ECE 255, PN Junction

11 Jan 2018

When semiconductor materials are doped, they become extrinsic semiconductors displaying different properties. Due to the abundance presence of holes or electrons, they become much better conductors than intrinsic semiconductors. The holes and electrons are carriers of electricity when they move about. These carriers move around in a random walk fashion, colliding with the lattice or impurities in the material. On the average, random walk motion does not give rise to a net flow of current. Their motion is like the Brownian motion of molecules.

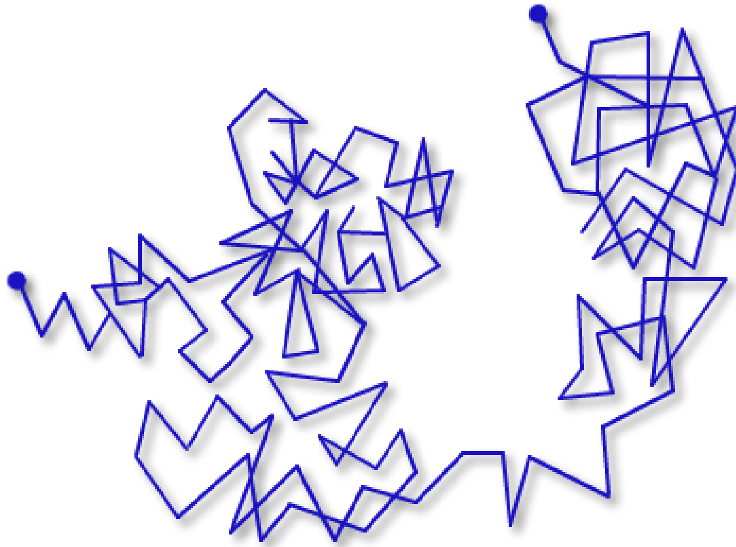

Figure 1: A random walk motion of a carrier particle.

There are two ways to make these carriers move in a preferred direction on average: one by applying an electric field giving rise to **drift current**; and second, by causing the concentration of carriers to be non-uniform yielding **diffusion current**. These two kinds of current will be discussed next.

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1 Drift Current

An electric field applies a positive force on a positively charged particle. A hole effectively behaves like a positively charged particle, and will be pushed in the direction of the applied electric field. On the other hand, an electron will be driven in the opposite direction when an electric field is applied.

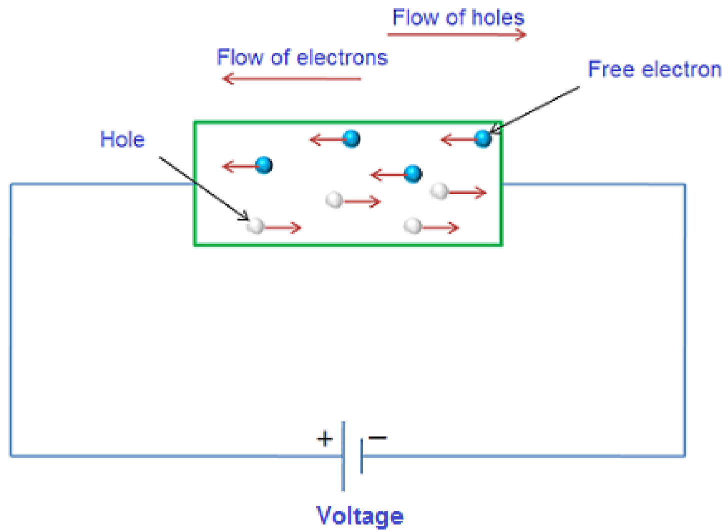

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Figure 2: The electrons and holes flow in opposite directions, on the average, in the presence of an electric field, or an applied potential difference.

Because of this, the holes and electrons will acquire an average drift velocity v_p and v_n proportional to the strength of the electric field E. In other words,

$$v_p = \mu_p E \tag{1.1}$$
$$v_n = -\mu_n E \tag{1.2}$$

where μ_p and μ_n are the **mobilities** of the holes and electrons respectively. They are indicators of how mobile the holes and electrons are. Typically, holes have mobility of ≈ 480 cm^2/V·s, while electrons have mobility of ≈ 1350 cm^2/V·s. Electrons are seen to be more mobile than holes, due to their lighter weight and no bond-breaking is associated with their movements.

