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ECE 350 Lecture Notes

11. Lossy Transmission Lines.

When R and G are not zero, we have a lossy transmission line. In this case,

$$V(z) = V_0(e^{-\gamma z} + \rho_v e^{+\gamma z}) \tag{1}$$

where

$$\gamma = \sqrt{ZY} = \sqrt{(j\omega L + R)(j\omega C + G)} = \alpha + j\beta.$$

The current is derived using the telegrapher's equation to be

$$I(z) = \frac{V_0}{Z_0}(e^{-\gamma z} - \rho_v e^{\gamma z}), \tag{2}$$

where

$$Z_0 = \sqrt{\frac{Z}{Y}} = \sqrt{\frac{j\omega L + R}{j\omega C + G}}.$$

When $\frac{R}{L} = \frac{G}{C}$, then Z_0 becomes frequency independent, and $Z_0 = \sqrt{\frac{L}{C}}$. Also,

$$\gamma = j\omega\sqrt{LC}\left(1 + \frac{R}{j\omega L}\right)^{\frac{1}{2}}\left(1 + \frac{G}{j\omega L}\right)^{\frac{1}{2}} = j\omega\sqrt{LC}\left(1 + \frac{R}{j\omega L}\right) \tag{3}$$

From (3), we see that $\alpha = R\sqrt{\frac{C}{L}} = \frac{R}{Z_0}$ while $\beta = \omega\sqrt{LC}$. Since α is frequency independent, and the $v = \frac{\omega}{\beta} = \frac{1}{\sqrt{LC}}$ is also frequency independent, the transmission line is a distortionless line because any pulse that propagates on the line will not be distorted. This is because a pulse can be thought of as a superposition of Fourier harmonics. Each Fourier harmonic is a time harmonic signal. On a distortionless line, all the Fourier harmonics propagate at the same velocity and suffer the same attenuation. Hence the pulse is not distorted but only diminished in amplitude.

If we divide (1) by (2), we get

$$Z(z) = \frac{V(z)}{I(z)} = Z_0\frac{1 + \Gamma(z)}{1 - \Gamma(z)}, \tag{4}$$

where

$$\Gamma(z) = \rho_v e^{2\gamma z}. \tag{5}$$

Note that (4) also implies that

$$\Gamma(z) = \frac{Z(z) - Z_0}{Z(z) + Z_0} = \frac{Z_n(z) - 1}{Z_n(z) + 1}. \tag{6}$$

Equations (4) and (6) can be solved using a Smith Chart. However, now we have

$$|\Gamma(z)| = |\rho_v| \, e^{2\alpha z}. \tag{7}$$

The amplitude of $|\Gamma(z)|$ is diminishing when we move from the load to the source. From (5), we note that $\Gamma(z) \to 0$ when $z \to -\infty$, $Z(z) \to Z_0$ when $z \to -\infty$. Hence, a long lossy transmission line is always matched. The locus traced out by (7) is a spiral converging on the origin of the Smith Chart when we move from the load to the source.

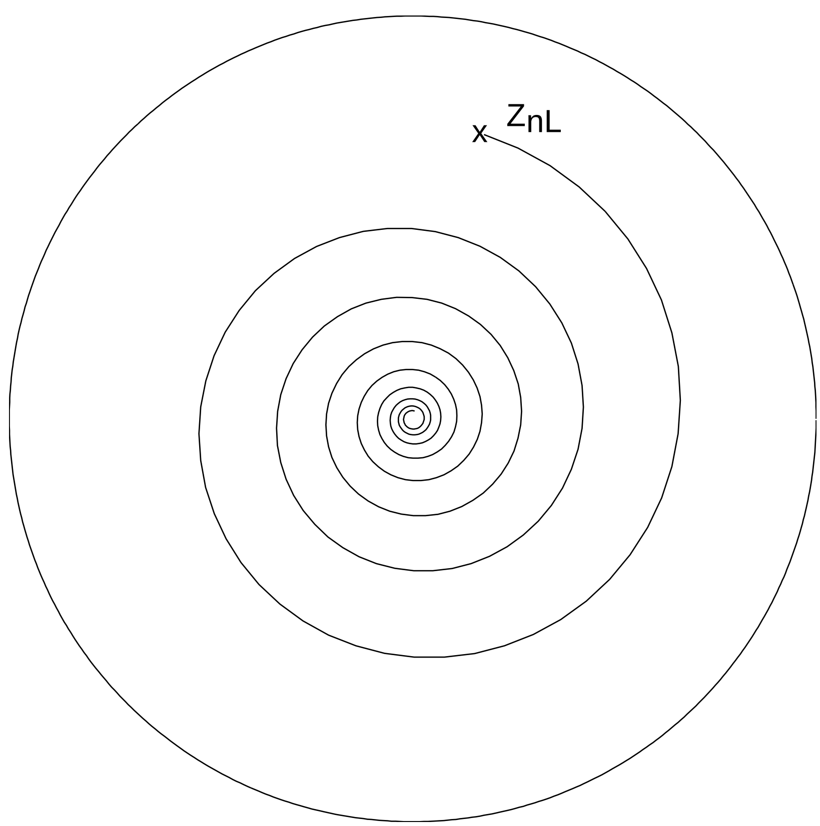
Also, the voltage standing wave pattern is given by

