ECE 255, Darlington Pair

7 November 2017

In order to understand the derivation of Prof. Chen on the Darlington pair in her notes, it will be prudent to do Exercise 8.30 from Sedra and Smith. You are asked to show that (see Figure 1):

1. $R_{\text{in}} = (\beta_1 + 1)\left[r_{e1} + (\beta_2 + 1)(r_{e2} + R_E)\right]$
2. $R_{\text{out}} = R_E \parallel \left[r_{e2} + \frac{r_{e1} + [R_{\text{sig}}/(\beta_1+1)]}{\beta_2+1}\right]$
3. $\frac{v_o}{v_{sig}} = \frac{R_E}{R_E + r_{e2} + [r_{e1} + R_{sig}/(\beta_1+1)]/(\beta+2+1)}$

Answers:

1. The answer is to use the resistance-reflection rule twice. If Q_1 has an emitter resistance of R_{E1}, then using formula (7.107) of S&S, one gets

   $$R_{\text{in}} = (\beta_1 + 1)(r_{e1} + R_{E1})$$

   But R_{E1} is the input resistance looking into the base of Q_2. Using the reflection formula again to get R_{E1} from R_E, or

   $$R_{E1} = (\beta_2 + 1)(r_{e2} + R_E)$$

   one gets the above formula in Part 1.

   Note: Since $(\beta_1+1)r_{e1} = r_{\pi 1}$ which can be gotten by comparing the input impedance of the hybrid-π model and the T model, the above reflection formula can also be written as

   $$R_{\text{in}} = (\beta_1 + 1)r_{e1}(1 + R_{E1}/r_{e1}) = r_{\pi 1}(1 + R_{E1}/r_{e1})$$

   Since $g_m = \alpha/r_{e1} \approx 1/r_{e1}$, the above formula is often approximated as

   $$R_{\text{in}} \approx r_{\pi 1}(1 + g_{m1}R_{E1})$$

   The above formula is often used in Prof. Chen's notes.

   Again, using the approximate reflection formula, one can rewrite R_{E1} as

   $$R_{E1} = r_{\pi 2}(1 + g_{m2}R_E)$$

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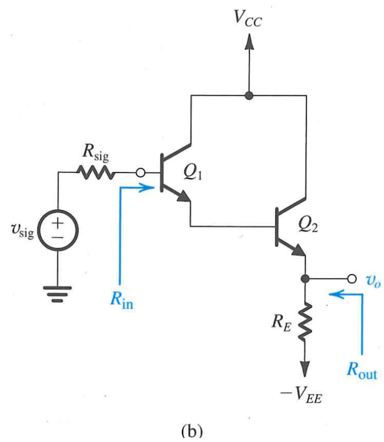

Figure 1: The Darlington pair or configuration for the analysis in Exercise 8.30 of S&S (Courtesy of Sedra and Smith).

2. First, one needs to find the Thevenin resistor of the Thevenin equivalence of the source to the left of R_E. Then $R_{\text{out}} = R_E \parallel R_{\text{Thev}}$. To find R_{Thev}, one shorts out the voltage source, and uses the inverse reflection rule two times to get
$$R_{\text{Thev}} = \left[r_{e2} + \frac{r_{e1} + [R_{sig}/(\beta_1 + 1)]}{\beta_2 + 1} \right]$$
The inverse reflection formula is based on that, for a BJT, every unit of current that flows in the base, there is $\beta + 1$ unit of current that flows in the emitter to arrive at the formula in Part 2.

   Using the fact that $g_m = \alpha/r_e \approx 1/r_e$, and that $\beta + 1 \approx \beta$, the above formula can also be approximated as
$$R_{\text{out}} \approx R_E \parallel \left[1/g_{m2} + \frac{1/g_{m1} + [R_{\text{sig}}/(\beta_1)]}{\beta_2} \right]$$
as is found in Prof. Chen's notes.

3. For this problem, it is necessary to find the Thevenin resistor looking to the left of R_E from the emitter of Q_2. This resistor is obtained by short-circuiting the source, and looking at the impedance to the left of R_E. This Thevenin resistor is similar to that in Part 2. Next, the equivalent Thevenin voltage source needs to be found. This can be done by open-circuiting the load by letting $R_E \to \infty$. It is seen that $R_{\text{in}} \to \infty$ in this case. With the small voltage drops between the base and the emitters, it is seen that $v_{\text{Thev}} \approx v_{\text{sig}}$. Hence formula in Part 3 above can be derived usig the Thevenin equivalent circuit and the voltage divider formula.