ECE 255, Differential Amplifiers, Cont.

9 November 2017

In this lecture, we will focus on the common-mode rejection of differential amplifiers.

1 Common-Mode Rejection

For a perfectly matched differential amplifier, the common-mode signals are exactly canceled out. We will study the case when the design of the amplifier is not completely symmetrical or matched, and how the imperfections will influence the common-mode rejection.

1.1 The MOS Case

As shown in Figure 1, for small signal analysis, we can replace the voltage source with short circuits in the analysis, and assume a small v_{icm} on top the the background common-mode voltage V_{CM}. Using the equivalent circuit model, it is seen that

$$v_{icm} = \frac{i}{g_m} + 2iR_{SS}, \text{ or } i = \frac{v_{icm}}{1/g_m + 2R_{SS}} \tag{1.1}$$

We can next find that

$$v_{o1} = v_{o2} = -R_D i = -\frac{R_D}{1/g_m + 2R_{SS}} v_{icm} \approx -\frac{R_D}{2R_{SS}} v_{icm} \tag{1.2}$$

where $2R_{SS} \gg 1/g_m$ has been assumed.

1.1.1 Effect of Mismatch

The critical thing to study here is that if the two R_D's are mismatched, namely, the load of Q_1 is R_D while that of Q_2 is $R_D + \Delta R_D$. Then following the above analysis,

$$v_{o1} \approx -\frac{R_D}{2R_{SS}} v_{icm}, \text{ and } v_{o2} \approx -\frac{R_D + \Delta R_D}{2R_{SS}} v_{icm} \tag{1.3}$$

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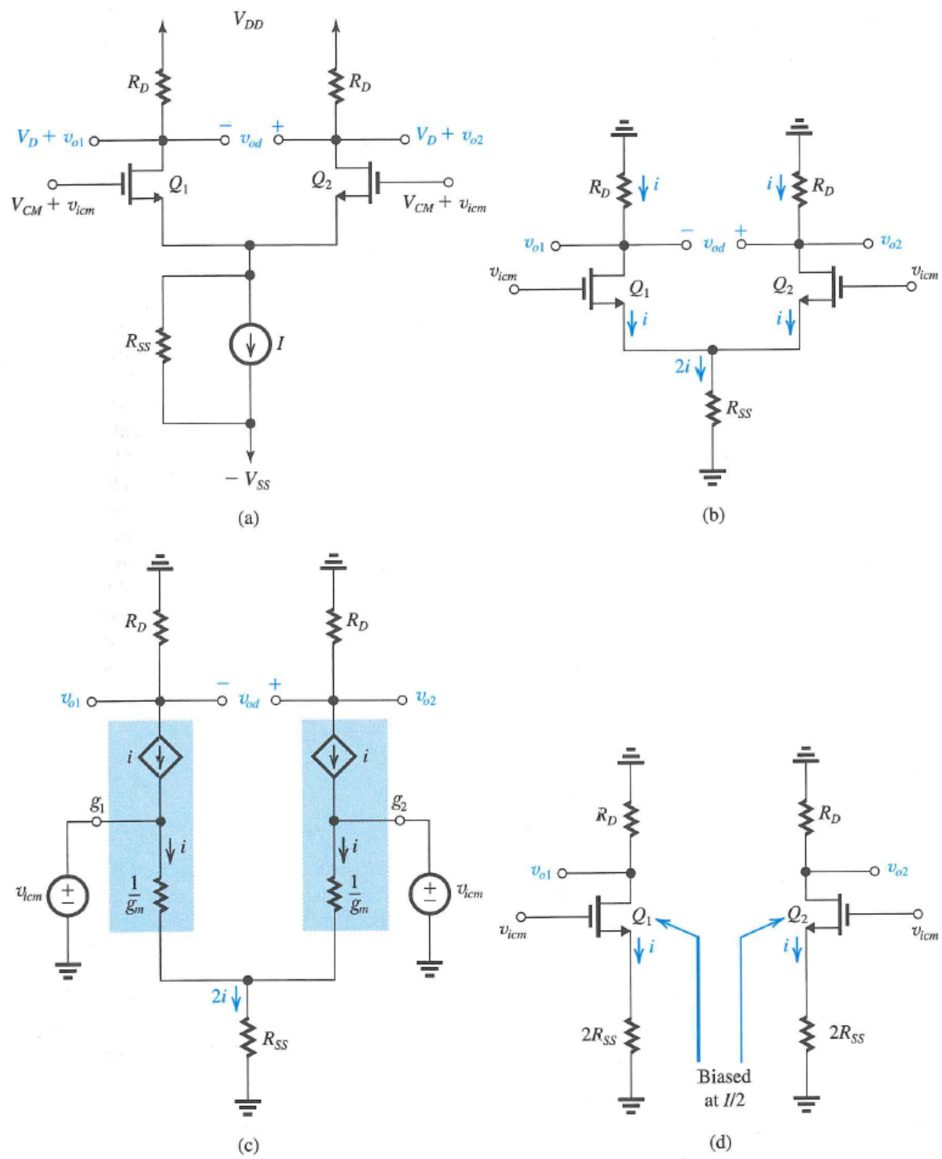


