ECE 255

31 August 2017

In this lecture, the use of the *pn* junction as a diode will be discussed. The diode is one of the simplest semiconductor device, and finds applications in many modern electronic gadgets. The marked feature of a diode is that its I-V relationship is nonlinear, making its analysis challenging. However, we will exploit as much of our knowledge in linear circuit analysis to analyze circuits with nonlinear diodes in them. It has been shown that as the voltage increases, the current cannot be linearly related to the applied voltage. This is unlike a resistor where one has $V = IR$ where V is linearly related to I. For a diode made from a *pn* junction, this relation, as shown previously, is

$$I = I_S\left(e^{\frac{V}{V_T}} - 1\right) \tag{0.1}$$

where V is the applied of biasing voltage and I is the diode current. This physical model is derived from semiconductor physics. It has the property that when $V \gg V_T$, I tends to infinity. But when $V \ll -1$, $I \approx I_S$ which is very small. Here, I_S is varyingly called the saturation current, the generation current, the leakage current, or the scale current; the last name follows from that this current scales as the cross-sectional area of the diode. This source of this saturation current is due to EHP generation in the semiconductor material and hence is small.

1 An Ideal Diode

The *i*-*v* relation[1] of an ideal diode is such that the current becomes infinitely large when it is **on**, or in forward bias. Conversely, the current is zero when the diode is **off**, or in reverse bias. This idealization can be used to simplify the analysis of some simple nonlinear circuits. As shown in Figure 1, the use of such idealization turns a nonlinear circuit into set of linear circuits that can be analyze easily.

Figure 2 shows the use of an ideal diode, where the on-off states are replaced with open and short circuits respectively. Then the circuit can be analyzed simply as such according to the state of the diode using linear circuit analysis. The behavior of the circuit can also be easily understood as such.

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[1]Following the text book, the lower cases will be used for current and voltage in this lecture.

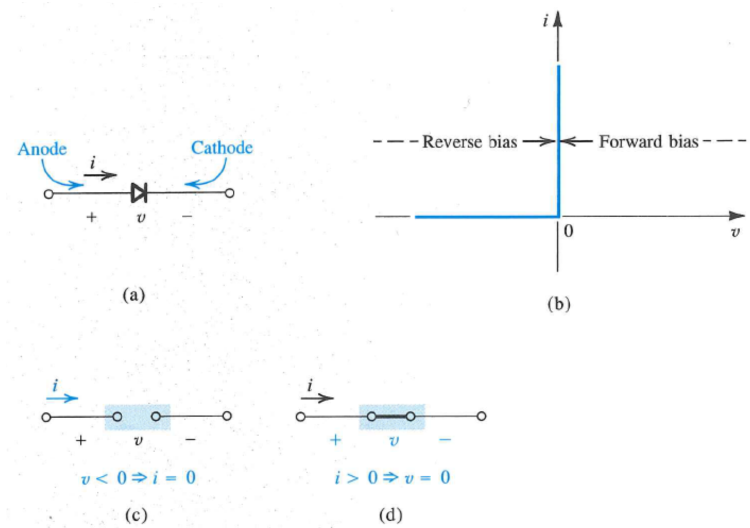

Figure 1: The i-v characteristic of a pn junction ideal diode (a) is shown in (b). In (c) it is an open circuit or **off** in reverse bias. In (d) it is a short circuit or **on** in forward bias (Courtesy of Sedra and Smith).

The turning on of an ideal diode can be delayed by biasing it with the DC voltage source as shown in Figure 3. This also resembles the charging of a 12 V battery by an AC source whose peak voltage is 24 V.

Example 1.[2]

For an ideal diode, it only has two states, the on or the off state. One will analyze the circuit by assuming that the diodes are either on or off, and analyze the voltage and current across the diodes accordingly using linear circuit analysis. In the on state, the voltage drop across the diode is zero, while in the off state, the current flow through the diode is zero. If the answer contradicts the assumption, it implies that the assumption is wrong, and it has to be revised.

For example in Figure 4(a), one assumes that both diodes are on. Then the voltage at node B is zero and

$$I_{D2} = \frac{10 - 0}{10} = 1\,\text{mA} \tag{1.1}$$

Writing KCL (Kirchhoff Current Law) at node B, then

$$I + 1 = \frac{0 - (-10)}{5} \tag{1.2}$$

giving $I = 1$ mA. Hence, both diodes are on, and not contradicting the assumption.

