ECE 255

12 October 2017

In this lecture, the basic configurations of BJT amplifiers will be studied. Previously, it has been shown that with the transistor DC biased at the appropriate point, linear relations can be derived between the small voltage and current signals. With these linear relations, the principles of linear systems can be applied to solve for the node voltages and branch currents. Moreover, they can be easily handled by commercial software such as SPICE for highly complex circuits as long as they are linear. These large complex circuit problems can be cast into solving a set of linear algebraic equations which can be solved efficiently by computers.

The take home message here is that linear problems are a lot simpler to solve compared to nonlinear problems. Hence, nonlinear problems are linearized with the small signal approximations before they are solved.

1 The Three Basic Configurations

The three basic configurations of a BJT are (a) common emitter (CE), (b) common base (CB), (c) common collector (CC) or emitter follower. These basic configurations are shown in Figure 1.

The replacement of the basic configurations with equivalent circuit models and the small signal approximations convert the original nonlinear problems into linear ones, greatly simplifying their analysis.

2 Characterizing Amplifiers

An amplifier can be denoted by a functional block as expressed in Figure 2(a), where a triangle block encapsulates the details of the small-signal equivalent circuit model as shown in Figure 2(b). The equivalent circuit model indicates that the amplifier has a finite internal impedance, R_{in}. Moreover, it has a finite output resistance R_o, which can be found by the test-current method, as shown in Figure Figure 2(c).

The internal input resistance can be found from

$$R_{\mathrm{in}} = \frac{v_i}{i_i} \tag{2.1}$$

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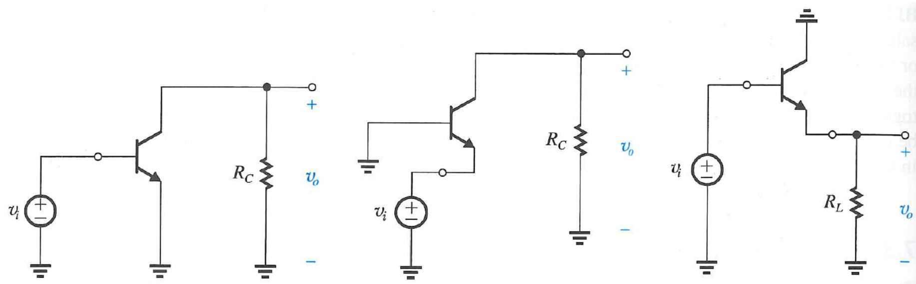

Figure 1: Basic configurations of a transistor amplifier: (a) common emitter (CE), (b) common base (CB), and (c) common collector (CC) (Courtesy of Sedra and Smith).

The amplifier is also defined with an **open-circuit voltage gain** A_{vo} defined as

$$A_{vo} = \frac{v_o}{v_i}\bigg|_{R_L=\infty} \tag{2.2}$$

The output resistance can be measured by setting $v_i = 0$ using the test-current method, and it is given by

$$R_o = \frac{v_x}{i_x} \tag{2.3}$$

With the load resistor R_L connected, the actual output voltage in Figure 2(b) is

$$v_o = \frac{R_L}{R_L + R_o} A_{vo} v_i \tag{2.4}$$

Hence, the actual **voltage gain of the amplifier proper** also called the **terminal voltage gain**, when a finite load R_L is added, is A_v given by

$$A_v = \frac{v_o}{v_i} = A_{vo} \frac{R_L}{R_L + R_o} \tag{2.5}$$

and the **overall voltage gain** of the entire circuit is given by

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_o}{v_i} \frac{v_i}{v_{\text{sig}}} \tag{2.6}$$

Using (2.5) and the fact that

$$\frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}$$

give

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} \frac{R_L}{R_L + R_o} A_{vo} \tag{2.7}$$