Figure 1: Small-signl analysis of the MOS differential amplifier: (a) The circuit with DC biases in place. (b) The small-signal circuit of the differential amplifier with DC biased removed. (c) The T-model equivalent circuit of the differential amplifier (Courtesy of Sedra and Smith).

Thus the differential voltage output now, due to a small common mode signal, v_{icm}, is

$$v_{od} = v_{o2} - v_{o1} = -\frac{\Delta R_D}{2R_{SS}} v_{icm} \tag{1.4}$$

The **common-mode gain** due to this imperfection is then

$$A_{cm} = \frac{v_{od}}{v_{icm}} = -\frac{\Delta R_D}{2R_{SS}} = -\left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta R_D}{R_D}\right) \tag{1.5}$$

The desirable signal is the output of the differential signal, while the undesirable signal is the output of the common-mode disturbance. The term **common-mode rejection ratio (CMRR)** is defined as

$$\text{CMRR} = \frac{|A_d|}{|A_{cm}|} \tag{1.6}$$

and when expressed in decibel, it is

$$\text{CMRR(dB)} = 20 \log_{10} \frac{|A_d|}{|A_{cm}|} \tag{1.7}$$

Recall that the differential mode voltage gain is $g_m R_D$, then the common-mode rejection ratio can be written as

$$\text{CMRR} = (2g_m R_{SS}) \Big/ \left(\frac{\Delta R_D}{R_D}\right) \tag{1.8}$$

It is seen that a source with large R_{SS} will increase this ratio, as well as a design with small ΔR_D or small mismatched.

1.1.2 Effect of g_m Mismatch on CMRR

If the transconductance of the two transistors are not matched, it can be shown that the gain for the common-mode disturbance is

$$A_{cm} \approx \left(\frac{R_D}{2R_{SS}}\right)\left(\frac{\Delta g_m}{g_m}\right) \tag{1.9}$$

and the corresponding CMRR will be

$$\text{CMRR} = (2g_m R_{SS}) \Big/ \left(\frac{\Delta g_m}{g_m}\right) \tag{1.10}$$

