ECE 255, BJT and Operating Point

30 January 2018

In this lecture, the current-voltage characteristics of BJT will be discussed.

1 Circuit Symbols and Conventions

Transistors are represented by three terminal devices show in Figure 1. The emitter, denoted by an arrowhead, shows the directions of current for *npn* and *pnp* transistors, which are different. They are usually shown as drawn with currents flowing from top to down.

In their active modes, the transistors are rigged up as shown in Figure 2. Note that in the above, for the *npn* and *pnp* transistors, the EBJ are both forward biased, namely, V_{BE} and V_{EB} are both positive. To remain in reverse bias, V_{CB} for *npn* transistor should be larger that -0.4 V. Similarly for *pnp* transistor, V_{BC} should be below 0.4 V. This biasing need is shown in Figure 3. Moreover, the BJT current-voltage relationships are shown in Figure 4 (or Table 6.2 of textbook).

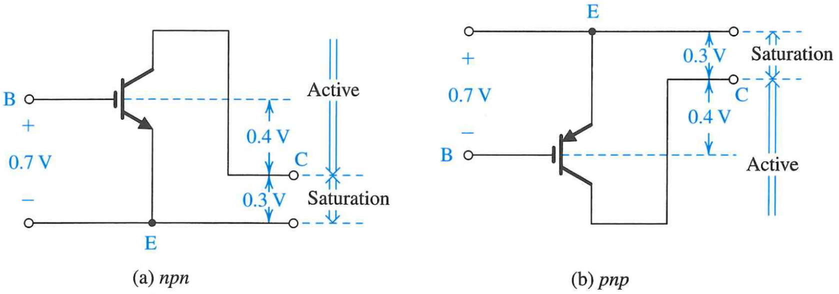
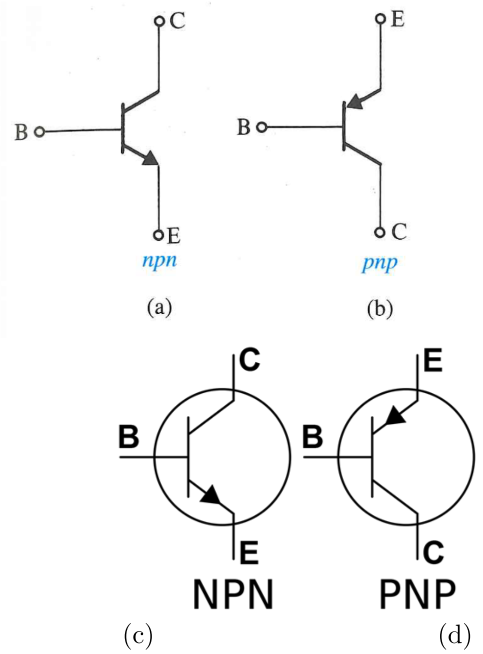


Figure 3: Biasing needs of *npn* and *pnp* transistors to remain in active mode (Courtesy of Sedra and Smith).

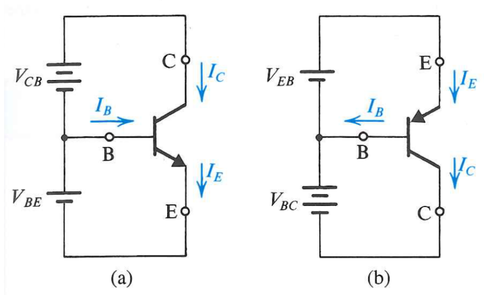
Printed on March 14, 2018 at 10 : 32: W.C. Chew and S.K. Gupta.



(a) (b)

(c) (d)

Figure 1: Circuit symbols for BJTs (Courtesy of Sedra and Smith, and Quora).



(a) (b)

Figure 2: Transistors rigged up in their active mode of operation, (a) for *npn* transistors, and (b) for *pnp* transistors (Courtesy of Sedra and Smith).

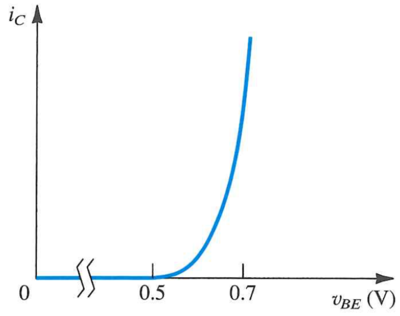


Figure 5: The i_C-v_{BE} relation of an *npn* transistor (Courtesy of Sedra and Smith).