The average current density is given by I/A in terms of amperes per unit area. If the average charge density Q is known, then the average current density due to the charge moving at velocity v is Qv. The drift current due to electrons

is then

$$\begin{aligned} J_n^{\text{drift}} = Q_n v_n &= (-qn)(-\mu_n E) \\ &= q\mu_n nE \quad \text{A/cm}^2 \end{aligned} \tag{1.3}$$

$$\begin{aligned} J_p^{\text{drift}} = Q_p v_p &= (+qp)(+\mu_p E) \\ &= q\mu_p pE \quad \text{A/cm}^2 \end{aligned} \tag{1.4}$$

where q is one electron charge which is 1.6×10^{-19} coulomb. The total current then is the sum of these two currents, namely,

$$J_{\text{total}}^{\text{drift}} = J_n^{\text{drift}} + J_p^{\text{drift}} = q(n\mu_n + p\mu_p)E \tag{1.5}$$

Consequently, one can define

$$J = \sigma E \tag{1.6}$$

where J is the total current driven by the electric field E, and σ is the effective conductivity of the medium. The conductivity is then

$$\sigma = q(n\mu_n + p\mu_p) \quad (\Omega \cdot \text{cm})^{-1} \tag{1.7}$$

Similarly, the resistivity is

$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)} \quad (\Omega \cdot \text{cm}) \tag{1.8}$$

Note that both the electrons and holes contribute to the total current. Whichever is the dominant carrier of the current depends on the concentrations of the carriers n or p. For a p-doped material, $p \gg n$, and for an n-doped material, $n \gg p$.

For intrinsic, undoped silicon, Si, $n = p = n_i = p_i \approx 10^{10}/\text{cm}^3$ at room temperature. Using the mobility that $\mu_p = 480$ cm^2/(V·s), and that $\mu_n = 1350$ cm^2/(V·s), one arrives at

$$\rho = \frac{1}{q(n\mu_n + p\mu_p)} \approx 2 \times 10^5 \quad \Omega \cdot \text{cm} \tag{1.9}$$

For extrinsic doped silicon, let $N_A = 10^{16}/\text{cm}^3$, and that $N_D = 2 \times 10^{15}/\text{cm}^3$, then

$$p \approx N_A \approx 10^{16}/\text{cm}^3, \qquad n = \frac{n_i^2}{p} \approx 10^4/\text{cm}^3 \tag{1.10}$$

Consequently,

$$\rho = \frac{1}{q(n\mu_n + p\mu_p)} \approx 1.6 \quad \Omega \cdot \text{cm} \tag{1.11}$$

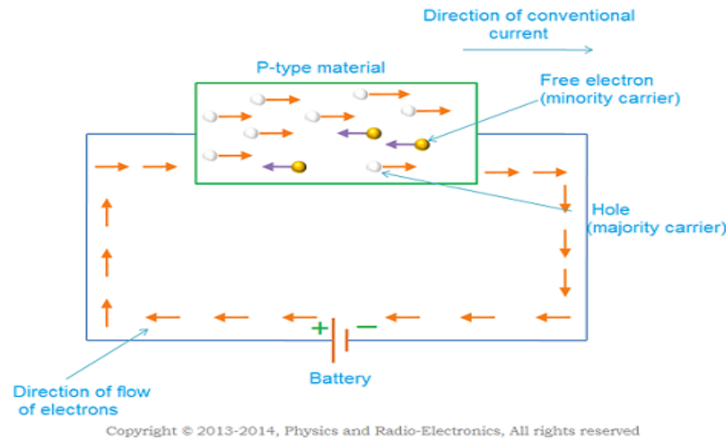

Figure 3: The pictorial representation of the total current in a doped p-type semiconductor. The majority carrier is hole here, even though both holes and electrons contribute to the total current.

2 Diffusion Current

Another way that the carriers can move about in a preferred direction is due to diffusion. Diffusion is a common phenomenon seen in gases. In gases, the kinetic theory of gas is used to explain this phenomenon. Gas molecules are constantly moving around and colliding with each other. In a region where the gas concentration is high, the rate at which gas molecules collide with each other is higher, and consequently, forces the gas molecules to move away from regions of high concentration to regions of low concentration.

