- Gauss's Law — Differential form
- Faraday's law — Integral form
  — Differential form
- Constitutive Relations
- Electric Scalar Potential ϕ
- Poisson's and Laplace's Equations

Interested Readers:

- Textbook 1.7, 1.8, 1.11, 1.12

Gauss's Law - Differential form

For continuous media

$$\because \oint\!\!\oint_S \vec{D} \cdot d\vec{s} = \iiint_V \nabla \cdot \vec{D} \, dV \quad \text{Gauss's theorem}$$

$$\therefore \iiint_V \nabla \cdot \vec{D} \, dV = \iiint_V \rho \, dV$$

$$\therefore \boxed{\nabla \cdot \vec{D} = \rho}$$

In Rectangular Coordinate System

$$\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z} = \rho$$

Example

If $\vec{D} = (2y^2 + z)\vec{a}_x + 4xy\,\vec{a}_y + x\vec{a}_z$

Find: 1 volume charge density ρ_V at $(-1, 0, 3)$

2 Electric flux through the cube defined by $0 \leq x \leq 1$, $0 \leq y \leq 1$, $0 \leq z \leq 1$

3 Total charge enclosed by the cube

Faraday's law – static
Integral Form

$$\oint \vec{E} \cdot d\vec{l} = 0$$

Electro-static system is conservative

Faraday's law – Differential Form

$\because$ $\oint_C \vec{E} \cdot d\vec{l} = \iint_S \nabla \times \vec{E} \cdot d\vec{s}$ Stokes's theorem

$\therefore$ $\iint_S \nabla \times \vec{E} \cdot d\vec{s} = 0$

$\because$ S is arbitrary

$\therefore$ $\boxed{\nabla \times \vec{E} = 0}$

Example

Suppose $\vec{E} = \hat{x}3y + \hat{y}x$. Calculate $\int \vec{E} \cdot d\vec{l}$ along a straight line in the x-y plane joining (0,0) to (3,1)

Consitutive Relations

$$\boxed{\vec{D} = \varepsilon \vec{E}}$$

ε: Permittivity (F/m)

In free space:

$$\varepsilon = \varepsilon_0 = 8.854 \times 10^{-12} = \frac{10^{-9}}{36\pi} \text{ F/m}$$

$$\boxed{\vec{J} = \sigma E}$$

σ: conductivity (S/m)

Electric Scalar Potential ϕ

$\because \nabla \times \vec{E} = 0$

$$\vec{E} = -\nabla \phi$$

Example

Fields of a sphere of uniform charge density ρ

Assuming $\phi \big|_{r=\infty} = 0$

Question: What is ϕ at $r \leq a$?

ϕ at $r > a$?

Poisson's and Laplace's Equations

$$\vec{E} = -\nabla\phi$$

$$\nabla\cdot\vec{D} = \rho$$

$$\Rightarrow \nabla\cdot(\varepsilon\vec{E}) = \rho$$

$$\Rightarrow -\nabla\cdot(\varepsilon\nabla\phi) = \rho$$

If ε is not position dependent

$$\boxed{\nabla^2\phi = -\frac{\rho}{\varepsilon}}$$ Poisson's Equation

In soure-free region

$$\boxed{\nabla^2\phi = 0}$$ Laplace's Equation

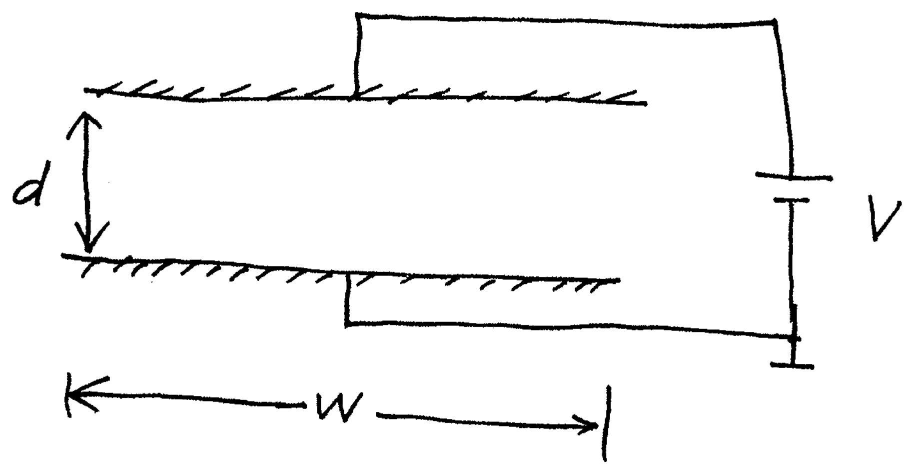
For Poisson's Equation

in homogeneous medium of ε

$$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon}\iiint_V \frac{\rho(\vec{r}')}{R}\,dV' \qquad R = |\vec{r}-\vec{r}'|$$

Example

A Capacitor with two parallel plates attached to a battery



Question: what is $\vec{E}$ inside the capacitor?