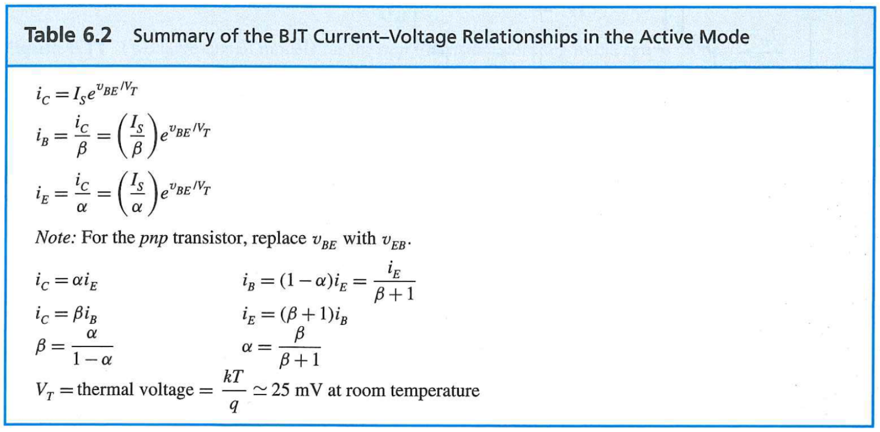


Table 6.2 Summary of the BJT Current–Voltage Relationships in the Active Mode

$i_C = I_S e^{v_{BE}/V_T}$

$i_B = \dfrac{i_C}{\beta} = \left(\dfrac{I_S}{\beta}\right) e^{v_{BE}/V_T}$

$i_E = \dfrac{i_C}{\alpha} = \left(\dfrac{I_S}{\alpha}\right) e^{v_{BE}/V_T}$

Note: For the *pnp* transistor, replace v_{BE} with v_{EB}.

$i_C = \alpha i_E$

$i_C = \beta i_B$

$\beta = \dfrac{\alpha}{1-\alpha}$

$i_B = (1-\alpha) i_E = \dfrac{i_E}{\beta+1}$

$i_E = (\beta+1) i_B$

$\alpha = \dfrac{\beta}{\beta+1}$

$V_T = \text{thermal voltage} = \dfrac{kT}{q} \simeq 25\text{ mV at room temperature}$

Figure 4: Current-voltage relationships of transistor in active mode (Courtesy of Sedra and Smith).

2 Graphical Representation of Transistor Characteristics

Since we are gifted in surmising the physical characteristics of many data by glancing at a graph, it is expedient to display the current-voltage characteristics with a graph. As has been shown before, the current i_C is related to the biasing voltage v_{BE} by the formula

$$i_C = I_S e^{v_{BE}/V_T} \tag{2.1}$$

A plot of such a relationship is shown in Figure 5.

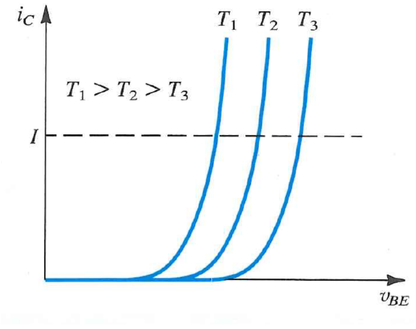


Figure 6: The temperature effect on i_C-v_{BE} curve. At constant emitter current, v_{BE} changes by -2 mV/°C (Courtesy of Sedra and Smith).

The collector current i_C is proportional to I_S, the saturation current. In our previous lecture, it has been discussed previously that the saturation current I_S is proportional to n_i^2, but $n_i \sim T^{3/2} e^{-E_g/2k_BT}$. Therefore,

$$i_C \sim T^3 e^{-E_g/(k_BT)} e^{v_{BE}/V_T} = T^3 e^{-(E_g/q)/V_T} e^{v_{BE}/V_T} \tag{2.2}$$

Since the bandgap of silicon is around 1.1 eV, $E_g/q \approx 1.1$ V. Defining $V_g = E_g/q$, the above can be written as

$$i_C \sim T^3 e^{-(V_g - v_{BE})/V_T} \tag{2.3}$$

Since $v_{BE} \approx 0.7$ V, then $V_g - v_{BE}$ is a positive number. And then i_C increases when V_T increases, where $V_T = k_BT/q$, or i_C increases when the temperature T increases. Alternatively, one can take the natural log of the above to arrive at

$$V_T \ln\left(\frac{i_C}{CT^3}\right) = -\left(V_g - v_{BE}\right) \tag{2.4}$$

Since the right-hand side is a negative number, the left-hand side is also a negative number. Rewriting the above,

$$v_{BE} = V_g + V_T \ln\left(\frac{i_C}{CT^3}\right) \tag{2.5}$$

where the second term becomes more negative as T increases. In a word, for a fixed i_C, v_{BE} becomes smaller as T increases.