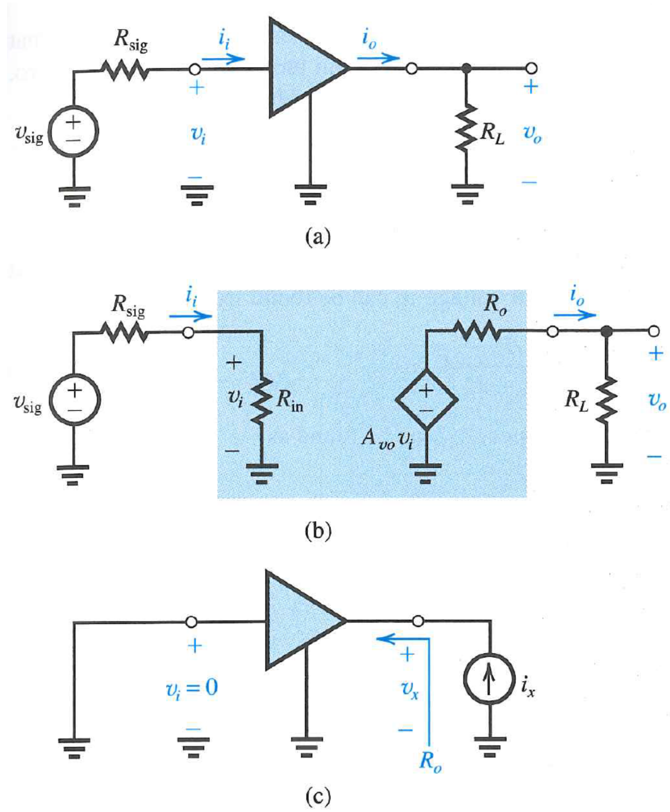

Figure 2: (a) The characterization of an amplifier, with a functional block representing the small-signal model, including R_{sig} for the source and R_L for the load. (b) The equivalent small signal model, and (c) the definition of the output resistance R_o using the test-current method (Courtesy of Sedra and Smith).

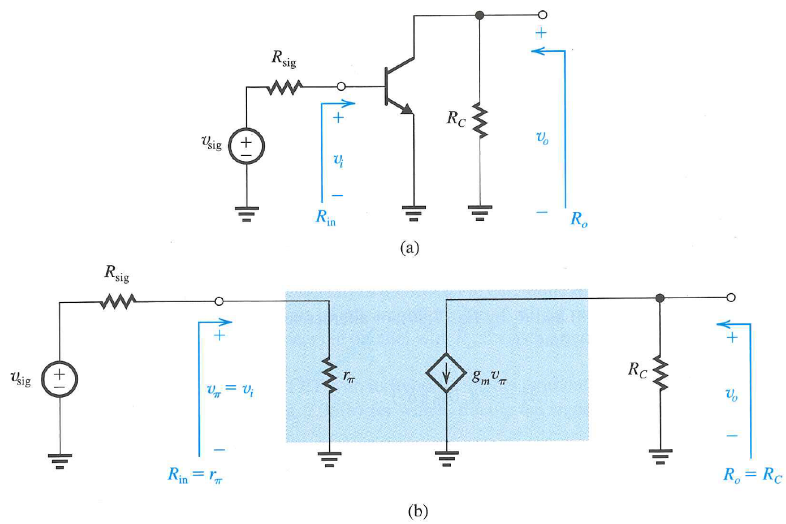

Figure 3: A common emitter (CE) amplifier (a) and its equivalent circuit hybrid-π model (b) (Courtesy of Sedra and Smith).

3 Common Emitter Amplifier

This is the most popular amplifier design, and by cascading a number of them, the aggregate gain of the amplifier circuit can be greatly increased.

3.1 Characteristic Parameters of the CE Amplifier

Figure 3 shows the BJT CE amplifier and its small-signal equivalent circuit model. It is seen, after using the voltage-divider formula, that

$$v_i = \frac{r_\pi}{r_\pi + R_{\text{sig}}} v_{\text{sig}}, \quad \text{and } v_o = -g_m v_i R_C \tag{3.1}$$

where g_m, the transconductance, is given by I_C/V_T, and $i_c = g_m v_i$ have been used. Then

$$A_{vo} = \frac{v_o}{v_i} = -g_m R_C \tag{3.2}$$

With the load resistance R_L, then the voltage gain proper (terminal voltage gain)

$$A_v = -g_m (R_C || R_L) \tag{3.3}$$

and the overall voltage gain

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \frac{v_o}{v_i} = -\frac{r_\pi}{r_\pi + R_{\text{sig}}} g_m (R_C || R_L) \tag{3.4}$$

When the transconductance g_m of a transistor is large, this can be a large number. In the above $v_i = v_{be}$, the base-emitter voltage. This voltage is reduced when R_{sig} is large, decreasing G_v.

3.2 Final Remarks on CE Amplifier

1. The CE amplifiers gives high input impedance (draws little current), and a moderately high output resistance (easier to match), and high voltage gain (a desirable feature of an amplifier).
2. The input resistance of the CE amplifier is $R_{\text{in}} = r_\pi = \beta/g_m = \beta V_T/I_C$ is inversely proportional to I_C. Hence, R_{in} can be increased by decreasing I_C, but that will lower g_m and reduce the gain of the amplifier, a trade-off.
3. Reducing R_C reduces the output resistance of a CE amplifier, but unfortunately, the voltage gain is also reduced. Alternate design can be employed to reduce the output resistance (to be discussed later).
4. A CE amplifier suffers from poor high frequency performance, as most transistor amplifiers do.