[2]Example 4.1 of textbook.

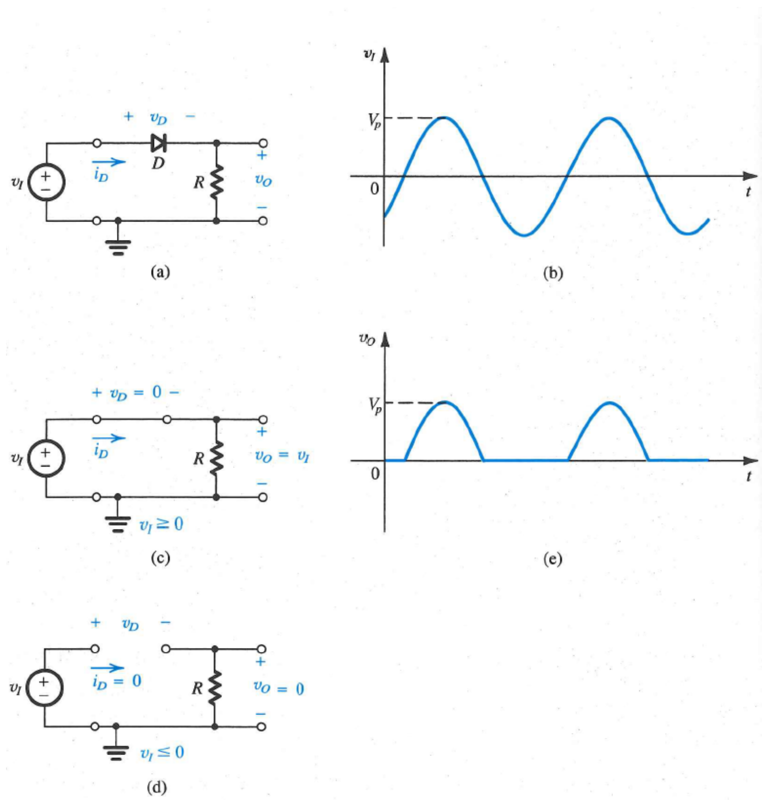

Figure 2: (a) The symbol of the diode. (b) The i-v characteristic of a pn junction as an ideal diode. (c) When the diode is reverse bias, it is an open circuit or off. (b) When the diode is forward bias, it is a short circuit or on (Courtesy of Sedra and Smith).

For the case in Figure 4(b), one first assumes that both diodes are on. Then $V_B = 0$ and $V = 0$, and

$$I_{D2} = \frac{10-0}{5} = 2\,\text{mA} \tag{1.3}$$

Applying KCL at node B,

$$I + 2 = \frac{0-(-10)}{10} \tag{1.4}$$

giving $I = -1$ mA. This contradicts our assumption that the diode D_1 is on.

To revise the wrong assumption, one assumes that D_1 is off while D_2 is on. Then the current through D_2 is

$$I_{D2} = \frac{10-(-10)}{15} = 1.33\,\text{mA} \tag{1.5}$$

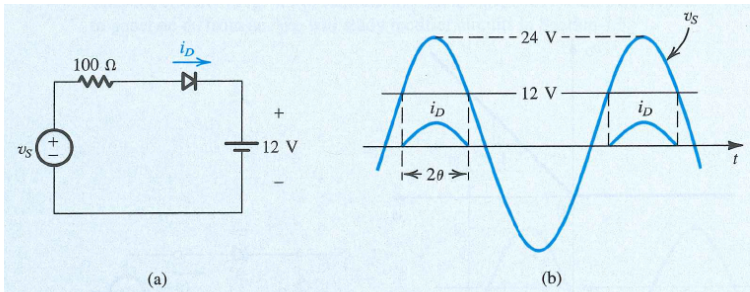

Figure 3: A DC voltage source can be used to delay the turning on of the ideal diode (Courtesy of Sedra and Smith).