This effect is shown in Figure 6. Alternatively, v_{BE} drops by 2 mV for every °C.

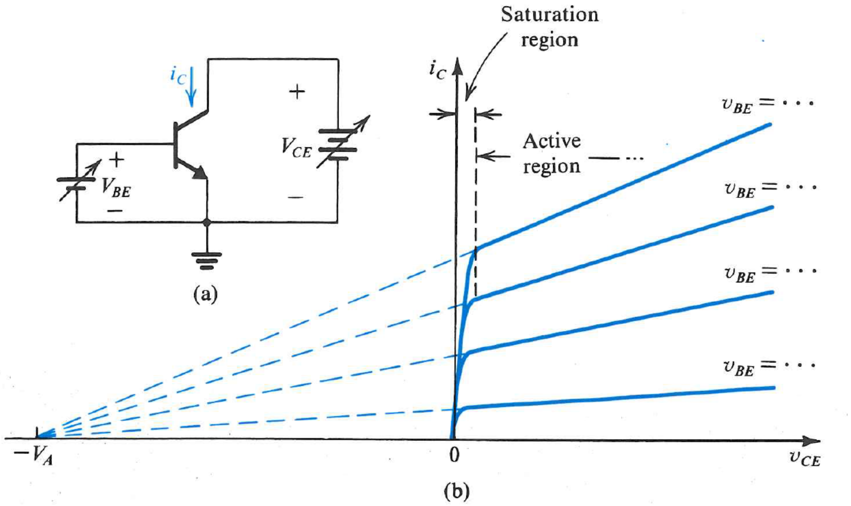


Figure 7: Circuit for measuring the i_C-v_{CE} curve and the resultant curves (Courtesy of Sedra and Smith).

3 The Early Effect–The Collector Current Effect on i_C

This effect was first observed by J.M. Early, and hence the name. It is also called the **base-narrowing**, or **base-width modulation effect**. As the reverse bias in the CBJ is increased, the depletion layer becomes larger. Therefore, the base region becomes thinner, or its width W is smaller. Again it is noted that

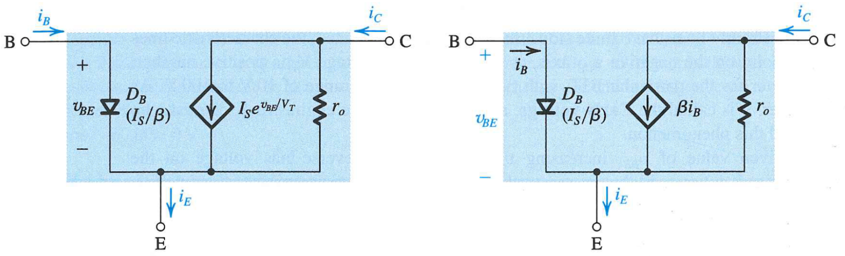
$$i_C = I_S e^{v_{BE}/V_T} \tag{3.1}$$

where

$$I_S = \frac{A_E q D_n n_i^2}{N_A W}$$

The reason why I_S increases as the base region width W is thinner because the gradient of the injected minority carrier in the base region is inversely proportional to W. Accordingly, as v_{CE} increases, the depletion region gets larger, and W decreases. As a result, i_C increases with v_{CE} for a fixed v_{BE} as shown in Figure 7(b). The configuration of the transistor in Figure 7(a) is called the **common-emitter configuration**. The characteristic curve is shown in Figure 7(b), is called the **common-emitter characteristics**.

When the v_{CE} gets exceedingly large, the depletion region can spread across the entire base region, and this is called the **punch-through** effect. This happens if the base region is thin, but the transistor may experience a breakdown when v_{CE} gets exceedingly large.