4 Common Emitter Amplifier with an Emitter Resistance

It is seen that the input resistance of the circuit is

$$R_{\text{in}} = \frac{v_i}{i_b}, \quad \text{with} \quad i_b = \frac{i_e}{\beta + 1} \tag{4.1}$$

With $i_e = v_i/(r_e + R_e)$, then

$$R_{\text{in}} = (\beta + 1)(r_e + R_e) \tag{4.2}$$

This is known as the **resistance-reflection rule**, because the input resistance is amplified by the factor $\beta + 1$, which can be large. Therefore, including an emitter resistance R_e can greatly increase R_{in}.

More specifically,

$$\frac{R_{\text{in}}(\text{with } R_e)}{R_{\text{in}}(\text{without } R_e)} = \frac{r_e + R_e}{r_e} = 1 + \frac{R_e}{r_e} \approx 1 + g_m R_e \tag{4.3}$$

after using the fact that $r_e = \alpha/g_m \approx 1/g_m$. Hence the input resistance R_{in} can be greatly increased with R_e.

The open-circuit voltage gain is given by $A_{vo} = v_o/v_i$. Since

$$v_o = -i_c R_C = -\alpha i_e R_C \tag{4.4}$$

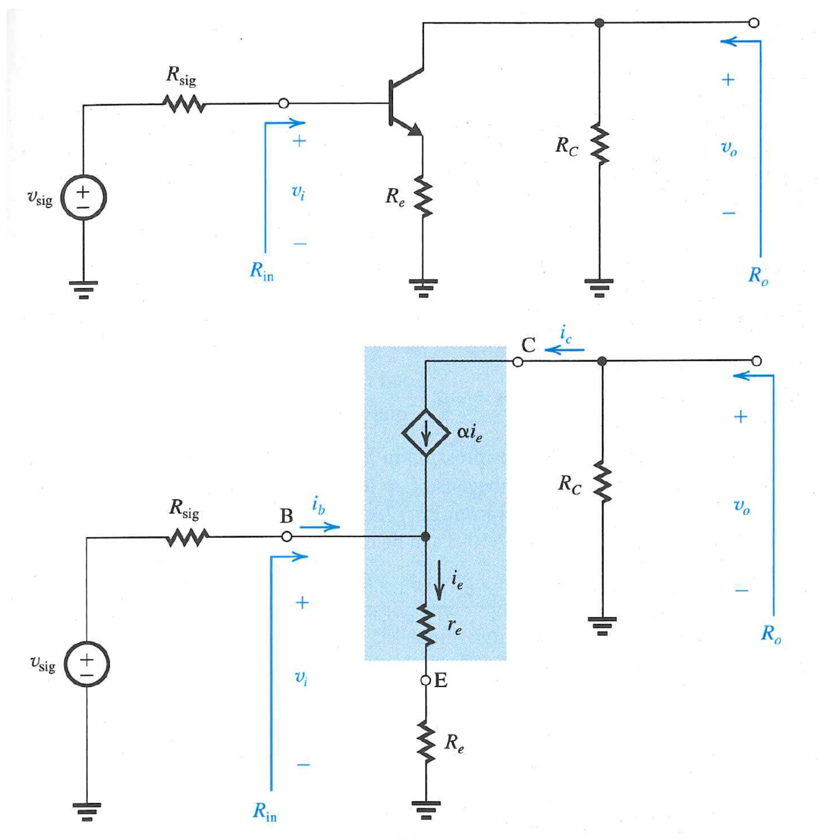

Figure 4: A CE amplifier with an emitter resistance: (top) detail circuit, (bottom) equivalent circuit hybrid-π model (Courtesy of Sedra and Smith).

and that $i_e = v_i/(r_e + R_e)$, then

$$A_{vo} = -\alpha \frac{R_C}{r_e + R_e} \tag{4.5}$$

Using $g_m = \alpha/r_e$, the above can be rewritten as

$$A_{vo} = -\frac{\alpha}{r_e}\left(\frac{R_C}{1 + \frac{R_C}{r_e}}\right) = -\frac{g_m R_C}{1 + R_e/r_e} \approx -\frac{g_m R_C}{1 + g_m R_e} \tag{4.6}$$

where $g_m \approx 1/r_e$ has been used in the last approximation. Hence, including R_e reduces the voltage gain, which also reduces R_{in}, a tradeoff for designers.

