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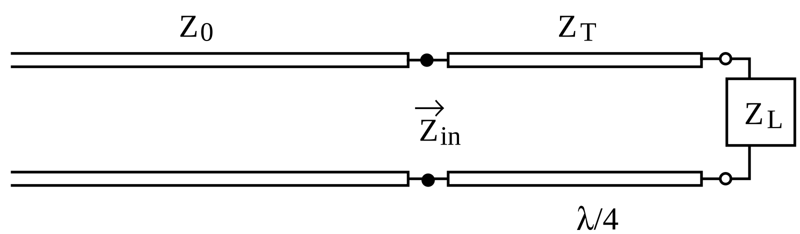
ECE 350 Lecture Notes

10. Impedance Matching on a Transmission Line.

We note that when the impedance of a load is the same as the characteristic impedance of the transmission line, there is no reflected wave, and all the forward going power is dissipated in the load. There are various ways to achieve this ***impedance matching*** and we will discuss some of them below.

(a) Quarter-Wave Transformer

A quarter wave transformer, like low-frequency transformers, changes the impedance of the load to another value so that matching is possible.



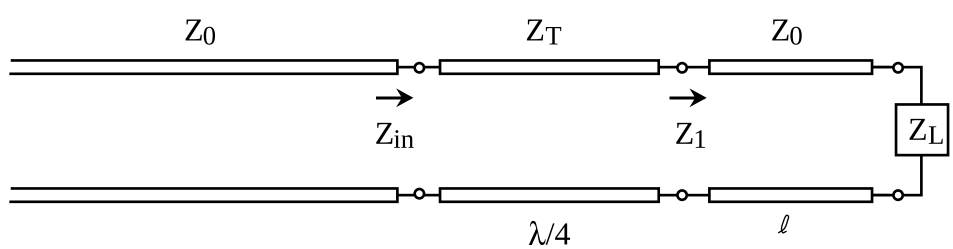
A quarter-wave transformer uses a section of line of characterstic impedance Z_T of $\frac{\lambda}{4}$ long. To have a matching condition, we want $Z_{in} = Z_0$. From Equation (6.11) we have

$$Z_{in} = Z_T \frac{Z_L + jZ_T \tan\frac{\pi}{2}}{Z_T + jZ_L \tan\frac{\pi}{2}} = \frac{Z_T^2}{Z_L}, \tag{1}$$

since $\tan\beta l = \tan\frac{2\pi}{\lambda}\frac{\lambda}{4} = \tan\frac{\pi}{2} = \infty$. In order for $Z_{in} = Z_0$, we need that

$$Z_T^2 = Z_0 Z_L \Rightarrow Z_T = \sqrt{Z_0 Z_l}. \tag{2}$$

If Z_0 and Z_L are both real, then Z_T is real, and we can use a lossless line to perform the matching. If Z_L is complex, it can be made real by adding a section of line to it.



Example

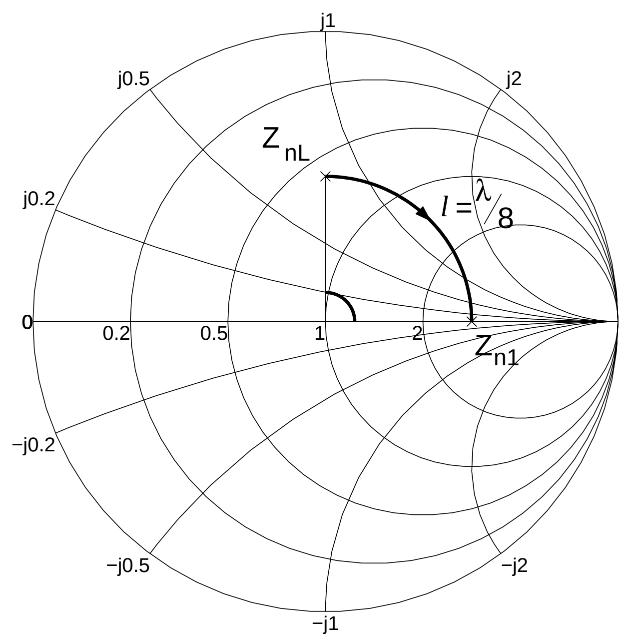
Given that $Z_L = (30 + j40)\Omega$, $Z_0 = 50\Omega$, find the shortest l and Z_T so that the above circuit is matched. Assume that Z_T is real and lossless.