(a) (b)

Figure 8: The equivalent circuits of *npn* BJT transistor in the active mode with common-emitter configuration with the output resistance r_O. (a) For voltage controlled current source. (b) For current controlled current source (Courtesy of Sedra and Smith).

Also, when V_{CE} is lower than about 0.3 V, there is not enough reverse bias voltage across CBJ, the the transistor lapses into the saturation mode.

Assuming a linear dependence of i_C on v_{CE}, one can write

$$i_C = I_S e^{v_{BE}/V_T}\left(1+\frac{v_{CE}}{V_A}\right) \tag{3.2}$$

Defining an incremental **output resistance** as

$$r_O = \left(\left.\frac{\partial i_C}{\partial v_{CE}}\right|_{v_{BE}=\text{constant}}\right)^{-1} \tag{3.3}$$

then it can be shown that

$$r_O = \frac{V_A}{I'_C}, \qquad I'_C = I_S e^{V_{BE}/V_T} \tag{3.4}$$

The above can also be written as

$$r_O = \frac{V_A + V_{CE}}{I_C}, \qquad I_C = I'_C/\left(1+\frac{V_{CE}}{V_A}\right) \tag{3.5}$$

The output resistance r_O can be incorporated into the circuit model of the transistor as shown in Figure 8.

4 Alternative Way to Characterize a Transistor

A transistor can also be characterize by feeding it with a constant i_B or base current. This is shown in Figures 9(a) and 9(b). At the operating point Q, one can calculate the transistor $\beta = \frac{I_C}{I_B}$.

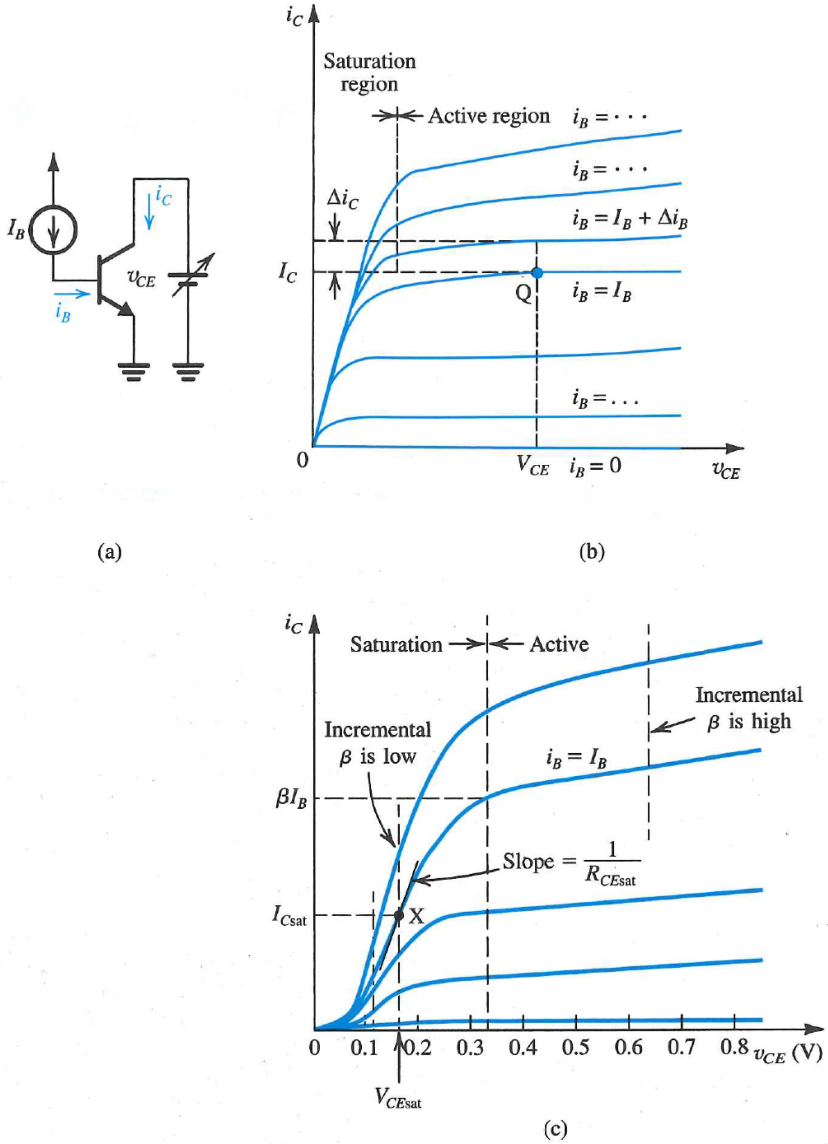


Figure 9: Another way to characterize a transistor by injecting it with a constant base current i_B (Courtesy of Sedra and Smith).