The output resistance

$$R_o = R_C \tag{4.7}$$

With a load resistor R_L connected to the amplifier output, then the voltage gain proper

$$A_v = A_{vo}\frac{R_L}{R_L + R_o} = -\alpha\frac{R_C}{r_e + R_e}\frac{R_L}{R_L + R_o} = -\alpha\frac{R_C||R_L}{r_e + R_e} \tag{4.8}$$

which is similar in spirit to (4.5) where R_C is now replaced by $R_C||R_L$.

The overall voltage gain G_v is

$$G_v = \frac{v_i}{v_{\text{sig}}}A_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}}\left(-\alpha\frac{R_C||R_L}{r_e + R_e}\right) = -\beta\frac{R_C||R_L}{R_{\text{sig}} + (\beta+1)(r_e + R_e)} \tag{4.9}$$

where (4.2) for R_{in} and that $\alpha = \beta/(\beta + 1)$ have been used. Notice that the voltage gain is lowered with the presence of R_e.

Moreover, the presence of R_e reduces nonlinearity, because v_π, which is the base-emitter voltage v_{be}, is reduced, namely,

$$\frac{v_\pi}{v_i} = \frac{r_e}{r_e + R_e} \approx \frac{1}{1 + g_m R_e} \tag{4.10}$$

where $g_m \approx 1/r_e$ has been used. Hence, it seen that v_π compared to v_i is reduced by a substantial factor, improving the linearity of the amplifier. This is because $i_C = I_C e^{v_{be}/V_T}$, a nonlinear relationship that has been linearized with a small signal approximation. The smaller v_{be}/V_t is, the better the small-signal or linearization approximation.

4.1 Summary of the CE Amplifier with Emitter Resistance

1. The input resistance R_{in} is increased by a factor of $1 + g_m R_e$ as seen in (4.3).

2. The base to collector voltage gain, A_{vo}, is reduced by a factor of $1 + g_m R_e$ as seen in (4.6).

3. For the same nonlinear distortion, the input signal can be increased by a factor of $1 + g_m R_e$ compared to without R_e.

4. The voltage gain is less dependent on β.

5. As shall be shown later, the high-frequency response of this design is improved.

In general, the addition of the emitter resistance R_e gives rise to a "negative" feedback factor $1 + g_m R_e$ that reduces voltage gain, but improves linearity, and high-frequency response. Because of the negative-feedback action of R_e, it is also called the **emitter degenerate resistance.**

5 Common-Base (CB) Amplifier

The common-base amplifier is shown in Figure 5. The input resistance R_{in} is given by

$$R_{\text{in}} = r_e = \frac{\alpha}{g_m} \approx 1/g_m \tag{5.1}$$

The voltage gain is

$$A_{vo} = \frac{v_o}{v_i} = \frac{\alpha i_e R_C}{r_e i_e} = \frac{\alpha}{r_e} R_C = g_m R_C \tag{5.2}$$

The output resistance here is $R_o = R_C$. When a load resistance R_L is connected, then the voltage gain proper (terminal voltage gain)

$$A_v = \frac{v_o}{v_i} = g_m R_C || R_L \tag{5.3}$$

and the overall voltage gain is

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \frac{v_o}{v_i} = \frac{r_e}{R_{\text{sig}} + r_e} \frac{\alpha}{r_e} R_C || R_L = \alpha \frac{R_C || R_L}{R_{\text{sig}} + r_e} \approx \frac{R_C || R_L}{R_{\text{sig}} + r_e} \tag{5.4}$$

5.1 Summary of the CB Amplifier

1. The CB amplifier has a low input resistance $\approx 1/g_m$. This is undesirable as it will draw large current when driven by a voltage input.

2. The voltage gain of the CB amplifier can be made similar in magnitude to that of the CE amplifier if $R_C || R_L$ can be made large compared to $R_{\text{sig}} + R_e$.

