- Polarization

Interested Readers:

- Textbook 6.3, Balanis 4.4

Polarization

1. $E_x = A e^{-j\beta z}$, $\quad H_y = \frac{A}{\eta} e^{-j\beta z}$

$\Longrightarrow$ linearly polarized in the x-direction

2. $E_y = B e^{-j\beta z}$, $\quad H_x = -\frac{B}{\eta} e^{-j\beta z}$

$\Longrightarrow$ linearly polarized in the y-direction

3. Consider their combination:

$$\vec{E} = \hat{x} A e^{-j\beta z} + \hat{y} B e^{-j\beta z} = (\hat{x} A + \hat{y} B) e^{-j\beta z}$$

$$\vec{H} = \hat{y} \frac{A}{\eta} e^{-j\beta z} - \hat{x} \frac{B}{\eta} e^{-j\beta z} = (\hat{y} A - \hat{x} B) \frac{1}{\eta} e^{-j\beta z}$$

In time domain:

$$E_x(z,t) = Re(E_x e^{j\omega t}) = Re(A e^{j(\omega t - \beta z)})$$

$$= Re(|A| e^{j(\omega t - \beta z + \alpha)})$$

$$= |A| \cos(\omega t - \beta z + \alpha)$$

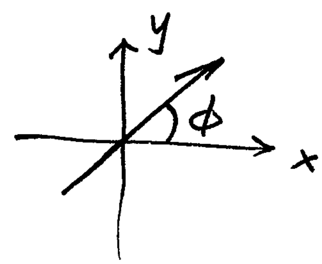
$$E_y(z,t) = |B|\cos(\omega t - \beta z + b)$$

$$\vec{E}(z,t) = \hat{x}|A|\cos(\omega t - \beta z + a) + \hat{y}|B|\cos(\omega t - \beta z + b)$$

* If $a=b$, then $\vec{E}(z,t) = (\hat{x}|A| + \hat{y}|B|)\cos(\omega t - \beta z + a)$

$\Rightarrow$ linearly polarized in the direction

$$\phi = \tan^{-1}\left(\frac{|B|}{|A|}\right)$$



* If $a=0$, $b=\frac{\pi}{2}$, then

$$\vec{E}(z,t) = \hat{x}|A|\cos(\omega t - \beta z) - \hat{y}|B|\sin(\omega t - \beta z)$$

$E_x(z,t) = |A|\cos(\omega t - \beta z)$, $E_y(z,t) = -|B|\sin(\omega t - \beta z)$

$$\left(\frac{E_x}{|A|}\right)^2 + \left(\frac{E_y}{|B|}\right)^2 = 1$$

$\Rightarrow$ elliptically polarized $<$ left-hand, counter-clockwise

If $|A| = |B|$ $\Rightarrow$ circularly polarized

* If $a = \frac{\pi}{2}$, $b = 0$, then

$$\vec{E}(z,t) = -\hat{x}|A|\sin(\omega t - \beta z) + \hat{y}|B|\cos(\omega t - \beta z)$$

$\Rightarrow$ elliptically polarized < right-hand, clockwise

$\boxed{\text{Summary}}$

1. Linearly polarized.

$$\vec{E} = (\hat{x}|A| \pm \hat{y}|B|)e^{-j\beta z} \cdot E_0$$

2. Circularly polarized:

$$\vec{E} = (\hat{x} - j\hat{y})E_0 e^{-j\beta z} \quad \text{right-hand}$$

$$\vec{E} = (\hat{x} + j\hat{y})E_0 e^{-j\beta z} \quad \text{left-hand}$$

3. Elliptically polarized.

$$\vec{E} = (\hat{x}|A| - j\hat{y}|B|)E_0 e^{-j\beta z} \quad \text{right-hand}$$

$$\vec{E} = (\hat{x}|A| + j\hat{y}|B|)E_0 e^{-j\beta z} \quad \text{left-hand}$$

Observation #1.

An elliptically (circularly) polarized wave can be decomposed to two linearly polarized waves

Observation #2:

A linearly polarized wave can be decomposed into two elliptically (circularly) polarized waves:

$$\vec{E} = \hat{x} A e^{-j\beta z} = \left(\hat{x}\frac{|A|}{2} + j\hat{y}\frac{|B|}{2} + \hat{x}\frac{|A|}{2} - j\hat{y}\frac{|B|}{2}\right)e^{-j\beta z}$$

$$= \left(\hat{x}\frac{|A|}{2} + j\hat{y}\frac{|B|}{2}\right)e^{-j\beta z} \quad \leftarrow \text{left-hand}$$

$$+ \left(\hat{x}\frac{|A|}{2} - j\hat{y}\frac{|B|}{2}\right)e^{-j\beta z} \quad \leftarrow \text{right-hand}$$

Observation #3:

For a linearly polarized wave:

$$\vec{S}(t) = \vec{E} \times \vec{H} = \hat{z}\frac{A^2}{\eta}\cos^2(\omega t - \beta z)$$

For a circularly polarized wave:

$$\vec{E} = (\hat{x} \pm j\hat{y})Ae^{-j\beta z}$$

$$\vec{H} = (\mp\hat{x} - j\hat{y})j\frac{A}{\eta}e^{-j\beta z}$$

$$\vec{E}(t) = \hat{x}A\cos(\omega t - \beta z) \pm \hat{y}A\sin(\omega t - \beta z)$$

$$\vec{H}(t) = \mp\hat{x}\frac{A}{\eta}\sin(\omega t - \beta z) + \hat{y}\frac{A}{\eta}\cos(\omega t - \beta z)$$

$$\vec{S}(t) = \vec{E} \times \vec{H} = \hat{z}\frac{A^2}{\eta}\cos^2(\omega t - \beta z) + \hat{z}\frac{A^2}{\eta}\sin^2(\omega t - \beta z)$$

$$= \hat{z}\,\frac{A^2}{\eta}$$

$\Longrightarrow$ circularly polarized waves have a steady power flow