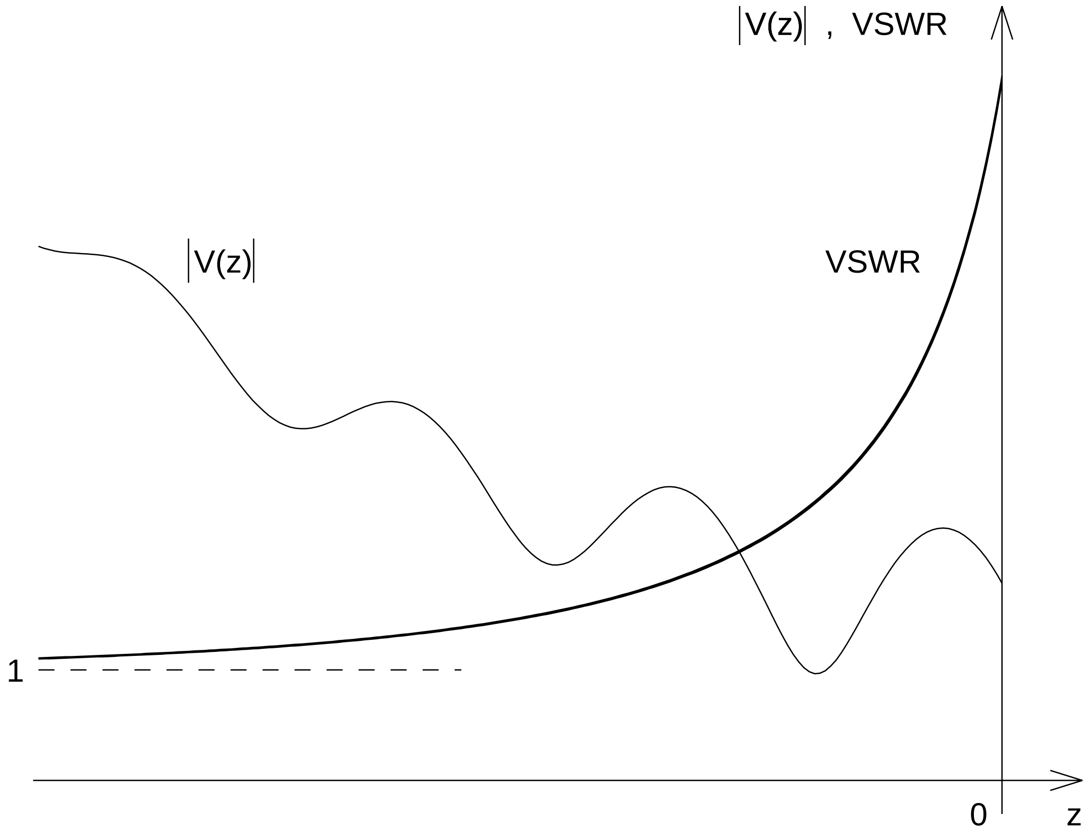
$$|V(z)| = |V_0| \, e^{-\alpha z} \, |1 + \Gamma(z)| \, . \tag{8}$$

A plot of $\Gamma(z)$ and $|V(z)|$ are as shown. Furthermore, we can define an ad hoc VSWR given to be

$$\text{VSWR} = \frac{1 + |\Gamma(z)|}{1 - |\Gamma(z)|} = \frac{1 + |\rho_v| \, e^{2\alpha z}}{1 - |\rho_v| \, e^{2\alpha z}}, \tag{9}$$

which is dependent on z.





Power on a Lossy Line

With $V(z)$ and $I(z)$ given by (1) and (2), one can define a complex power on a lossy line to be

$$\underline{P}(z) = V(z) I^{\star}(z), \tag{10}$$

where

$$V(z) = V_0 e^{-\gamma z}\big(1 + \Gamma(z)\big), \tag{11}$$

and

$$I(z) = \frac{V_0 e^{-\gamma z}}{Z_0}\big(1 - \Gamma(z)\big). \tag{12}$$

Hence,

$$\underline{P}(z) = \frac{|V_0|^2}{Z_0^{\star}} e^{-\gamma z - \gamma^{\star} z}\big(1 + \Gamma(z)\big)\big(1 - \Gamma^{\star}(z)\big), \tag{13}$$

which is equal to

$$\underline{P}(z) = \frac{|V_0|^2}{Z_0^{\star}} e^{-2\alpha\, z}\left[1 - |\Gamma(z)|^2 + 2j\Im m \Gamma(z)\right]. \tag{14}$$

Since $|\Gamma(z)| = |\rho_v|\, e^{2\alpha\, z}$, we have

$$\underline{P}(z) = \frac{|V_0|^2}{Z_0^{\star}} e^{-2\alpha\, z}\left[1 - |\rho_v|^2\, e^{4\alpha\, z} + 2j\Im m \Gamma(z)\right]. \tag{15}$$

We see that both the real part and the imaginary part of the complex power are functions of position. This is because real power is dissipated as the wave propagates. Hence, the real power at one point is not equal to the real power at another point.