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ECE 350 Lecture Notes

12. Transients on a Transmission Line.

When we do not have a time harmonic signal on a transmission line, we have to use transient analysis to understand the waves on a transmission line. A pulse waveform is an example of a transient waveform.

We have shown previously that if we have a forward going wave for a voltage on a transmission line, the voltage is

$$V(z,t) = f(z - vt). \tag{1}$$

The corresponding current can be derived via the telegrapher's equation

$$I(z,t) = \frac{1}{Z_0} f(z - vt). \tag{2}$$

If instead, we have a wave going in the negative direction,

$$V(z,t) = g(z + vt), \tag{3}$$

then the current from the telegrapher's equations, is

$$I(z,t) = -\frac{1}{Z_0} g(z + vt). \tag{4}$$

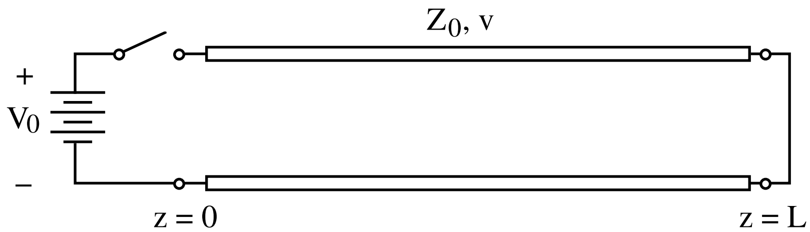
Hence, in general, if

$$V(z,t) = V_+(z,t) + V_-(z,t), \tag{5}$$

$$I(z,t) = Y_0 \big[V_+(z,t) - V_-(z,t) \big], \tag{6}$$

where $Y_0 = \frac{1}{Z_0}$, and the subscript $+$ indicates a positive going wave, while the subscript $-$ indicates a negative going wave.

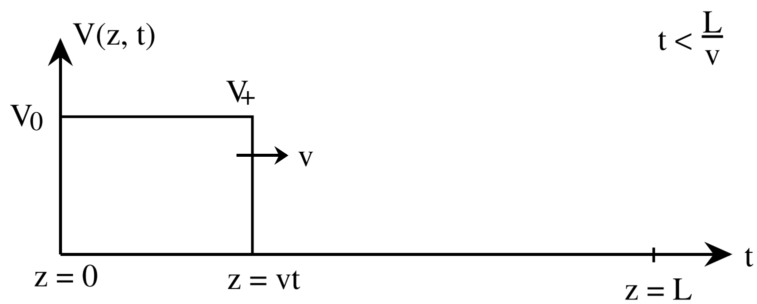
(a) **Reflection of a Transient Signal from a Shorted Termination**

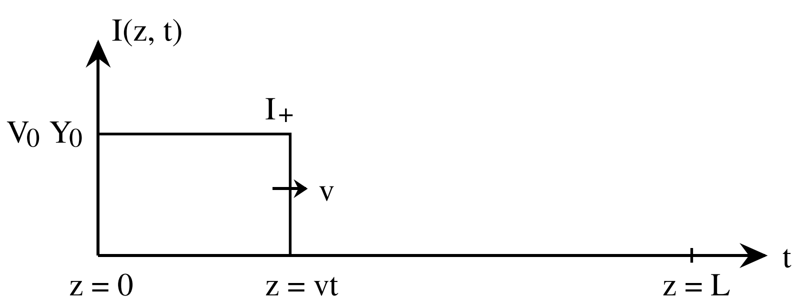


If we switch on the voltage of the above network at $t = 0$, the voltage at $z = 0$ has the form