1.2 The BJT Case

The BJT differential amplifier is shown in Figure 2. It can be shown that

$$v_{o1} = v_{o2} = -\frac{\alpha R_C}{r_e + 2R_{EE}} v_{icm} \tag{1.11}$$

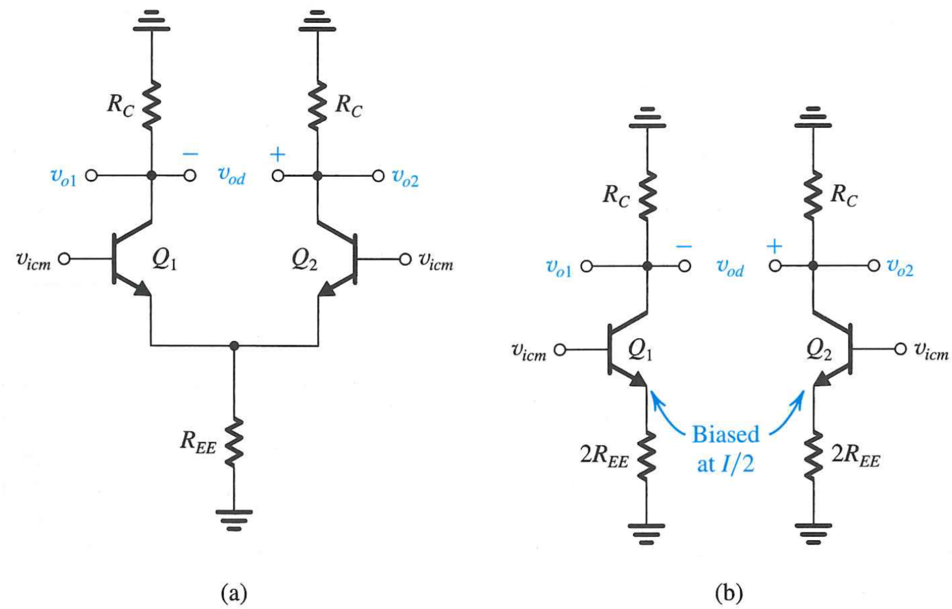


Figure 2: Small-signl analysis of the MOS differential amplifier: (a) The circuit with DC biases in place. (b) The small-signal circuit of the differential amplifier with DC biased removed. (c) The T-model equivalent circuit of the differential amplifier (Courtesy of Sedra and Smith).

Hence, the common mode signal is completely rejected when a differential signal is taken at the output. However, when the two circuits are not perfectly matched, for instance, having a slightly mismatched R_C, then the differential output is not zero, and one gets

$$A_{cm} = \frac{v_{od}}{v_{icm}} = -\frac{\alpha \Delta R_C}{2R_{EE} + r_e} \tag{1.12}$$

With $\alpha \approx 1$, the above becomes

$$A_{cm} \approx -\left(\frac{R_C}{2R_{EE}}\right)\left(\frac{\Delta R_C}{R_C}\right) \tag{1.13}$$

Using the fact that the differential gain is

$$A_d = \frac{\alpha(2R_C)}{2r_e + 2R_{EE}} \approx \frac{R_C}{R_{EE}} \tag{1.14}$$

then the CMRR is[1]

$$\text{CMRR} = 2 \bigg/ \left(\frac{\Delta R_C}{R_C}\right) \tag{1.15}$$

2 Making Sense out of Formulas

Since there are numerous formulas to remember for this course, it may be taxing for your memory bank. Hence, it is best to distill the knowledge in these formulas and look at them from a different light. Often time, with a new look angle, one can cut through the chaste, and arrive at these formulas using the back-of-the-envelop calculations.

The formula for transconductance, $g_m = \frac{I_C}{V_T}$, is derived by perturbation expansion. We shall not question its deeper physical meaning. However, the other formulas in Table 7.3 can be derived quite easily. For instance, the formula

$$r_\pi = \frac{\beta}{g_m} \tag{2.1}$$

can be gotten by comparing the controlled current sources in the two hybrid-π models in the Table. The equivalence of the models imply that

$$\beta i_b = g_m v_\pi \tag{2.2}$$

Dividing the above i_b, and using the definition that $r_\pi = \frac{v_\pi}{i_b}$, we get formula in (2.1).

To get the formula

$$r_e = \frac{\alpha}{g_m} \tag{2.3}$$

[1] There seems to be an error with the textbook formula (9.95).

we compare the current sources in the T models. From this comparison, one gets

$$g_m v_\pi = \alpha i \tag{2.4}$$

but from the figures, $v_\pi / i = r_e$, and hence, we get formula (2.3).

With these basic formulas, the derivation of the other formulas are quite straightforward. For instance, using the previous two formulas, one can show that

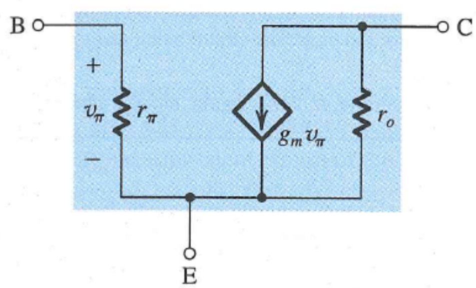
$$r_\pi = (\beta + 1) r_e \tag{2.5}$$

On the other hand, the above is just the resistance reflection formula for the input impedance when the T model is used, or one can get it by comparing the input impedance of the hybrid-π model and the T model.