We want Z_1 to be real and Z_{in} to be $Z_0 = 50\Omega$ in order for Z_T to be real and the matching condition satisfied. We find that $Z_{nL} = 0.6 + j0.8$. In order to make Z_{n1} real, the shortest l from the Smith Chart is $\frac{\lambda}{8}$. Then $Z_{n1} = 3.0$, and $Z_1 = 150\Omega$. Since $Z_{in} = 50\Omega$, we need

$$Z_T = \sqrt{Z_{in} Z_1} = \sqrt{50 \times 150} = 86.6\Omega$$

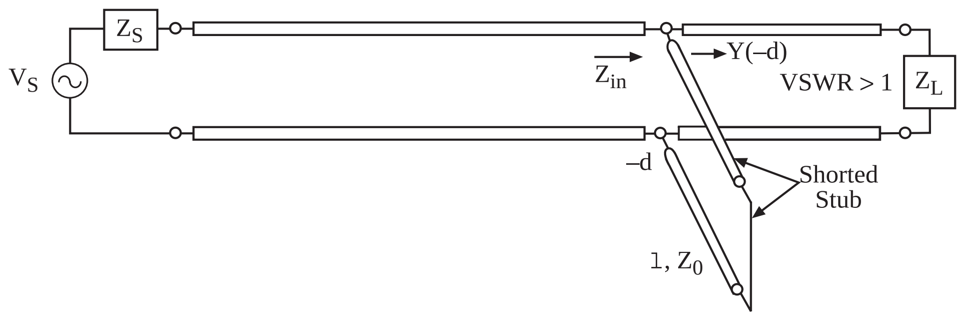
in order for matching condition to be satisfied.

Note that the quarter wave transformer only matches the circuit at one frequency. Often time, it has a small bandwidth of operation, i.e., it only works in the frequencies in a small neighborhood of the matching frequency. Sometimes, a cascade of two or more quarter-wave transformers are used in order to broaden the bandwidth of operation of the transformer.



(b) **Single Stub Tuning**

Another device for performing matching is a single stub (either shorted or opened at one end) which is shunted across the transmission line at $z = -d$ from the load.



The location d is chosen so that the admittance $Y(-d)$ looking toward the load is $Y_0 + jB$ ($Y_0 = \frac{1}{Z_0}$). The length l of the shorted stub is chosen so that its admittance is $-jB$. Hence, when the stub is connected in parallel to the transmission line at $z = -d$, the impedance $Z_{in} = Z_0$, so that matching condition is achieved.

A shorted stub has impedance and admittance given by

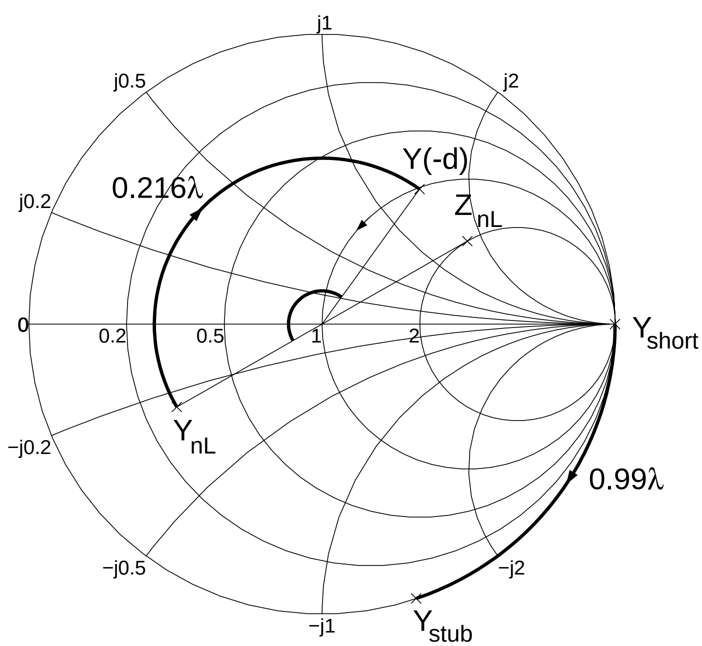
$$Z_s = jZ_0 \tan \beta l, \tag{3}$$

$$Y_s = -jY_0 \cot \beta l. \tag{4}$$

An open-circuited stub can also be used, and the impedance and admittance are given by

$$Z_{op} = -jZ_0 \cot \beta l, \tag{5}$$

$$Y_{op} = jY_0 \tan \beta l. \tag{6}$$



Example

Let $Z_L = (100 + j85)\Omega$, find the minimum d and l that will reduce the VSWR of the main line to 1. Assume that $Z_0 = 50\Omega$.