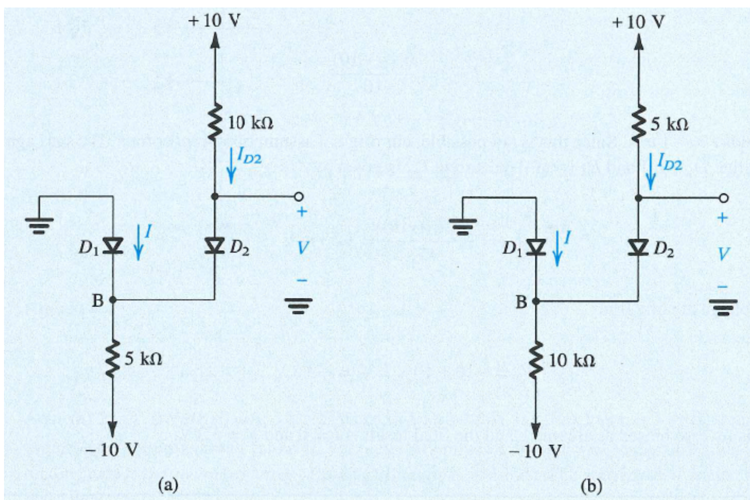

Figure 4: The circuit diagram for analyzing nonlinear circuits with ideal diodes for Example 1 (Courtesy of Sedra and Smith).

The voltage at node B is obtained by applying KVL (Kichhoff voltage law) then

$$V_B = -10 + 10 \times 1.33 = +3.3\,\text{V} \tag{1.6}$$

Hence, $V_B > 0$ implying that D_1 is off or reverse bias giving rise to $I = 0$ and $V = 3.3\,\text{V}$.

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2 More on I-V Relation of Junction Diodes

The i-v relation of a diode, as shown by using device physics, is given by

$$i = I_S\left(e^{\frac{v}{V_T}} - 1\right) \tag{2.1}$$

where $V_T = k_B T/q$ is about 25 mV at room temperature. When the voltage v is about 0.1 V, it is about 4 times V_T. And e^{v/V_T} is about $e^4 \approx 54.6$ which is quite large. When $v = 0.6$ V, a biasing voltage 6 times larger, then $e^{4\times 6} = e^{24} = 2.65\times 10^{10}$ which is a very large number. A small increase in v can overcome the smallness of I_S which can be of the order of 10^{-15}. Hence, the turn-on voltage of a diode is roughly 0.6-0.8 V. The above equation (2.1) can be inverted to give

$$v = V_T\left(\ln\frac{i}{I_S} + 1\right) \tag{2.2}$$

When the bias voltage v is large, the i-v relation (2.1) can be simplified and approximated as

$$i \approx I_S e^{\frac{v}{V_T}} \tag{2.3}$$

or

$$v \approx V_T \ln\frac{i}{I_S} \tag{2.4}$$

Revision of previous knowledge–The formula for the saturation current is

$$I_S = Aqn_i^2\left(\frac{D_p}{L_p N_D} + \frac{D_n}{L_n N_A}\right) \tag{2.5}$$

which is of the order of 10^{-15} A. As have been learned earlier, the formula for n_i is

$$n_i = BT^{3/2}e^{-E_g/(2k_B T)} \tag{2.6}$$

where

$$B = 7.3\times 10^{15}\mathrm{K}^{-3/2}\mathrm{cm}^{-3} \tag{2.7}$$

Hence, when the temperature T increases, there are more thermalized carriers n_i giving rise to larger I_S. It is generally assume that I_S doubles for every 5° C rise in temperature.

3 More on Diode Modeling

Assuming that the diode is connected to a resistor as shown in Figure 5, and one needs to find the diode current I_D and voltage V_D. Since the i-v relation of a diode is nonlinear, there is no simple way to solve this problem. The solution has to be sought graphically or numerically.

3.1 The Exponential Model

Let us assume that the bias voltage is large so that the I-V relationship for a *pn* junction diode can be approximated with high accuracy with the exponential model to be

$$I_D = I_S e^{V_D/V_T} \tag{3.1}$$

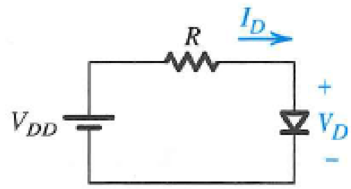

Figure 5: The case of a diode connected to a battery source via a load resistor (Courtesy of Sedra and Smith).