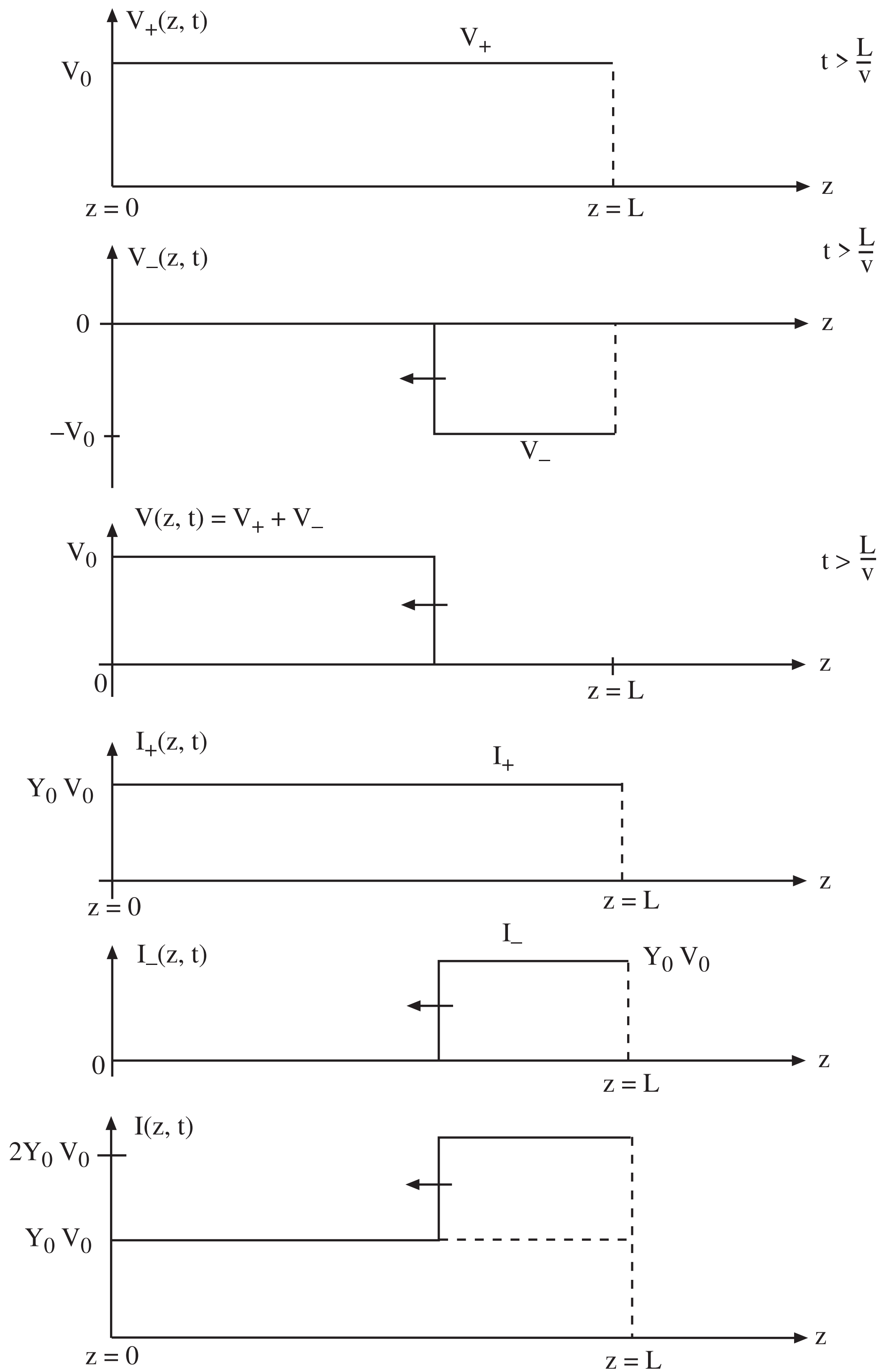
$$V(z = 0, t) = V_0 U(t). \tag{7}$$

The voltage on the transmission line is zero initially, the disturbance at $t = 0$ will create a wave front propagating to the right as t increases.