However, the carriers in a semiconductor do not actually collide with each other as in gas molecules. To begin, one notes that a semiconductor material is actually charge neutral. But instead, in a semiconductor, each carrier particle, like a hole (electron) is either positively (negatively) charged. Therefore, they repel each other, and in regions of high concentration of carriers of the same kind, they tend to repel each other more than the region of low concentration. Consequently, at equilibrium, the carrier concentration will be evenly distributed. The uneven distribution of carrier concentration gives rise to the flow of these carriers is attributed to diffusion.

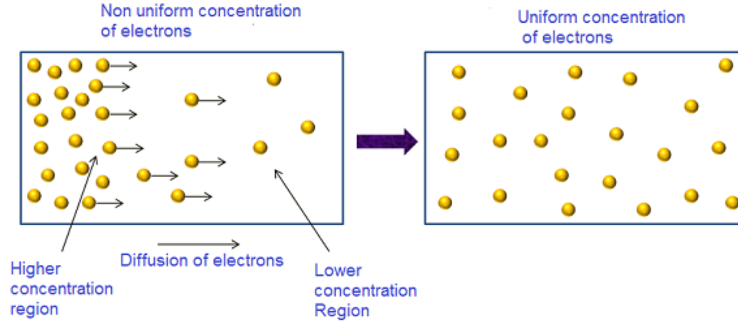

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Figure 4: The carriers flow from the region of high concentration to region of low concentration due to repulsion between the carriers.

Hence, one can safely assume that the diffusion current is due to the uneven concentration of the carriers or **concentration gradient**. In one dimension, if the electron carrier concentration is $n(x)$, then the diffusion current is proportional to $\frac{dn}{dx}$ the gradient of the concentration. Therefore,

$$J_n^{\text{diff}} = (-q)D_n\left(-\frac{dn}{dx}\right) = +qD_n\frac{dn}{dx} \tag{2.1}$$

$$J_p^{\text{diff}} = (+q)D_p\left(-\frac{dp}{dx}\right) = -qD_p\frac{dp}{dx} \tag{2.2}$$

$$\tag{2.3}$$

where D_n and D_p are **diffusion coefficients** (also called **diffusion constant** or **diffusivity**) for the electron and hole respectively. Diffusion coefficient has a unit of cm^2/s.

Diffusion Current and Recombination Rate
A possible uneven concentration profile of hole is

$$p(x) = p_0 e^{-x/L_p} \tag{2.4}$$

$$\frac{dp}{dx} = -\frac{1}{L_p}p_0 e^{-x/L_p} \tag{2.5}$$

This implies that the diffusion current, hole concentration, and the gradient are proportional to each other, or that $\frac{dp}{dx} \sim p(x) \sim J_p^{\text{diff}}$. Recombination of electrons and holes can give rise to a gradient of the hole concentration. Such recombination rate is proportional to np. If n is the **majority carrier**, recombination affects its concentration very little, but if p is a **minority carrier**, it is much affected by recombination. A case where the diffusion current is proportional to the concentration will have an exponentially decaying solution.

In general, the total current is again the contribution from both types of

carriers, namely,

$$J_{\text{total}}^{\text{diff}} = qD_n \frac{dn}{dx} - qD_p \frac{dp}{dx} \tag{2.6}$$

When drift and diffusion currents co-exist, the total current is then

$$J_n = J_n^{\text{drift}} + J_n^{\text{diff}} = q\mu_n nE + qD_n \frac{dn}{dx} \tag{2.7}$$

$$J_p = J_p^{\text{drift}} + J_p^{\text{diff}} = q\mu_p pE - qD_p \frac{dp}{dx} \tag{2.8}$$

The total current is the sum of these two currents, namely, $J = J_n + J_p$.

3 *pn* Junction

Now, one would like to know what happens when one puts a p-type material next to an n-type material. This is known as a pn-junction. The p-type material could have been doped with phosphorous, and the n-type material could have been doped with boron. At the junction of these two materials, interesting physics happens.

The first thing that happens when two junctions come into contact is the diffusion phenomenon. The preponderance of p-type carriers in the p-type material will cause their diffusion across the junction into the n-type material. The same for the n-type carries in the n-type material. A **carrier diffusion region** or a **space charge region** is formed.