This is also called the exponential model for the diode.

The other equation for I-V is governed by KVL for the voltage drop across the resistor R, or that

$$I_D = \frac{V_R}{R} = \frac{V_{DD} - V_D}{R} \tag{3.2}$$

The two unknowns to be sought are I_D and V_D, assuming the rest to be known constants. Here, I_D and V_D are common to both equations. Albeit simple, these equations have no closed form. For instance, one can eliminate V_D by inverting the first equation, and substituting into the second equation, one gets

$$I_D = \frac{V_{DD} - V_T \ln(I_D/I_S)}{R} \tag{3.3}$$

The above equation has one unknown I_D, but it is a transcendental equation that cannot be solved in closed form.

3.1.1 Graphical Analysis

One way to solve them is via graphical method, as shown in Figure 6. One plot in the graph is for equation (3.1) representing the diode characteristic using the exponential model. The second plot, called the load line, is from equation (3.2) due to the resistive load of the circuit. The solution is given by the point where the plots for the two equations meet. At this point, both equations share the same I_D and V_D, and is called the operating point. This is a method of finding the solution of two simultaneous equations, one of which can be nonlinear. When the number of equations is large, this method is unwieldy, and one resorts to a numerical method of solving these equations.

3.1.2 Iterative Analysis–Method of Successive Approximations

The graphical method can be used by humans easily because of our gifted visual intelligence. However, it is difficult to program a computer to pick out the operating point on a graph. For numerical or computer method, it is better to design an algorithm that can be converted to a program systematically: such is the spirit of iteration analysis or method. We will illustrate this with the method

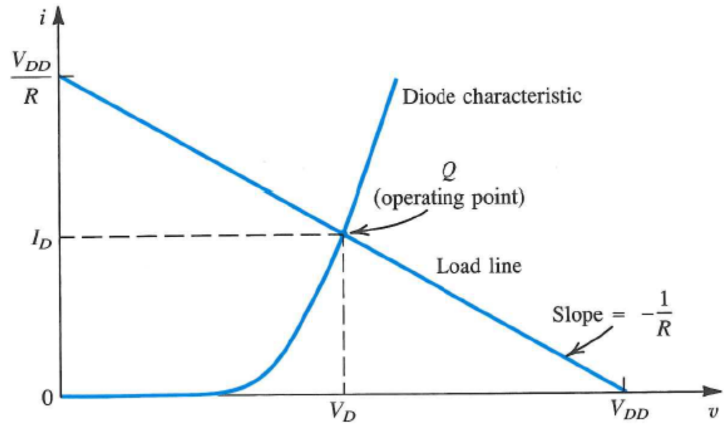

Figure 6: The graphical solution yields the solution of a transcendental equation quickly by visual inspection. (Courtesy of Sedra and Smith).

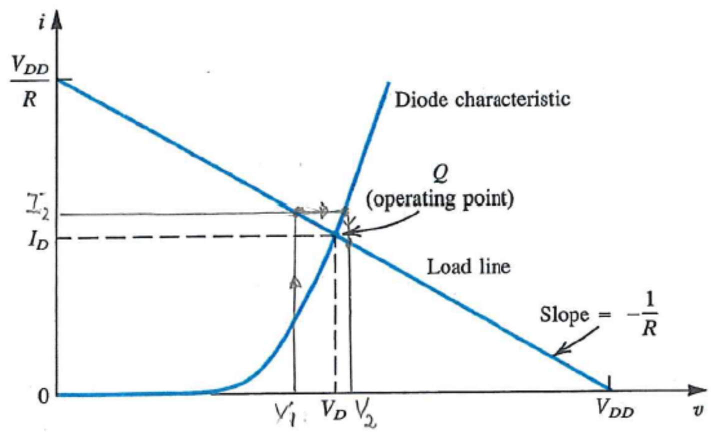

Figure 7: Iterative method such as the method of successive approximation can be converted into a computer program easily (Courtesy of Sedra and Smith).

of successive approximation in the following example, which can be converted to a computer program easily.