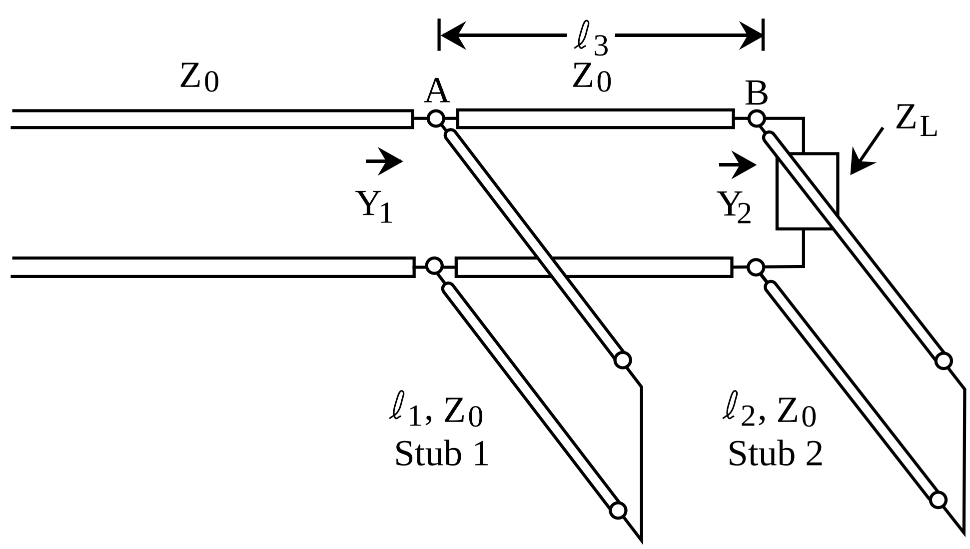
We find that the normalized load $Z_{nL} = 2 + 1.7j$ as shown on the Smith Chart. Since this problem involves parallel connections, it is more convenient to work with admittances. $Y_{nL} = \frac{1}{Z_{nL}}$ is as shown. When we move toward the generator, $Y_n(z)$ traces out a locus on the Smith Chart as shown. It intersects the $G = 1$ circle as shown, after moving through 0.216λ. Therefore, $d = 0.216\lambda$.

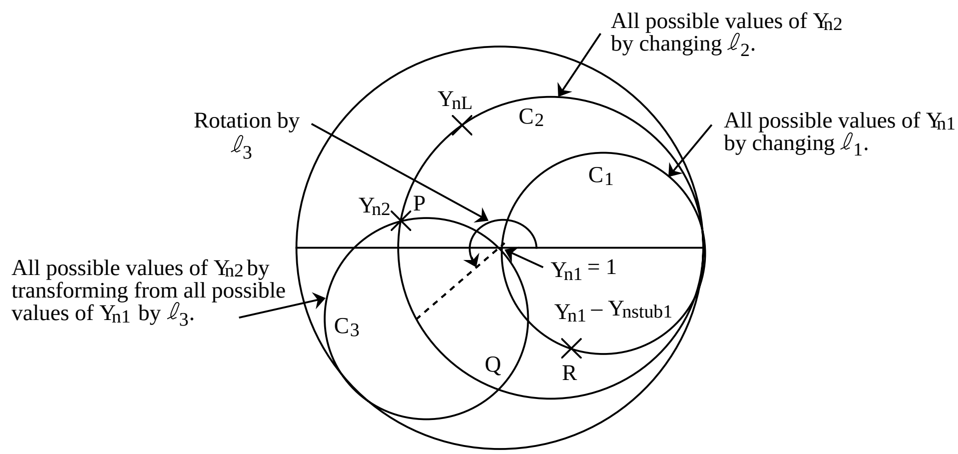
Now, $Y_n(-d) = 1 + j1.4$. Hence, $Y_{n\text{stub}} = -j1.4$. From the Smith Chart, we note that the admittance for a short is infinity, and is at the right end of the Smith Chart. To get a $Y_{n\text{stub}} = -j1.4$, we move toward the generator for 0.099λ. Hence, $l = 0.099\lambda$.