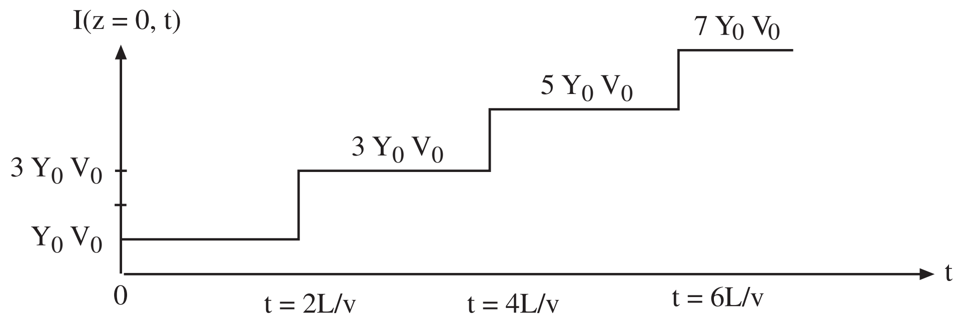


When the wave reaches the right end termination, the voltage and the current wave fronts will be reflected. However, the short at $z = L$ requires that $V(z = L, t) = 0$ always. Hence the reflected voltage wave, which is negative going, has an amplitude of $-V_0$. The corresponding current can be derived from (4) and is as shown.



$V_+(z, t)$
V_+
V_0
$t > \frac{L}{v}$
z
$z = 0$
$z = L$
$V_-(z, t)$
$t > \frac{L}{v}$
0
z
$-V_0$
V_-
$V(z, t) = V_+ + V_-$
V_0
$t > \frac{L}{v}$
z
0
$z = L$
$I_+(z, t)$
I_+
$Y_0 V_0$
z
$z = 0$
$z = L$
I_-
$I_-(z, t)$
$Y_0 V_0$
0
z
$z = L$
$I(z, t)$
$2Y_0 V_0$
$Y_0 V_0$
z
$z = 0$
$z = L$

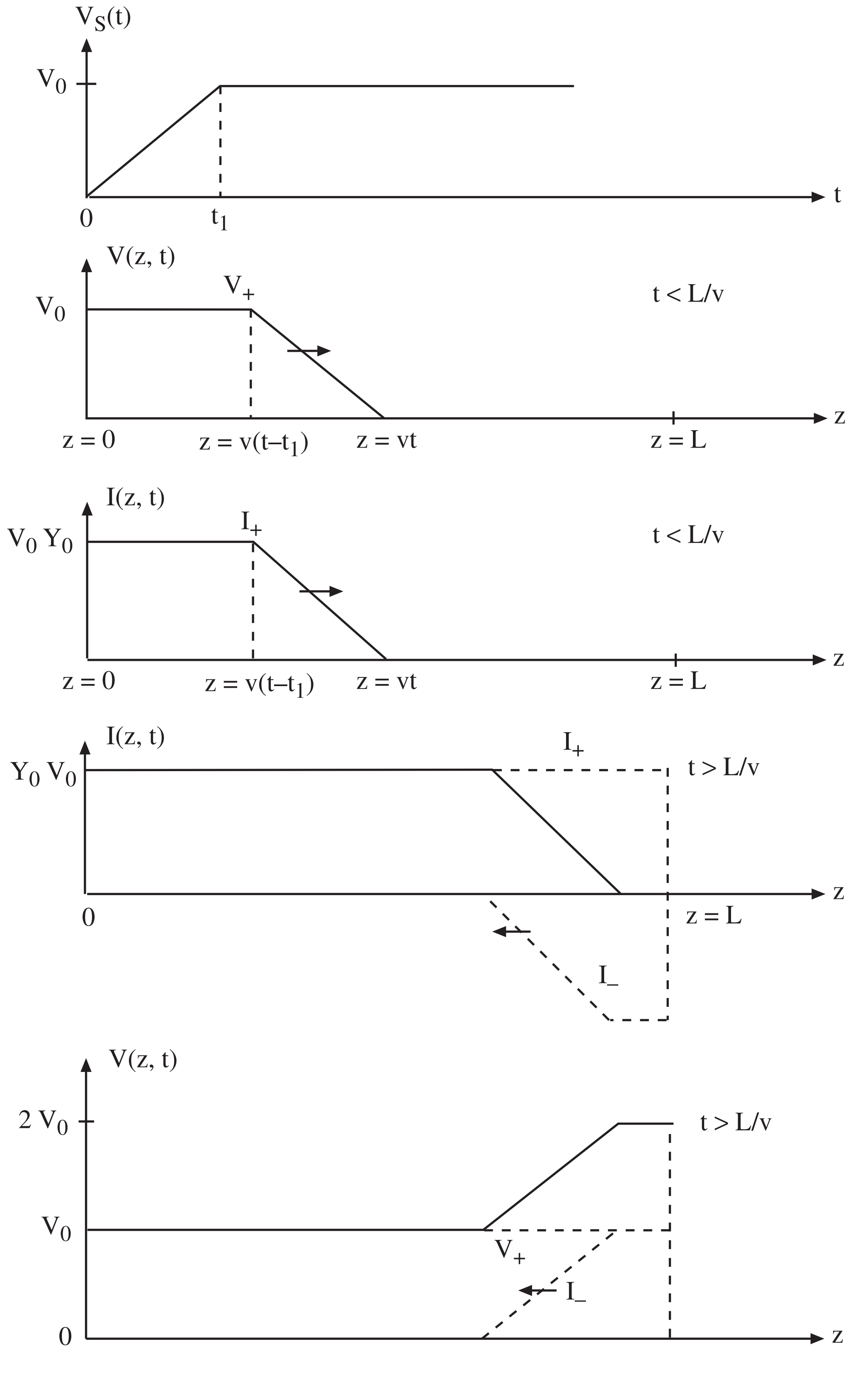
When the signal reaches the source end, it is being reflected again. A voltage source looks like a short circuit because the reflected voltage must cancel the incident voltage in order for the voltage across the voltage source remains unchanged. Hence the negative going voltage and current are again reflected like a short. Hence, if one is to measure the voltage at $z = 0$, it will always be V_0. However, the current at $z = 0$ will increase indefinitely with time as shown.