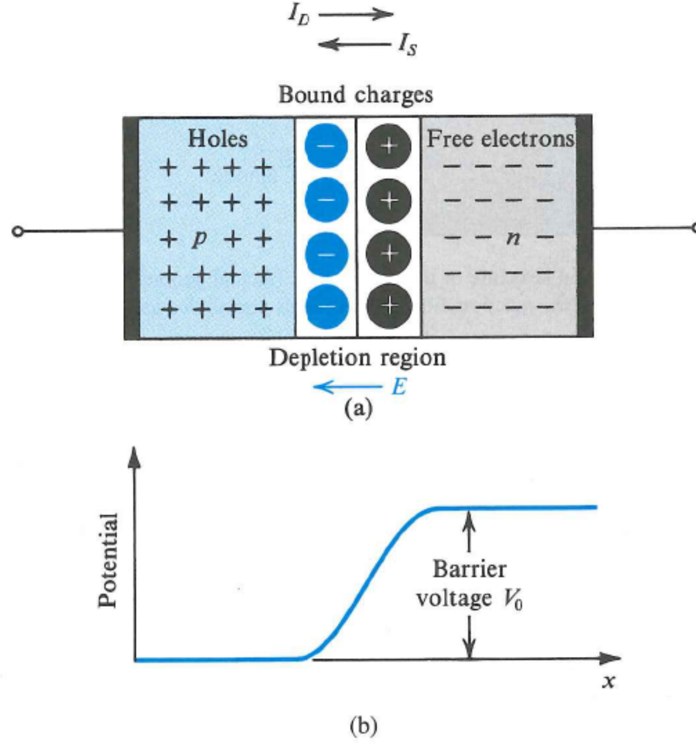

Figure 5: A pn junction of two different materials. The thicknesses of the depletion region need not be the same (Courtesy of Sedra and Smith).

Therefore, an internal electric field is established that prevents further migration of the carriers across the junction. At this juncture, $J^{\text{drift}} = -J^{\text{diff}}$ for both kinds of carriers. As shown in the Appendix, the hole density and the potential ϕ are related to each other according to Boltzmann's law, namely

$$p(x) \sim e^{-q\phi(x)/(k_B T)} \tag{3.1}$$

Hence, the voltage drop across a pn junction at equilibrium is given as

$$V_0 = \phi_j = \frac{k_B T}{q} \ln \frac{p(x = -x_p)}{p(x = +x_n)} \tag{3.2}$$

where $p(x = -x_p)$ is the hole density in the p region (see Figure 6) or the left edge of the transition region (also called depletion region) of the junction, while similarly $p(x = +x_n)$ in the n region. In the p region, holes are majority carriers and its density is given by

$$p(-x_p) = N_A \tag{3.3}$$

while in the n region, holes are minority carrier, and the hole density is given by

$$p(+x_n) = n_i^2/N_D \tag{3.4}$$

Substituting the above back in (3.2), the following equation is arrived at yielding a formula for the voltage or potential difference between the two materials

$$V_0 = \phi_j = \frac{k_B T}{q} \ln \frac{N_D N_A}{n_i^2} \tag{3.5}$$

The **thermal voltage** $V_T = k_B T/q$ which is about 25 mV at room temperature. Here, again, $k_B = 8.62 \times 10^{-5}$ K$^-$1eV is the Boltzmann constant, and T is in degree Kelvin. Moreover, because the diffusion current and the drift current cancel each other, it can be established that there is a relationship, called the **Einstein relationship**, between the diffusion coefficient D and mobility μ. Namely that (see Appendix)

$$\frac{D_n}{\mu_n} = \frac{D_p}{\mu_p} = \frac{k_B T}{q} = V_T \tag{3.6}$$

Since the thermal voltage is a constant for constant T, the above implies that once the mobility of a medium is known, the diffusivity is known. The above formulas also convey the notion that the higher the temperature T, the higher the diffusion coefficients. The higher the temperature, the more kinetic energy the electrons will acquire, and the faster they will diffuse.