3. The output resistance can be made large since $R_o = R_C$.

4. The CB amplifier has good high frequency performance as shall be shown later.

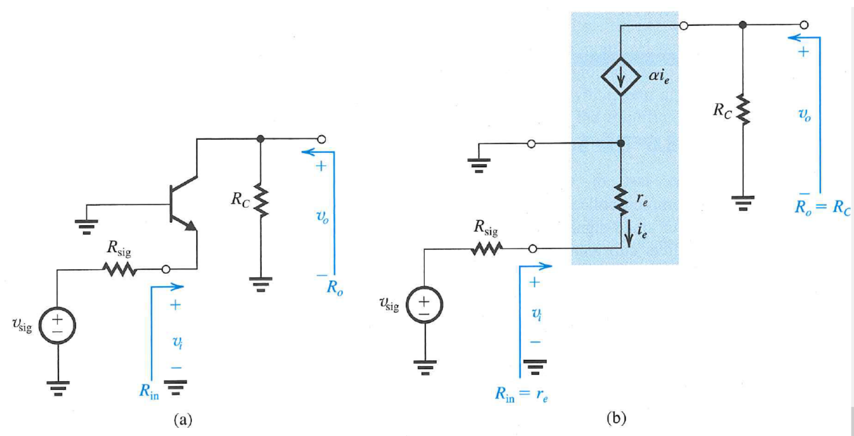

Figure 5: A common-base (CB) amplifier (a) with biasing detail omitted, (b) equivalent circuit using the T model for the BJT (Courtesy of Sedra and Smith).

6 The Emitter Follower

The emitter follower can be used as a voltage buffer. It is also the common-collector amplifier: one of the three basic configurations of transistor amplifiers. Figure 6 shows the use of a voltage buffer concept in a circuit with an amplifier of unit gain.

Figure 7 shows the circuit diagram for the the common-collector amplifier. The input impedance is given by

$$R_{\text{in}} = \frac{v_i}{i_b} \tag{6.1}$$

Using that $i_b = i_e/(\beta + 1)$ and that $i_e = v_i/(r_e + R_L)$, one gets

$$R_{\text{in}} = (\beta + 1)(r_e + R_L) \tag{6.2}$$

which also agrees with the resistance-reflection rule. It is this reflection rule that makes the amplifier or the emitter follower to have a large input impedance, and hence, its usefulness as a buffer.

The voltage gain proper (terminal voltage gain) A_v is

$$A_v = \frac{v_o}{v_i} = \frac{R_L}{R_L + r_e} \sim 1, \quad R_L \to \infty \tag{6.3}$$

(When an amplifier is loaded with an output resistor R_L, the above voltage gain is usually called the voltage gain proper or the terminal voltage gain.) Hence, it works well as an emitter follower since $v_o = v_i$.

To find R_o, one finds the Thévenin equivalence of the amplifier when it is driven by an input voltage source v_i. Then R_o is just the Thévenin resistor,

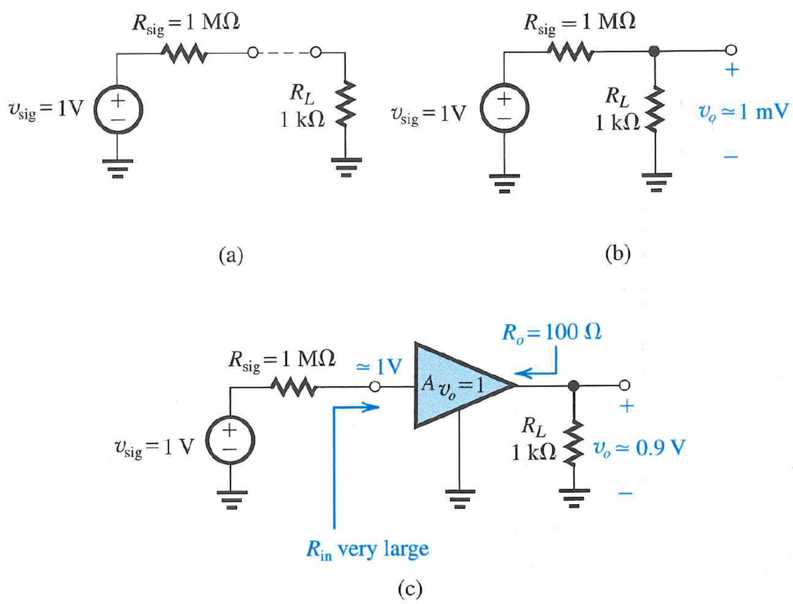

Figure 6: The schematics of a unit gain voltage buffer amplifier (Courtesy of Sedra and Smith).

which in this case is r_e. Hence,

$$R_o = r_e \tag{6.4}$$

Since r_e is usually small, this amplifier has a low output impedance.