Example 2.[3]

In this method, first, we guess a V_D which is not correct, unless we have clairvoyance. Say, one starts with $V_D = 0.7$ V, and we can find out where on the load line the current should be if it were to satisfy (3.2). To this end, the

[3]This is Example 4.2 of the textbook.

correspond current through the resistor is given by

$$I_D = \frac{V_{DD} - V_D}{R} = \frac{5 - 0.7}{1} = 0.43\,\text{mA} \tag{3.4}$$

Next, given this new I_D, one needs to ascertain what V_D should be from the diode equation. One can invert equation (3.1) to obtain V_D, given I_D. However, we do not know I_S, but it can be found since it is given, in this example, that $I_D = 1$ mA when V_D is 0.7 V. Alternatively, one can use the fact that

$$I_1 = I_S e^{V_1/V_T}, \qquad I_2 = I_S e^{V_2/V_T} \tag{3.5}$$

then

$$\frac{I_2}{I_1} = e^{(V_2 - V_1)/V_T} \tag{3.6}$$

Inverting the above gives

$$V_2 - V_1 = V_T \ln \frac{I_2}{I_1}, \quad V_2 - V_1 = 2.3 V_T \log_{10} \frac{I_2}{I_1} \tag{3.7}$$

Using the above, and using that $2.3V_T = 60$ mV, with $V_1 = 0.7$ V, $I_1 = 1$ mA, and $I_2 = 4.3$ mA, gives $V_2 = 0.738$ V. This process can be repeated until the solution converges. When convergence is reached, the solution changes little with iteration number.

The method of successive approximation is also shown in Figure 7. A word of caution is that this method does not always converge. Then other iterative or numerical methods have to be used, for instance, the secant method, or the Newton-Raphson method.

4 The Constant-Voltage-Drop Model

As can be seen previously, when nonlinear equations are involved, their solutions are often difficult. One method is to approximate the I-V characteristics of a diode with piecewise linear approximation. When the diode is operating in the piecewise linear regime, simpler linear methods can be applied. The constant-voltage-drop model is such an attempt to simplify the analysis. The gist of this method is shown in Figure 8. When the diode is off, or the bias voltage is below 0.7 V, then it is replaced by an open circuit. When the diode is on, it is replaced by a short circuit with an internal battery with voltage of v_D as shown.

5 Small-Signal Model

Another way of making a linear approximation to a nonlinear equation is to use the small signal model. The schematic for this model is shown in Figure 9.

The Math Behind Small Signal Model

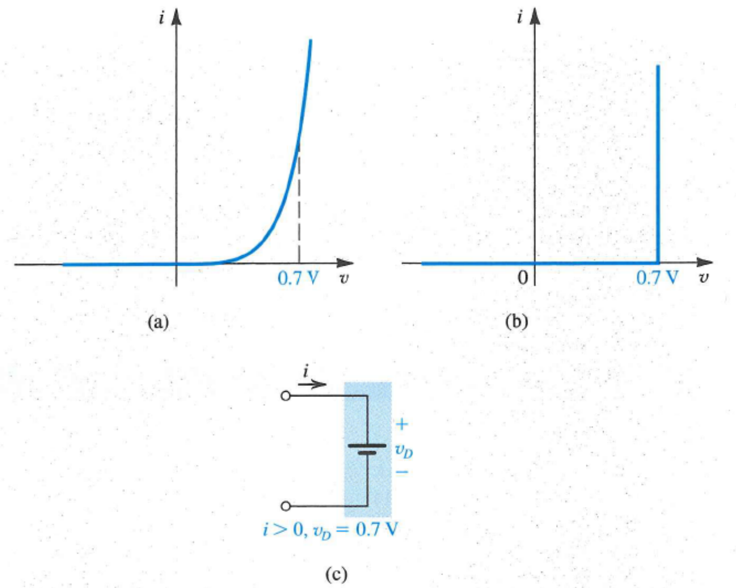

Figure 8: The constant-voltage-drop model can be use to simplify the analysis of diode circuits (Courtesy of Sedra and Smith).

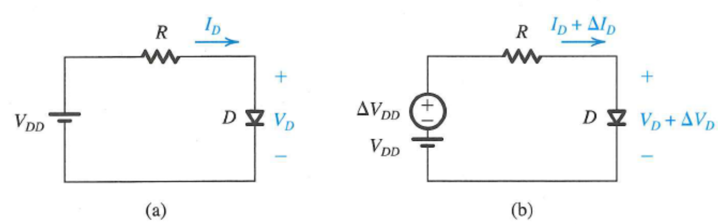

Figure 9: The circuit for a small signal model where a small voltage ΔV_{DD} is superposed on top of a large voltage V_{DD} (Courtesy of Sedra and Smith).

