions
- Electric Energy
- Capacitance

Interested Readers:

- 1.14, 1.22

Boundary Conditions

For a discontinuous interface



$$\oint \vec{E} \cdot d\vec{l} = 0$$

$$\vec{E}_1 \cdot \hat{t}\,\Delta l + \vec{E}_1 \cdot \hat{n}\,\frac{\Delta t}{2} + \vec{E}_2 \cdot \hat{n}\,\frac{\Delta t}{2} - \vec{E}_2 \cdot \hat{t}\,\Delta l$$
$$- \vec{E}_2 \cdot \hat{n}\,\frac{\Delta t}{2} - \vec{E}_1 \cdot \hat{n}\,\frac{\Delta t}{2} = 0$$

Let $\Delta t \to 0$

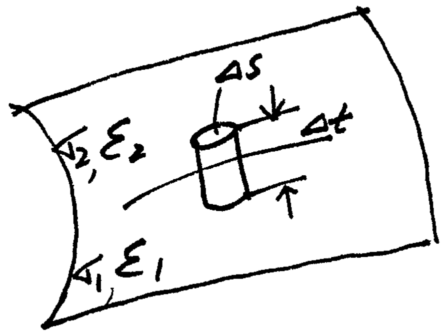
$$\vec{E}_1 \cdot \hat{t}\,\Delta l - \vec{E}_2 \cdot \hat{t}\,\Delta l = 0$$

$$\Rightarrow \quad \vec{E}_1 \cdot \hat{t} = \vec{E}_2 \cdot \hat{t}$$

$$\boxed{E_{1t} = E_{2t}}$$

<u>tangential component of $\vec{E}$ is always continuous</u>

How about $\vec{D}$?



$$\oint\!\!\oint_S \vec{D} \cdot d\vec{s} = Q$$

$$\vec{D}_2 \cdot \hat{n} \Delta s + \vec{D}_1 \cdot (-\hat{n}) \Delta s + \vec{D} \cdot \hat{\rho}\, 2\pi\rho \Delta t = \rho_s \Delta s$$

Let $\Delta t \to 0$

$$\boxed{\hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s}$$

Boundary conditions

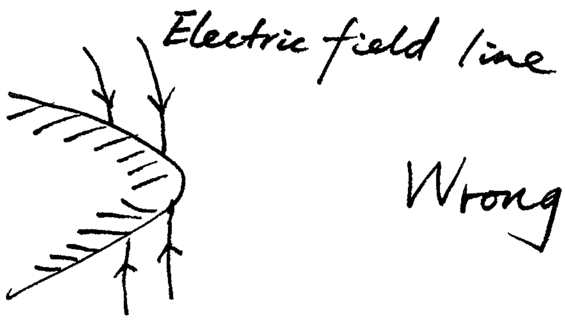
$$E_{1t} = E_{2t}$$

$$\hat{n} \cdot (\vec{D}_2 - \vec{D}_1) = \rho_s$$

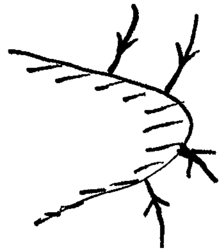
If medium 1 is a perfect conductor then $\vec{E}_1 = \vec{D}_1 = 0$

$$E_t = 0 \quad \hat{n} \cdot \vec{D} = \rho_s$$

Example



Wrong !

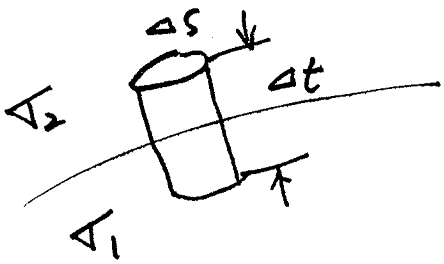


Correct !

How about $\vec{J}$ in a conductive medium?

$$\oint\!\!\oint_S \vec{J} \cdot d\vec{s} = -\frac{dQ}{dt}$$

Static cases: $\oint\!\!\oint_s \vec{J} \cdot d\vec{s} = 0$



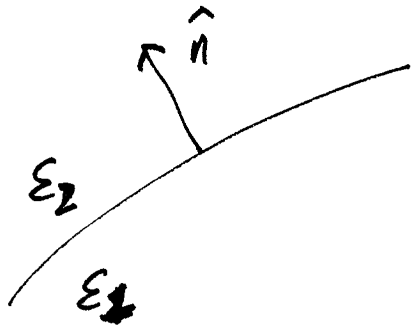
$$\vec{J_2} \cdot \hat{n} \Delta s + \vec{J_1} \cdot (-\hat{n}) \Delta s + \vec{J} \cdot \hat{\rho} 2\pi\rho \Delta t = 0$$

Let $\Delta t \to 0$

$$\Rightarrow \quad \hat{n} \cdot (\vec{J_2} - \vec{J_1}) = 0$$

$$\Rightarrow \quad \boxed{\sigma_2 E_{2n} = \sigma_1 E_{1n}}$$

Boundary Conditions in terms of ϕ

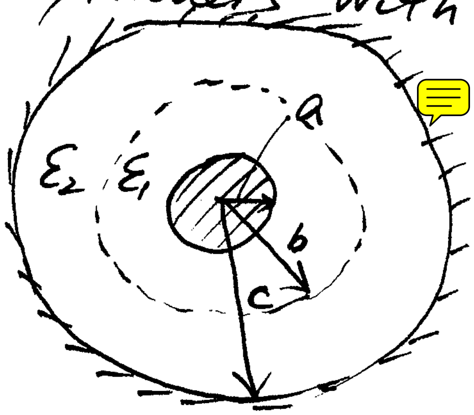


$$\begin{cases} \phi_1 = \phi_2 \\ \varepsilon_1 \dfrac{\partial \phi_1}{\partial n} = \varepsilon_2 \dfrac{\partial \phi_2}{\partial n} \end{cases}$$ when $\rho_s = 0$

Question: why?

Example

Coaxial cylinders with two dielectrics



Question: What is $\phi \begin{cases} a < r < b \\ b < r < c \end{cases}$?

Electric Energy

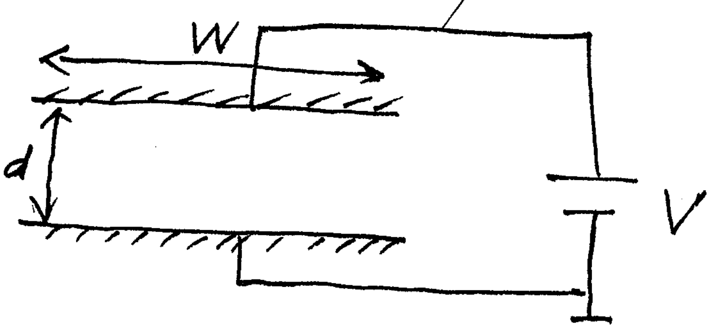
$$W_e = \iiint_V \frac{1}{2} \varepsilon |\vec{E}|^2 \, dV$$

Capacitance

$$C = \frac{Q}{V}$$

Example

A capacitor with two parallel plates attached to a battery



Question: What is C per unit length?

What is We .. .. .. ?