To find the overall voltage gain G_v, first one finds that

$$\frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} = \frac{(\beta + 1)(r_e + R_L)}{(\beta + 1)(r_e + R_L) + R_{\text{sig}}} \tag{6.5}$$

Then the overall voltage gain is

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}}\frac{v_o}{v_i} = \frac{v_i}{v_{\text{sig}}}A_v = \frac{(\beta + 1)R_L}{(\beta + 1)(r_e + R_L) + R_{\text{sig}}} \sim 1, \quad R_L \to \infty \tag{6.6}$$

This clearly indicates that G_v, the overall voltage gain is less than one but can approach one for large R_L.

One can think of the emitter follower just as a voltage divider. Two simpler versions of the equivalent voltage divider are presented in Figure 8.

6.1 Thévenin Equivalence of the Emitter Follower

The Thévenin equivalence of the emitter follower can be found. Let us first assume that we do not know what the voltage source is. Since this is a linear problem in small signals in Figure 9(a), the voltage source is indicated by $G_{vo}v_{\text{sig}}$ with R_{out} yet to be determined as indicated in the figure. By setting R_L to be infinitely large, making it an open circuit, and using the equivalent circuit as shown in Figure 8(b), it is seen that $G_{vo} = 1$ and that

$$R_{\text{out}} = r_e + R_{\text{sig}}/(\beta + 1) \tag{6.7}$$

as shown in Figures 9(b) and Figures 9(c). The more detail circuit is shown in Figure 9(d).

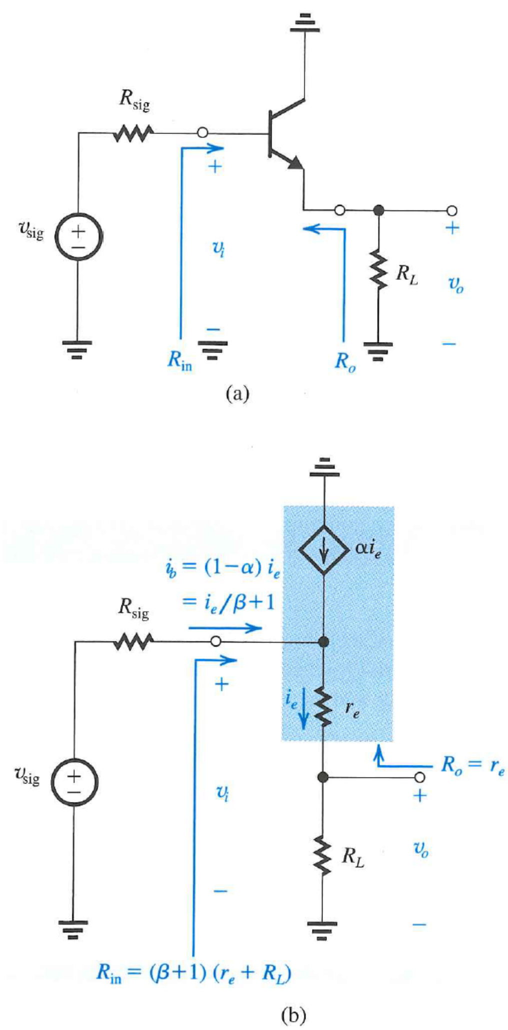

Figure 7: A common-collector (CC) amplifier or emitter follower: (a) with biasing detail omitted, (b) equivalent circuit using the T model for the BJT (Courtesy of Sedra and Smith).

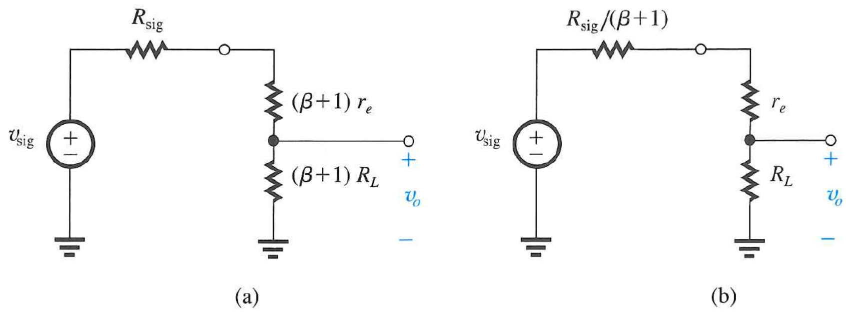

Figure 8: Two ways of representing the emitter follower as a voltage divider (Courtesy of Sedra and Smith).