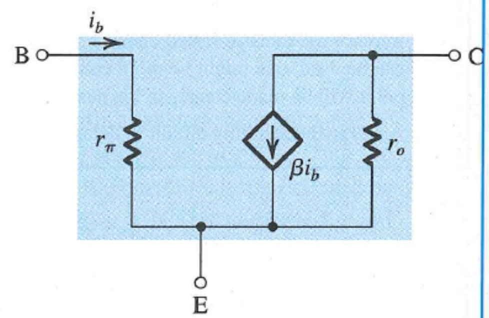
Table 7.3 Small-Signal Models of the BJT

Hybrid-π Model

■ $(g_m v_\pi)$ Version

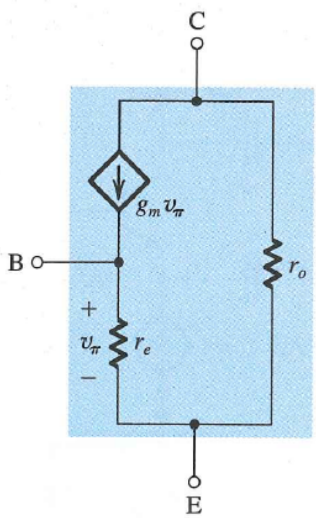


■ (βi_b) Version

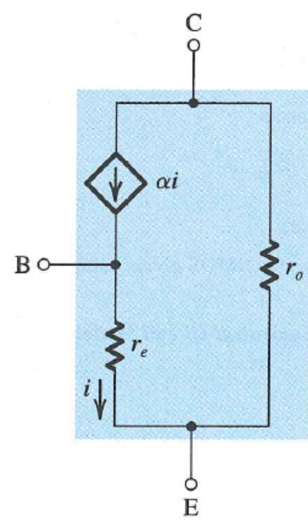


T Model

■ $(g_m v_\pi)$ Version



■ (αi) Version



Model Parameters in Terms of DC Bias Currents

$$g_m = \frac{I_C}{V_T} \qquad r_e = \frac{V_T}{I_E} = \alpha \frac{V_T}{I_C} \qquad r_\pi = \frac{V_T}{I_B} = \beta \frac{V_T}{I_C} \qquad r_o = \frac{|V_A|}{I_C}$$

In Terms of g_m

$$r_e = \frac{\alpha}{g_m} \qquad r_\pi = \frac{\beta}{g_m}$$

In Terms of r_e

$$g_m = \frac{\alpha}{r_e} \qquad r_\pi = (\beta + 1) r_e \qquad g_m + \frac{1}{r_\pi} = \frac{1}{r_e}$$

Relationships between α and β

$$\beta = \frac{\alpha}{1-\alpha} \qquad \alpha = \frac{\beta}{\beta+1} \qquad \beta + 1 = \frac{1}{1-\alpha}$$

The formulas in Table 7.5 can also be easily derived. The formulas in the first row of Table 7.5, corresponding to the common emitter case, can be easily derived, except for the last formula in the row for G_v. The last formula for G_v is always equal to the voltage divider formula times A_v, namely, that

$$G_v = \frac{R_{\text{in}}}{R_{\text{sig}} + R_{\text{in}}} A_v \tag{2.6}$$

With a little bit of manipulation, one gets to the last formula in the first row.

For the second row, the common-emitter case with an emitter resistor R_e, then part of the input voltage v_i is shared by R_e. The formula for A_{vo} in this row is an approximation. Before the approximation, the exact formula is

$$A_{vo} = \frac{v_o}{v_i} = -\frac{i_c R_C}{v_i} = -\frac{\alpha i_e R_C}{v_i} = -\frac{\alpha R_C}{r_e + R_e} \tag{2.7}$$

where one has used the formula that $i_e = v_i/(r_e + R_e)$ in the last equality. This equation clearly shows that R_e reduces the emitter current because only a fraction of v_i is applied across v_{be} or v_π. The factor in the above

$$\frac{\alpha}{r_e + R_e} = \frac{g_m}{1 + R_e/r_e} \approx \frac{g_m}{1 + g_m R_e} \tag{2.8}$$

as is seen in the formula above.