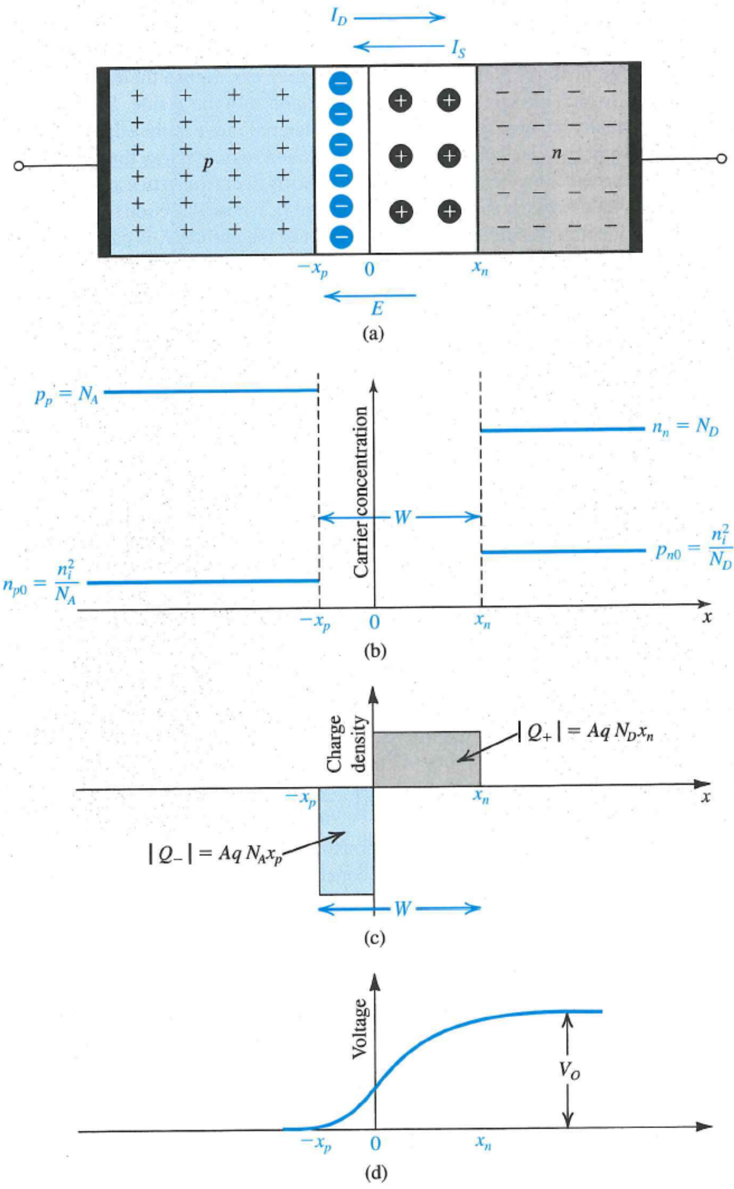

Figure 6: A pn junction of two different materials in the open circuit case. The thicknesses of the depletion region are not the same with x_n denoting the thickness of the n region, and x_p denoting the thickness of the p region (Courtesy of Sedra and Smith).

Appendix A Derivation of the Einstein Relationship

The Einstein relationship is obtain by relatinng the equilibrium state of a *pn* junction to a state in thermal equilibrium, which should obey Boltzmann's law. When a junction is in equilibrium, the drift current cancels the diffusion current. In other words, looking at the hole current alone,

$$J_p(x) = q\left[\mu_p p(x)E(x) - D_p\frac{dp(x)}{dx}\right] = 0 \tag{A.1}$$

In the above, one can define $E(x) = -\frac{d\phi(x)}{dx}$ to obtain the equation

$$-\frac{\mu_p}{D_p}\frac{d\phi(x)}{dx} = \frac{1}{p(x)}\frac{dp(x)}{dx} \tag{A.2}$$

The right-hand side can be rewritten as

$$-\frac{\mu_p}{D_p}\frac{d\phi(x)}{dx} = \frac{d\ln p(x)}{dx} \tag{A.3}$$

After defining

$$\frac{q}{k_B T} = \frac{\mu_p}{D_p} \tag{A.4}$$

the above equation can be integrated to yield

$$p(x) = Ce^{-q\phi(x)/(k_B T)} \tag{A.5}$$

where C is an arbitrary constant independent of x. The above is just Boltzmann's law, and the derivation can be repeated for electron carriers. Moreover, (A.4) can be used to derive (3.6).

The above derivation indicates that the diffusion process is in fact a "thermalized" process. The fact that carriers diffuse is because they acquire kinetic energy from the thermal environment. The heat bath supplies energy to the material giving rise to lattice vibrations. These lattice vibrations in turn transfer kinetic energy to the electrons causing them to diffuse. The diffusion of the charged carriers then gives rise to uneven potential in the environment, yielding internal electric field that stops the diffusion process. The equilibrium of the diffusion current and the drift current is due to "thermal equilibrium". Hence, the carrier distributions must obey Boltzmann's law.