The current will eventually become infinitely large because the transmission line will become like a short circuit to the D.C. voltage source. Therefore, the current becomes infinite.

(b) **Open-Circuited Termination**

If we have an open-circuited termination at $z = L$, then the current has to be zero always. In this case, the reflected current is negative that of the incident current such that $I(z = L, t) = 0$ always. For example, if the source waveform looks like as shown below, the reflected waveform will behave as shown.



$V_S(t)$
V_0
t
0
t_1
$V(z, t)$
V_+
V_0
$t < L/v$
z
$z = 0$
$z = v(t-t_1)$
$z = vt$
$z = L$
$I(z, t)$
I_+
$V_0\ Y_0$
$t < L/v$
z
$z = 0$
$z = v(t-t_1)$
$z = vt$
$z = L$
$I(z, t)$
I_+
$Y_0\ V_0$
$t > L/v$
z
0
$z = L$
I_-
$V(z, t)$
$2\ V_0$
$t > L/v$
V_0
V_+
I_-
0
z

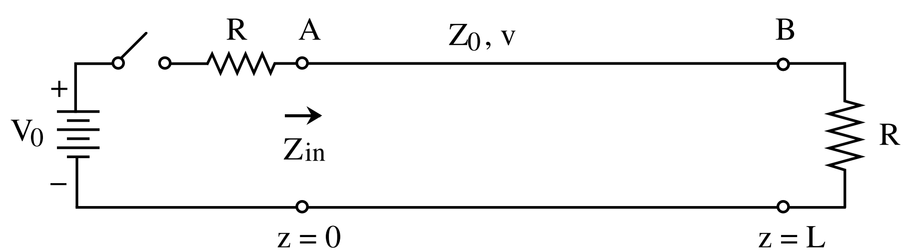
(c) **Resistive Termination**

We can think of transient signals as superpositions of time harmonic signals. This is a consequence of Fourier analysis. We see that the voltage reflection coefficient is -1 for a shorted termination for all frequencies. Hence, the voltage reflection coefficient is -1 for a transient signal. By a similar argument, the voltage reflection coefficient for an open-circuited termination is $+1$.

