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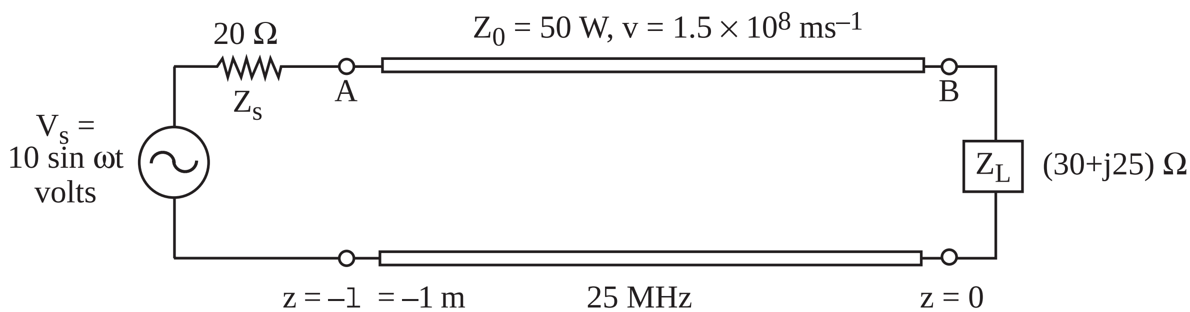
ECE 350 Lecture Notes

8. Examples on Using the Smith Chart

(a) Find the voltages at A on the transmission line.



The voltage source sets up a forward going and a backward going wave on the transmission lines. Hence,

$$V(z) = V_0 e^{-j\beta z} + \rho_v V_0 e^{j\beta z}. \tag{1}$$

The corresponding current is

$$I(z) = \frac{V_0}{Z_0} e^{-j\beta z} - \rho_v \frac{V_0}{Z_0} e^{j\beta z}. \tag{2}$$

In impedance at position Z is

$$Z(z) = \frac{V(z)}{I(z)} = Z_0 \frac{e^{-j\beta z} + \rho_v e^{j\beta z}}{e^{-j\beta z} - \rho_v e^{j\beta z}} = Z_0 \frac{1+\Gamma(z)}{1-\Gamma(z)}, \tag{3}$$

where

$$\Gamma(z) = \rho_v e^{2j\beta z}, \quad \rho_v = \frac{Z_L - Z_0}{Z_l + Z_0}. \tag{4}$$

We can use the Smith Chart to find $Z(-l)$. To use the Smith Chart, we have to normalize all the impedances with respect to the characteristic impedance of the line. Hence,

$$Z_{nL} = \frac{Z_L}{Z_0} = \frac{30 + j25}{50} = 0.6 + j0.5. \tag{5}$$

We can locate Z_{nL} on the Smith Chart which is the complex Γ plane. $\Gamma(0)$ or ρ_v can also be deduced from the Smith Chart. Since $\Gamma(z)$ is given by (4), at $z = -l$, we have

$$\Gamma(-l) = \rho_v e^{-2j\beta l}. \tag{6}$$

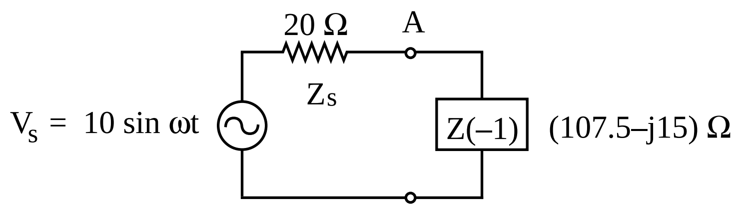
At $f = 25MHz$, and with $v = 1.5 \times 10^8$ ms^{-1}, $\lambda = v/f = 6$m. Then $\beta l = \frac{2\pi}{\lambda} l = \frac{\pi}{3} l$. Therefore,

$$\Gamma(-l) = \rho_v e^{-j\frac{2\pi}{3} l}. \tag{7}$$

At $z = -l = -1m, \Gamma(-1) = \rho_v e^{-j\frac{2\pi}{3}}$. From the Smith Chart, we can read

$$Z_n(-1) = 2.15 - j0.3, \qquad \text{or} \quad \mathrm{Z}(-1) = 107.5 - \mathrm{j}15\,\Omega. \tag{8}$$

So, an equivalent circuit for the point A is:



In phasor representation, $V_S = 10e^{-j\frac{\pi}{2}} = -j10$. Hence,

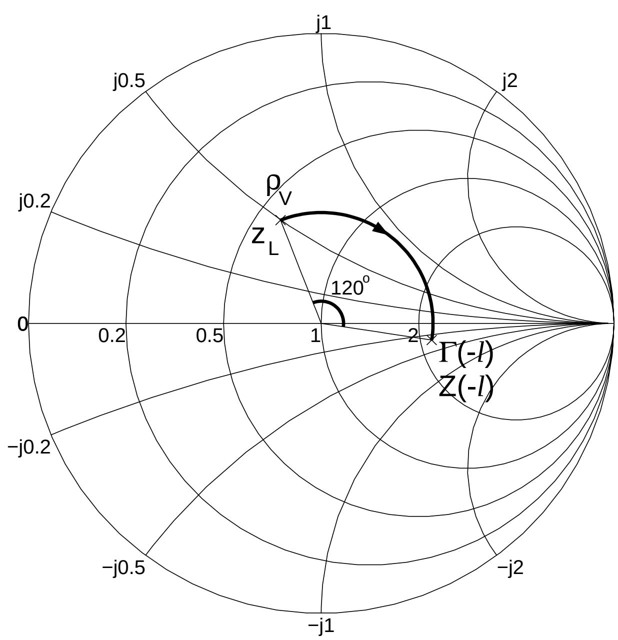
$$V_A = V_S \frac{Z(-1)}{Z_S + Z(-1)} = -j10 \frac{107.5 - j15}{127.5 - j15} = \frac{108.54e^{-j7.9^\circ}}{128.38e^{-j6.7}} e^{-j90^\circ} 10\text{V}$$
$$= 8.5e^{-j91.2^\circ}\text{V}. \tag{9}$$