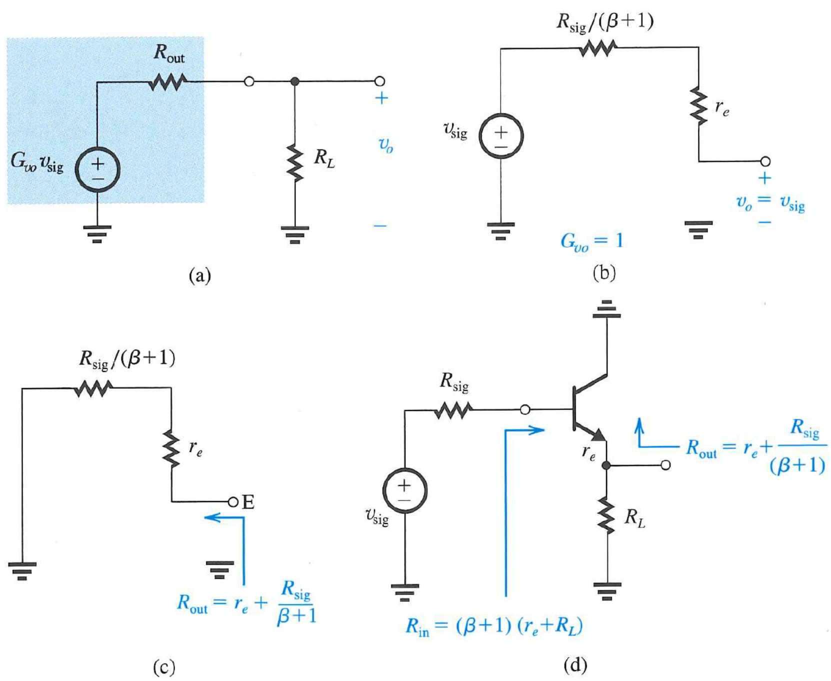

Figure 9: Way to find the Thévenin equivalent of the small-signal model of the emitter follower as shown in (a) and (b) to find the open circuit voltage, (c) to find the equivalent Thévenin resistance, and (d) the full circuit model for small signals. In (d), notice that the impedances are different looking to the right or to the left of the transistor (Courtesy of Sedra and Smith).

6.2 Summary Table

Table 7.5 Characteristics of BJT Amplifiers[a,b]

	R_{in}	A_{vo}	R_o	A_v	G_v
Common emitter (Fig. 7.36)	$(\beta+1)r_e$	$-g_m R_C$	R_C	$-g_m(R_C \parallel R_L)$ $-\alpha \frac{R_C \parallel R_L}{r_e}$	$-\beta \frac{R_C \parallel R_L}{R_{sig}+(\beta+1)r_e}$
Common emitter with R_e (Fig. 7.38)	$(\beta+1)(r_e+R_e)$	$-\frac{g_m R_C}{1+g_m R_e}$	R_C	$\frac{-g_m(R_C \parallel R_L)}{1+g_m R_e}$ $-\alpha \frac{R_C \parallel R_L}{r_e+R_e}$	$-\beta \frac{R_C \parallel R_L}{R_{sig}+(\beta+1)(r_e+R_e)}$
Common base (Fig. 7.40)	r_e	$g_m R_C$	R_C	$g_m(R_C \parallel R_L)$ $\alpha \frac{R_C \parallel R_L}{r_e}$	$\alpha \frac{R_C \parallel R_L}{R_{sig}+r_e}$
Emitter follower (Fig. 7.43)	$(\beta+1)(r_e+R_L)$	1	r_e	$\frac{R_L}{R_L+r_e}$	$\frac{R_L}{R_L+r_e+R_{sig}/(\beta+1)}$ $G_{vo}=1$ $R_{out}=r_e+\frac{R_{sig}}{\beta+1}$

[a] For the interpretation of R_{in}, A_{vo}, and R_o refer to Fig. 7.34.
[b] Setting $\beta=\infty$ $(\alpha=1)$ and replacing r_e with $1/g_m$, R_C with R_D, and R_e with R_s results in the corresponding formulas for MOSFET amplifiers (Table 7.4).

The summary table, Table 7.5 is from Sedra and Smith. Notice that A_v, the voltage gain proper (also called terminal voltage gain), becomes A_{vo}, the open-circuit voltage gain when $R_L \to \infty$. The CB amplifier has a low input impedance, while the emitter follower has a low output impedance.

Example 7.8

A CE amplifier utilizes a BJT with $\beta = 100$ is biased at $I_C = 1$ mA and has a collector resistance $R_C = 5\ \text{k}\Omega$. Find R_{in}, R_o, and A_{vo}. If the amplifier is fed with a signal source having a resistance of 5 kΩ, and a load resistance $R_L = 5\ \text{k}\Omega$ is connected to the output terminal, find the resulting A_v and G_v. If $\hat{v}_\pi$ is to be limited to 5 mV, what are the corresponding $\hat{v}_{\text{sig}}$ and $\hat{v}_o$ with the load connected?