When the termination is resistive on a lossless transmission line, we recall that the voltage reflection coefficient is

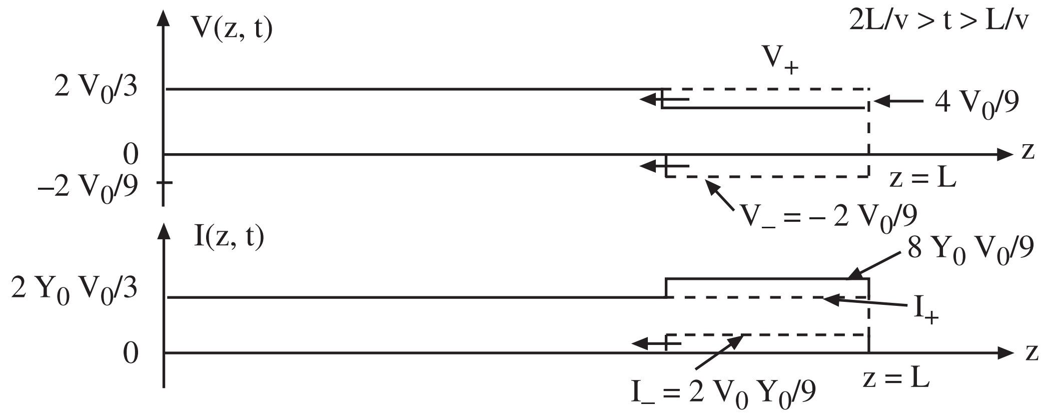
$$\rho_v = \frac{Z_L - Z_0}{Z_L + Z_0} = \frac{R_L - Z_0}{R_L + Z_0}. \tag{8}$$

Hence, the reflection coefficient is frequency independent. All frequency components in a transient signal will experience the same reflection. Hence, ρ_v is also the reflection coefficient for a voltage pulse.

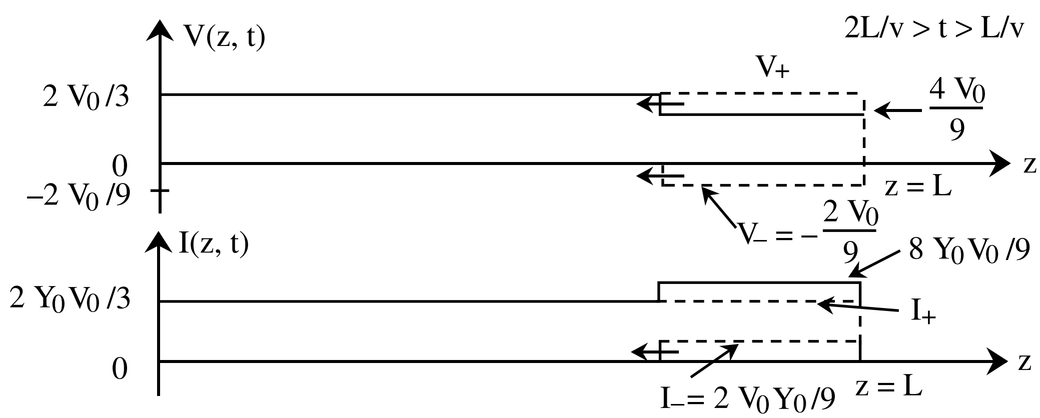


Consider, for example, a transmission line being driven via a source resistance R and a load termination R. If $R = \frac{1}{2}Z_0$, let us see what happens when we turn on the switch.

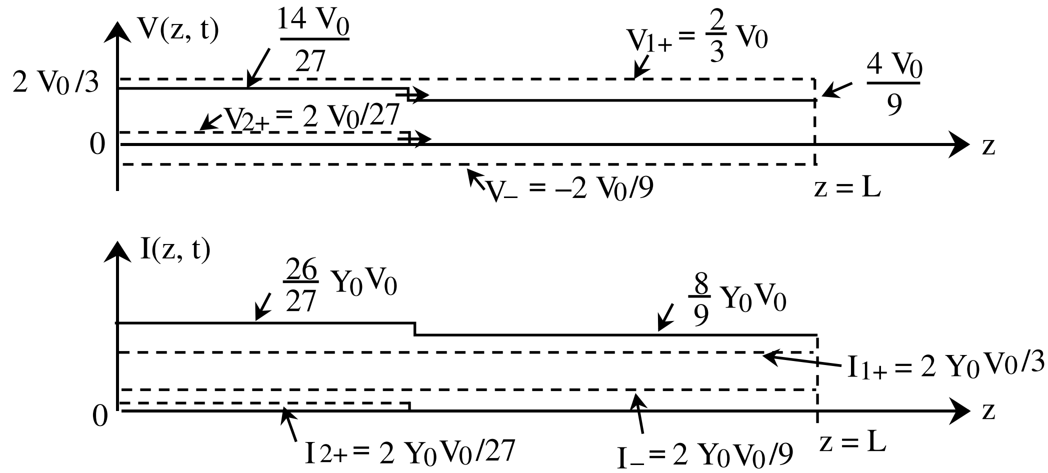
For $t < \frac{L}{V}$, the transmission line appears to be infinitely long to the source. Hence, Z_{in} looks like Z_0 to the source. Hence, $V_A = \frac{Z_0}{Z_0+R}V_0 = \frac{2}{3}V_0$ for $R = \frac{1}{2}Z_0$. Hence, we have a wavefront of $\frac{2}{3}V_0$ propagating to the right for $t < \frac{L}{V}$.