Often time, it is not easy to change d, but quite easy to change l. We note that both in the quarter wave transformer and the single stub tuner, we have to change 2 parameters for tuning. We can provide these 2 degrees of freedom by using two stubs, changing their length, but not their positions.

(c) Double Stub Tuning (optional reading)

Both single stub tuning and quarter wave transformer matching require changing the location of the stub or the transformer. In practice, this is difficult, and a double stub tuning removes the difficulty.





(1) In order to have a matched circuit, we should have $Y_1 = Y_0$ so that $Y_{n1} = 1$. However, if we change l_1, the possible values of Y_{n1} trace out a circle C_1 as shown.

(2) If Y_{nL} is as shown, by changing l_2, the possible values of Y_{n2} trace out a circle C_2 as shown.

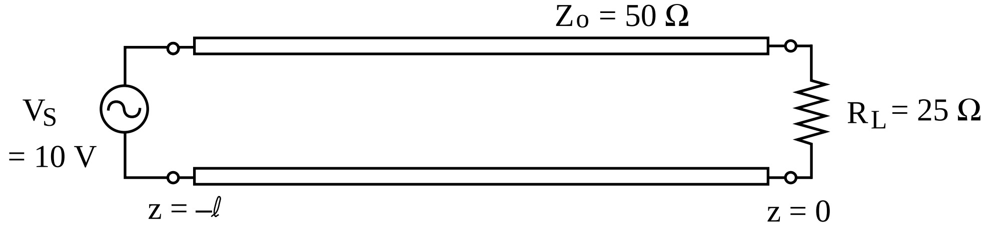
(3) When l_3 is added, all the possible values of Y_{n1} at A is transformed to B by a rotation according to the length of l_3. This constitute a circle C_3 which is all the possible values of Y_{n2} obtained from Y_{n1}. There are only two points, P and Q that the two circles C_2 and C_3 intersect. If we pick P, then this point should correspond to the value of Y_{n2}.

$$Y_{n2} = Y_{nl} + Y_{n\text{stub}2} \tag{25.1}$$

We can figure out $Y_{n\text{stub}2}$ and hence the length l_2.

(4) The length l_3 rotates the point P to the point R. Then R has the impedance $Y_{n1} - Y_{n\text{stub}1} = 1 - Y_{n\text{stub}1}$. We can figure out $Y_{n\text{stub}1}$ from the Smith Chart and hence the length l_1.

(d) **Ferranti Effect**

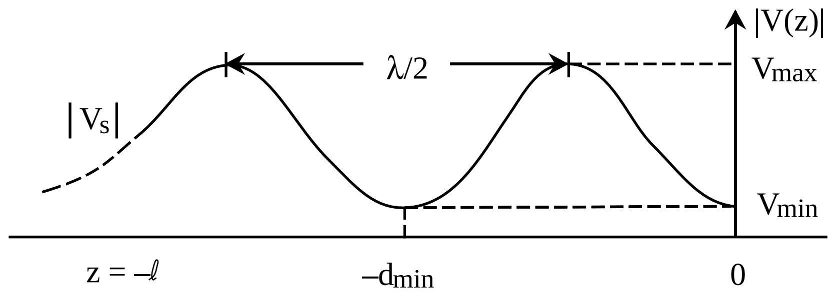


. Find VSWR on the line, and if l is allowed to vary arbitrarily, find the maximum voltage on the line.

We can find VSWR from the Smith Chart or by calculator.

$$P(0) = P_v = \frac{25 - 50}{25 + 50} = -\frac{1}{3},$$

$$\mathrm{VSWR} = \frac{1 + |P_v|}{1 - |P_v|} = \frac{\frac{4}{3}}{\frac{2}{3}} = 2.$$



The voltage at $Z = -l$ is always fixed to be V_s. Hence, we can see that $|V(z)|$ on parts of the transmission line can be longer than $|V_s|$. If l is chosen so that V_s is at V_{min}, then

$$V_{max} = \mathrm{VSWR} \times V_{min} = 10 \text{ volts} \times 2 = 20 \text{ volts}.$$

This amplification of voltage on a line is known as the Ferranti's effect. If the VSWR on the line is very high, V_{max} can be so large that it reaches the breakdown voltage of the line. This is something one should be cautious of in designing transmission line circuits.