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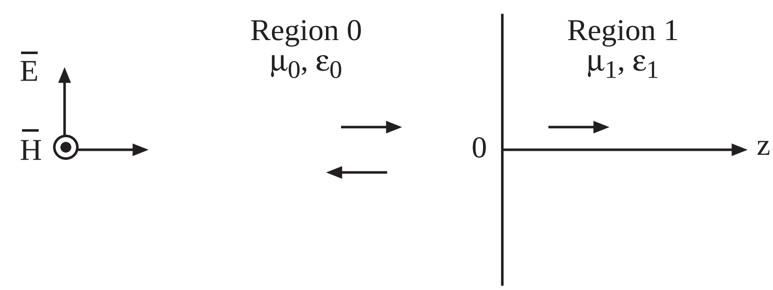
ECE 350 Lecture Notes

19a. Reflection and Transmission of a Simple Plane Wave Off an Interface.

We have learnt that in an infinite free space, a simple plane wave solution exists that is given by

$$\begin{aligned}\mathbf{E} &= \hat{x}E_x(z) = \hat{x}E_0 e^{-j\beta_0 z},\\ \mathbf{H} &= \hat{y}H_y(z) = \hat{y}H_0 e^{-j\beta_0 z} = \hat{y}\frac{E_0}{\eta_0}e^{-j\beta_0 z},\end{aligned} \tag{1}$$

where $\eta_0 = \sqrt{\mu_0/\epsilon_0}$ is the intrinsic impedance, and $\beta_0 = \omega\sqrt{\mu_0\epsilon_0}$ is the wavenumber. Also, $\beta_0 = 2\pi/\lambda_0$ where λ_0 is the free space wavelength.



When the simple plane wave is normally incident on a flat material interface, we expect to have a reflected wave in Region 0, and a transmitted wave in Region 1.

In Region 0, we can write the total fields as

$$\mathbf{E_0} = \hat{x}\left(E_0^+ e^{-j\beta_0 z} + E_0^- e^{+j\beta_0 z}\right), \tag{2}$$

$$\mathbf{H_0} = \hat{y}\left(\frac{E_0^+}{\eta_0}e^{-j\beta_0 z} - \frac{E_0^-}{\eta_0}e^{+j\beta_0 z}\right). \tag{3}$$

In Region 1, the total fields are

$$\mathbf{E_0} = \hat{x}E_1^+ e^{-j\beta_1 z}, \tag{4}$$

$$\mathbf{H_0} = \hat{x}\frac{E_1^+}{\eta_1}e^{-j\beta_1 z}, \tag{5}$$

where $\eta_1 = \sqrt{\mu_1/\epsilon_1}$ and $\beta_1 = \omega\sqrt{\mu_1\epsilon_1}$. There are two unknowns in the above expressions, E_0^- and H_0^+. E_0^+ is known because it is the amplitude if the incident field. We can set up two equations to find two unknowns by matching boundary conditions at $z = 0$. The requisite boundary conditions are that the tangential components of the **E** field and **H** field should be continuous.

By imposing tangential **E** continuous, we arrive at

$$E_0^+ + E_0^- = E_1^+, \tag{6}$$

whereas imposing tangential **H** conditions yields

$$\frac{E_0^+}{\eta_0} - \frac{E_0^-}{\eta_0} = \frac{E_1^+}{\eta_1}. \tag{7}$$

Solving these two equations expresses E_0^- and E_1^+ in terms of E_0^+:

$$E_0^- = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0} E_0^+, \tag{8}$$

$$E_1^- = \frac{2\eta_1}{\eta_1 + \eta_0} E_0^+. \tag{9}$$

We define the reflection coefficient to be

$$\Gamma = \frac{\eta_1 - \eta_0}{\eta_1 + \eta_0}, \tag{10}$$

and the transmission coefficient to be

$$T = \frac{2\eta_1}{\eta_1 + \eta_0}. \tag{11}$$

Notice that $1 + \Gamma = T$.

When there is a mismatch at the interface, we expect most of the wave to be reflected. This occurs when $\eta_1 \ll \eta_0$. In this case, $\Gamma \simeq -1$, and $T \simeq 0$. It also occurs when $\eta_1 \gg \eta_0$, for which case, $\Gamma \simeq +1$, $T \simeq 2$.

The above derivation also holds true when Region 1 is a conductive lossy region. In this case, we replace ϵ_1 with a comlex permittivity $\tilde{\epsilon}_1$ which is given by

$$\tilde{\epsilon}_1 = \epsilon_1 - j\frac{\sigma_1}{\omega}. \tag{12}$$

Then $\eta_1 = \sqrt{\mu_1/\tilde{\epsilon}}$ where η_1 would be a complex number. Also, $j\beta_1$ becomes $\gamma_1 = j\omega\sqrt{\mu_1\tilde{\epsilon}_1} = \alpha_1 + j\beta_1$ which is a complex number also.

For a highly conductive medium like copper, $\sigma_1/\omega \gg \epsilon_1$, $\tilde{\epsilon}_1 \simeq -j\sigma_1/\omega$, and $\eta_1 = (1+j)\sqrt{\omega\mu_1/(2\sigma_1)}$. Consequently, $\eta_1 \ll \eta_0$ and $\Gamma \simeq -1$, $T \simeq= 0$.