In this model, one assumes that the voltage across the diode

$$v_D(t) = V_D + v_d(t) \tag{5.1}$$

where $v_d(t)$ is small signal voltage compared to V_D, the quiescent DC voltage. The corresponding current through the diode is then

$$i_D(t) = I_S e^{v_D(t)/V_T} = I_S e^{(V_D + v_d(t)/V_T} \tag{5.2}$$

The above can be rewritten as

$$i_D(t) = I_S e^{V_D/V_T} e^{v_d(t)/V_T} = I_D e^{v_d(t)/V_T} \tag{5.3}$$

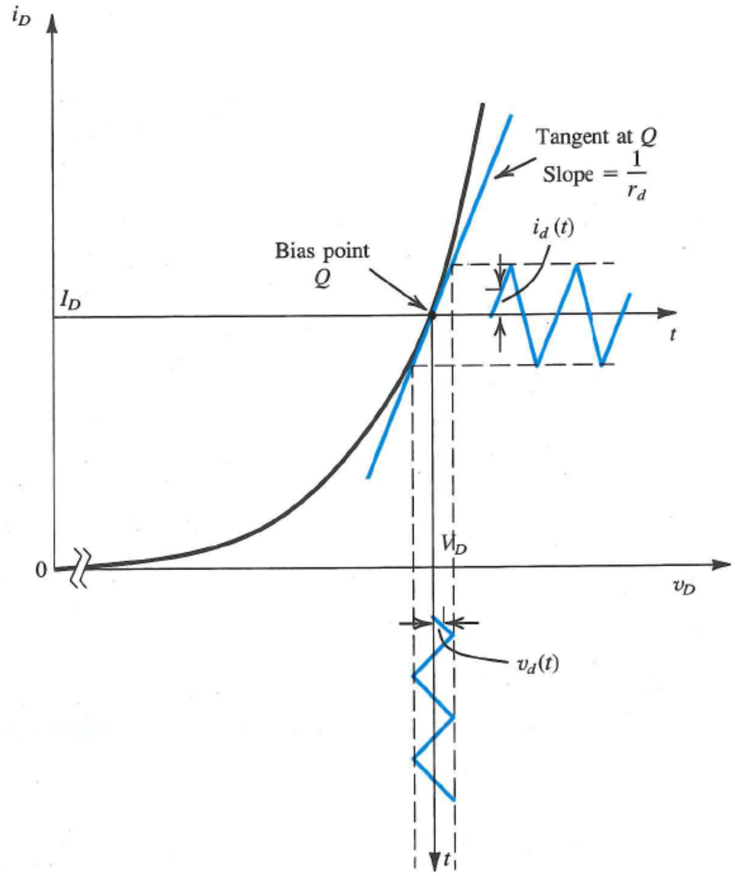

Figure 10: Graphical depiction of the small signal model where $v_d(t)$ is assumed to be much smaller than V_D (Courtesy of Sedra and Smith).

where I_D is time independent, and is

$$I_D = I_S e^{V_D/V_T} \tag{5.4}$$

Since $v_d(t)/V_T \ll 1$ always, using $e^x \approx 1+x$ when x is small, the above equation (5.3) can be approximated, namely,

$$i_D(t) \approx I_D \left(1 + \frac{v_d(t)}{V_T}\right) = I_D + \frac{I_D}{V_T} v_d(t) \tag{5.5}$$

Writing

$$i_D(t) = I_D + i_d(t) \tag{5.6}$$

where $i_d(t)$ is a small signal current, or that $i_d(t) \ll I_D$, then

$$i_d(t) = \frac{I_D}{V_T} v_d(t) \tag{5.7}$$

One can then define an incremental resistance, or small signal resistance

$$r_d = \frac{V_T}{I_D} \tag{5.8}$$

The above approximation converts a nonlinear problem into a linear problem at the DC bias point, or the **quiescent point**. The slope of the i-v curve at this point is also inversely proportional to the incremental resistance, namely that

$$\left[\frac{\partial i_D}{\partial v_D}\right]_{i_D=I_D} = \frac{1}{r_d} \tag{5.9}$$

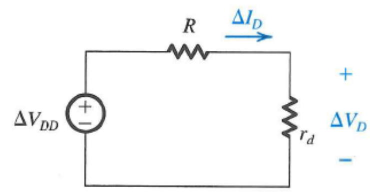

Figure 11: A nonlinear diode circuit can be replaced by a linear resistor circuit under the small signal approximation (Courtesy of Sedra and Smith).