Since

$$V_A = V(-1) = V_0 e^{j\beta}[1 + \Gamma(-1)], \tag{10}$$

we can find V_o from the above. Once V_o is found, we can find V_B from

$$V_B = V(0) = V_o[1 + \rho_v]. \tag{11}$$



(b) Find Z_L from VSWR and d_{min} using a Smith Chart

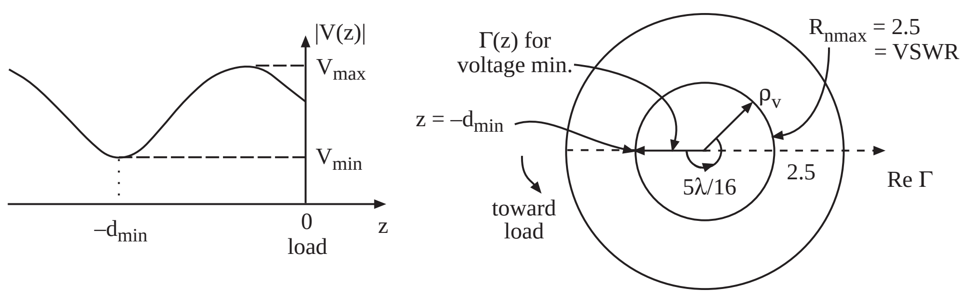
The voltage on the transmission line is

$$V(z) = V_o(e^{-j\beta z} + \rho_v e^{+j\beta z}) = V_o e^{-j\beta z}[1 + \Gamma(z)]. \tag{12}$$

If $V(z) = |V(z)|e^{j\theta(z)}$, the real time voltage can be written as

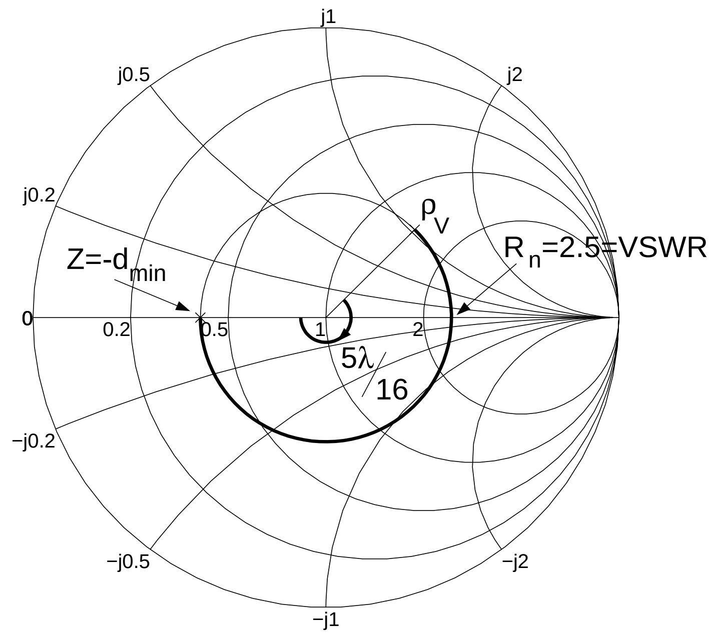
$$V(z,t) = \Re e[|V(z)|e^{j\theta(z)}e^{j\omega t}] = |V(z)|\cos[\,\omega t + \theta(t)]. \tag{13}$$

Hence the amplitude of the real time voltage is proportional to $|V(z)|$ which is the voltage standing wave pattern.



For example, we may be given that the $VSWR = 2.5$ on the line, $Z_o = 75\Omega$, and $d_{min} = 5\lambda/16$, in order to find Z_L.

First, we note that $|V(z)| \propto |1 + \Gamma(z)|$ where $\Gamma(z) = \rho_v e^{2j\beta z}$. Note that V_{min} occurs when $\Gamma(z)$ is purely negative. When z varies, $\Gamma(z)$ traces out a constant circle on the Smith Chart, since $|\Gamma(z)| = |\rho_v|$ is independent of z. Since the $|\Gamma(z)|$ circle must intersect the real Γ axis at $R_n = 2.5$ since the VSWR$= 2.5$, we can deduce that magnitude of $|\Gamma(z)| = |\rho_v|$. Since $z = -d_{min}$ point corresponds to $\Gamma(z)$ as shown above, and the load is $5\lambda/16$ from the d_{min} point, we can figure out ρ_v's location on the Smith Chart. We can read off $Z_{nL} = 1.4 + j1.1$ on the Smith Chart. Hence $Z_L = (105 - j82.5)\Omega$.



j1
j0.5
j2
j0.2
ρ
V
R
n
=2.5=VSWR
Z=-d
min
0
0.2
0.5
1
2
5λ
16
−j0.2
−j0.5
−j2
−j1