Solution

At $I_C = 1$ mA,

$$g_m = \frac{I_C}{V_T} = \frac{1\ \text{mA}}{0.025\ \text{V}} = 40\ \text{mA/V}$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40\ \text{mA/V}} = 2.5\ \text{k}\Omega$$

Example 7.8 *continued*

The amplifier characteristic parameters can now be found as

$$R_{\text{in}} = r_\pi = 2.5\ \text{k}\Omega$$
$$A_{vo} = -g_m R_C$$
$$= -40\ \text{mA/V} \times 5\ \text{k}\Omega$$
$$= -200\ \text{V/V}$$
$$R_o = R_C = 5\ \text{k}\Omega$$

With a load resistance $R_L = 5\ \text{k}\Omega$ connected at the output, we can find A_v by either of the following two approaches:

$$A_v = A_{vo}\frac{R_L}{R_L + R_o}$$
$$= -200 \times \frac{5}{5+5} = -100\ \text{V/V}$$

or

$$A_v = -g_m(R_C \parallel R_L)$$
$$= -40(5 \parallel 5) = -100\ \text{V/V}$$

The overall voltage gain G_v can now be determined as

$$G_v = \frac{R_{\text{in}}}{R_{\text{in}} + R_{\text{sig}}} A_v$$
$$= \frac{2.5}{2.5+5} \times -100 = -33.3\ \text{V/V}$$

If the maximum amplitude of v_π is to be 5 mV, the corresponding value of $\hat{v}_{\text{sig}}$ will be

$$\hat{v}_{\text{sig}} = \left(\frac{R_{\text{in}} + R_{\text{sig}}}{R_{\text{in}}}\right)\hat{v}_\pi = \frac{2.5+5}{2.5} \times 5 = 15\ \text{mV}$$

and the amplitude of the signal at the output will be

$$\hat{v}_o = G_v \hat{v}_{\text{sig}} = 33.3 \times 0.015 = 0.5\ \text{V}$$

Appendix A Maximum Power Transfer Theorem

Let us assume that a signal source can be modeled with a Thévenin equivalence. So when the signal source is connected to a load, it can be modeled by the circuit shown in Figure 10.

The current flowing in the circuit is

$$I = V_S/(R_S + R_L) \tag{A.1}$$

The power transferred to the load is given by

$$P_L = I^2 R_L = \frac{V_S^2 R_L}{(R_S + R_L)^2} \tag{A.2}$$

R_L in the above can be varied so that P_L is maximized. Without loss of generality, one can assume that $V_S = 1$, and find that

$$\frac{dP_L}{dR_L} = \frac{1}{(R_S + R_L)^2} - 2\frac{R_L}{(R_S + R_L)^3} = 0 \tag{A.3}$$

at the maximum point. The above evaluates to $R_L = R_S$. In other words, the load impedance should be matched to the source impedance.

The theorem can be generalized to time-harmonic sources and complex impedances. In this case, the matching condition is that

$$Z_L = Z_S^* \tag{A.4}$$

where * implies the complex conjugation, and Z_L is the complex load impedance, and Z_S is the complex source impedance.

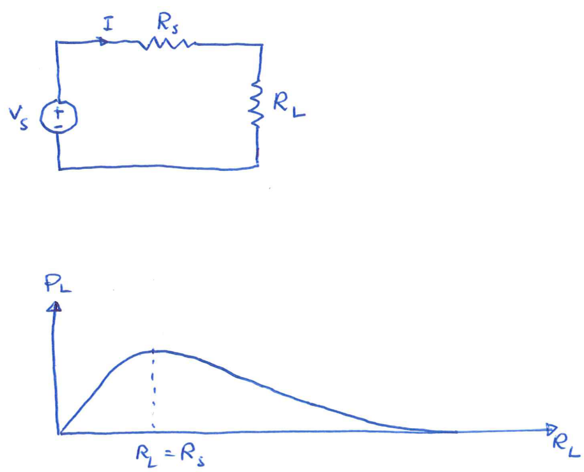

Figure 10: (Top) Simple circuit where a load R_L is connected to the Thévenin equivalence of the source. (Bottom) The plot of power transferred to the load P_L versus the change of R_L (Courtesy of Sedra and Smith).