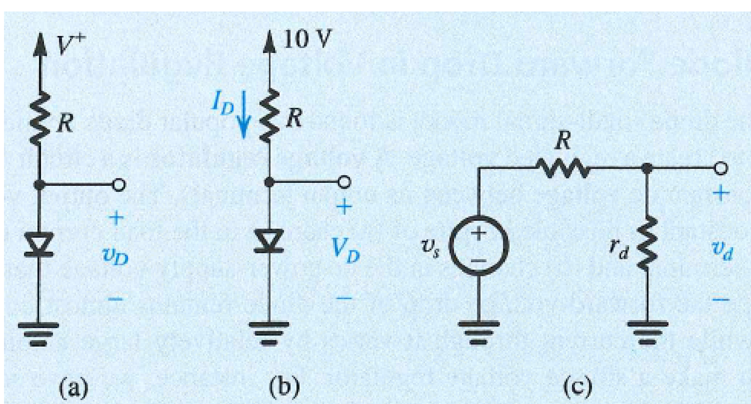

Figure 12: A nonlinear diode circuit can be replaced by a linear resistor circuit under the small signal approximation (Courtesy of Sedra and Smith).

General case of linearization approximation
In general, any nonlinear function can be linearized about some quiescent point if the function is smooth in the vicinity of that point. Consider a function $i_D(v_D)$ to be a general nonlinear function of v_D. Using the Taylor series approximation that any function $f(x)$ can be approximated linearly about a point $x = x_0$ as $f(x) = f(x_0) + (x - x_0)f'(x_0) + \ldots$, then above function of v_D can be approximated as

$$i_D(v_D) = i_D(V_D) + (v_D - V_D)i'_D(V_D) + \ldots \tag{5.10}$$

Then, defining $I_D = i_D(V_D)$,

$$i_D(v_D) - I_D = (v_D - V_D)i'_D(V_D) + \ldots \tag{5.11}$$

or letting $\Delta i_D = i_D(v_D) - I_D$, and $\Delta v_D = (v_D - V_D)$, gives

$$\Delta i_D = \Delta v_D i'_D(V_D) = \Delta v_D \frac{1}{r_D} \tag{5.12}$$

The above represents a general relationship between the incremental current Δi_D and incremental voltage Δv_D. The relationship between them is via the reciprocal resistance $1/r_D$.

Example 3.[4]
Consider a circuit shown in Figure 12 with $R = 10$ kΩ. The power source V^+ has a DC value of 10 V, on top of which is superposed an AC signal with 1-V peak amplitude at 60 Hz. This models the imperfection of the power supply ripple. Find the DC voltage of the diode, and the sinusoidal signal across it, assuming that V_D of the diode is 0.7 V at then $I_D = 1$ mA.
Answer:
Assuming DC signal only, and that $V_D = 0.7$ V, as shown in Figure 12(b), then

$$I_D = \frac{10 - 0.7}{1}0 = 0.93\,\text{mA} \tag{5.13}$$

Since we are in the small-signal regime, then the incremental resistance is given by

$$r_d = \frac{V_T}{I_D} = \frac{25}{0.93} = 26.9\,\Omega \tag{5.14}$$

The original circuit can now be replaced by the small signal model of Figure 12(c). Using the voltage divider rule, then

$$v_d(t) = v_s(t)\frac{r_d}{R + r_d} \tag{5.15}$$

[4]Same as Example 4.5 of the textbook.

The peak voltage is then

$$v_d(peak) = v_s(peak)\frac{r_d}{R + r_d} = 1\frac{0.0269}{10 + 0.0269} = 2.68\,\mathrm{mV} \tag{5.16}$$

This voltage is small compared to V_T affirming out small-signal assumption.