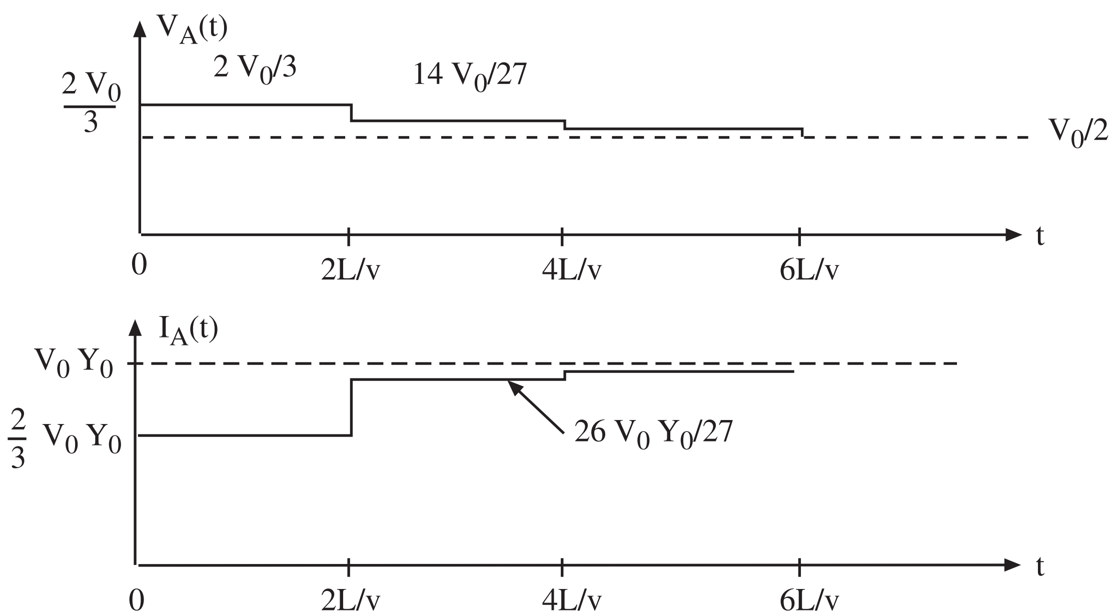
For $t > \frac{L}{V}$, a reflected voltage wave is generated at the termination and its amplitude is $\frac{2}{3}\rho_v V_0$. $\rho_v = -\frac{1}{3}$ for this termination.



For $t > 2\frac{L}{V}$, a voltage source looks like a short to the transient signal. The reflection from the left is again $-\frac{1}{3}$ for the voltage and $+\frac{1}{3}$ for the current.



When $t \to \infty$, the voltage and current on the line will settle down to a steady state. In that case, we have only DC signal on the line, and we need only to use DC circuit analysis to find the steady state solution. At DC, the transmission line becomes first two pieces of wires, $V_A = V_B = \frac{R}{2R} V_0 = \frac{1}{2} V_0$. The current through the circuit is $\frac{V_0}{Z_0}$. If one is to measure V_A as a function of time, it will look like



Transient analysis has important application to computer circuitry. We note that when we switch on a circuit with a delay line, we do not immediately arrive at the desired steady state value when we have a transmission line or a delay line. The settling time depends on the length of the line involved.