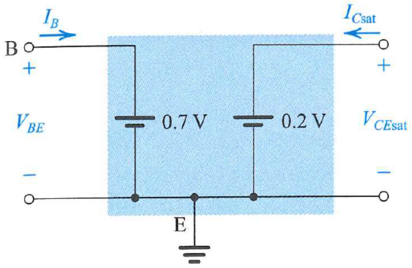


Figure 10: An equivalent circuit model of a saturated transistor where both junctions are forward biased (Courtesy of Sedra and Smith).

In the saturation region, i_C increases rapidly with v_{CE}. One can define an incremental resistance for this region called the transistor collector-to-emitter resistance $R_{CE\text{sat}}$ defined as

$$R_{CE\text{sat}} = \left.\frac{\partial v_{CE}}{\partial i_C}\right|_{i_\beta = I_\beta, i_C = I_{C\text{sat}}} \tag{4.1}$$

where $R_{CE\text{sat}}$ ranges from a few ohms to few tens of ohms.

Example 1.[1]

For a circuit shown in Figure 11, we need the value of V_{BB} for the transistor to operate in:

1. In the active mode with $V_{CE} = 5$ V.
2. At the edge of saturation.
3. Deep in saturation with $\beta_{\text{forced}} = 10$.

Assume that V_{BE} stays constant at 0.7 V and the transistor $\beta = 50$.

Solution

1. To operate in the active mode with $V_{CE} = 5$ V,

$$I_C = \frac{V_{CC} - V_{CE}}{R_C} = \frac{10 - 5}{1k\Omega} = 5 \text{ mA} \tag{4.2}$$

$$I_B = \frac{I_C}{\beta} = \frac{5}{50} = 0.1 \text{ mA} \tag{4.3}$$

Now the required value of V_{BB} can be found as follows:

$$V_{BB} = I_B R_B + V_{BE} = 0.1 \times 10 + 0.7 = 1.7 \text{ V} \tag{4.4}$$

---

[1] Example 6.3 of textbook.

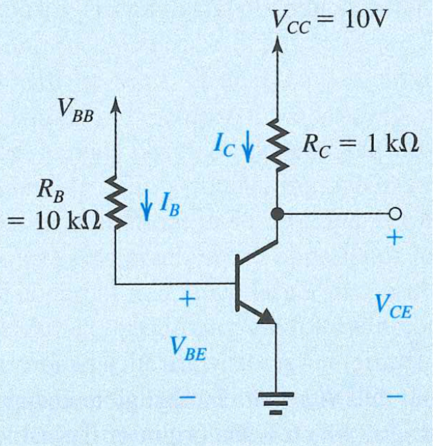


Figure 11: Circuit model for this example (Courtesy of Sedra and Smith).

2. Operation at the edge of saturation is obtained with $V_{CE} = 0.3$ V. Thus

   $$I_C = \frac{10 - 0.3}{1} = 9.7 \text{ mA} \tag{4.5}$$

   Since at the edge of saturation, I_C and I_B are still related by β,

   $$I_B = \frac{9.7}{50} = 0.194 \text{ mA} \tag{4.6}$$

   Then V_{BB} can be found as $V_{BB} = 0.194 \times 10 + 0.7 = 2.64$ V.

3. To operate in deep saturation, assume that $V_{CE} = V_{CE\text{sat}} \approx 0.2$ V, then

   $$I_C = \frac{10 - 0.2}{1} = 9.8 \text{ mA} \tag{4.7}$$

   The value of forced β can be used to determine the required I_B as

   $$I_B = \frac{I_C}{\beta_{\text{forced}}} = \frac{9.8}{10} = 0.98 \text{ mA} \tag{4.8}$$

   and the required V_{BB} can now be found as $V_{BB} = 0.98 \times 10 + 0.7 = 10.5$ V.

   Notice that I_C changes little before and after saturation. Hence, increasing V_{BB}, and thus I_B does not change I_C, and the transistor loses its ability to amplify I_B.