Using the voltage divider formula to convert v_{sig} to v_i, one can always get the formulas in the last column on the right. Namely,

$$\frac{v_i}{v_{\text{sig}}} = \frac{R_{\text{in}}}{R_{\text{sig}} + R_{\text{in}}} \tag{2.9}$$

and hence,

$$G_v = \frac{v_o}{v_{\text{sig}}} = \frac{v_i}{v_{\text{sig}}} \frac{v_o}{v_i} = \frac{R_{\text{in}}}{R_{\text{sig}} + R_{\text{in}}} A_v \tag{2.10}$$

With

$$A_v = -\alpha \frac{R_C \parallel R_L}{r_e + R_e} \tag{2.11}$$

and $R_{\text{in}} = (\beta + 1)(r_e + R_e)$, and substituting into the above, one gets

$$G_v = -\beta \frac{R_C \parallel R_L}{R_{\text{sig}} + (\beta + 1)(r_e + R_e)} \tag{2.12}$$

the formula we have in the right most column of the second row of the table.

Table 7.5 Characteristics of BJT Amplifiers[a,b]

	R_{in}	A_{vo}	R_o	A_v	G_v
Common emitter (Fig. 7.36)	$(\beta+1)r_e$	$-g_m R_C$	R_C	$-g_m(R_C \parallel R_L)$ $-\alpha \dfrac{R_C \parallel R_L}{r_e}$	$-\beta \dfrac{R_C \parallel R_L}{R_{sig}+(\beta+1)r_e}$
Common emitter with R_e (Fig. 7.38)	$(\beta+1)(r_e+R_e)$	$-\dfrac{g_m R_C}{1+g_m R_e}$	R_C	$\dfrac{-g_m(R_C \parallel R_L)}{1+g_m R_e}$ $-\alpha \dfrac{R_C \parallel R_L}{r_e+R_e}$	$-\beta \dfrac{R_C \parallel R_L}{R_{sig}+(\beta+1)(r_e+R_e)}$
Common base (Fig. 7.40)	r_e	$g_m R_C$	R_C	$g_m(R_C \parallel R_L)$ $\alpha \dfrac{R_C \parallel R_L}{r_e}$	$\alpha \dfrac{R_C \parallel R_L}{R_{sig}+r_e}$
Emitter follower (Fig. 7.43)	$(\beta+1)(r_e+R_L)$	1	r_e	$\dfrac{R_L}{R_L+r_e}$	$\dfrac{R_L}{R_L+r_e+R_{sig}/(\beta+1)}$ $G_{vo}=1$ $R_{out}=r_e+\dfrac{R_{sig}}{\beta+1}$

[a] For the interpretation of R_{in}, A_{vo}, and R_o refer to Fig. 7.34.
[b] Setting $\beta=\infty$ ($\alpha=1$) and replacing r_e with $1/g_m$, R_C with R_D, and R_e with R_s results in the corresponding formulas for MOSFET amplifiers (Table 7.4).

The same thing can be said of the MOSFET case. In this case, $\alpha = 1$ and $r_e = 1/g_m$. With these concepts in mind, these formulas can be derived easily and quickly.

Table 7.4 Characteristics of MOSFET Amplifiers

	Characteristics[a]				
Amplifier type	R_{in}	A_{vo}	R_o	A_v	G_v
Common source (Fig. 7.35)	∞	$-g_m R_D$	R_D	$-g_m(R_D \parallel R_L)$	$-g_m(R_D \parallel R_L)$
Common source with R_s (Fig. 7.37)	∞	$-\frac{g_m R_D}{1+g_m R_s}$	R_D	$\frac{-g_m(R_D \parallel R_L)}{1+g_m R_s}$ $-\frac{R_D \parallel R_L}{1/g_m+R_s}$	$-\frac{g_m(R_D \parallel R_L)}{1+g_m R_s}$ $-\frac{R_D \parallel R_L}{1/g_m+R_s}$
Common gate (Fig. 7.39)	$\frac{1}{g_m}$	$g_m R_D$	R_D	$g_m(R_D \parallel R_L)$	$\frac{R_D \parallel R_L}{R_{sig}+1/g_m}$
Source follower (Fig. 7.42)	∞	1	$\frac{1}{g_m}$	$\frac{R_L}{R_L+1/g_m}$	$\frac{R_L}{R_L+1/g_m}$

[a] For the interpretation of R_{in}, A_{vo}, and R_o, refer to Fig. 7.34(b).