Some useful integral identities for products of Bessel functions are as follows:

$$\int^z \left[(k^2-l^2)t - \frac{(\mu^2-\nu^2)}{t}\right] C_\mu(kt)D_\nu(lt)dt \\ = z\left[kC_{\mu+1}(kz)D_\nu(lz) - lC_\mu(kz)D_{\nu+1}(lz)\right] - (\mu-\nu)C_\mu(kz)D_\nu(lz) \tag{1}$$

where C and D are any Bessel functions. If $\mu = \nu$, it simplifies to:

$$\int^z (k^2-l^2)tC_\mu(kt)D_\mu(lt)dt \\ = z\left[kC_{\mu+1}(kz)D_\mu(lz) - lC_\mu(kz)D_{\mu+1}(lz)\right] \tag{2}$$

Using the recurrence relation for Bessel functions:

$$B_{\mu+1}(x) = -B'_\mu(x) + \frac{\mu}{x}B_\mu(x)$$

we obtain for Bessel functions:

$$\int_0^z tJ_\mu(kt)J_\mu(lt)dt \\ = z\left[-kJ'_\mu(kz)J_\mu(lz) + lJ_\mu(kz)J'_\mu(lz)\right]/(k^2-l^2) \tag{3}$$

When lz and kz are zeros of Bessel functions, or zeros of the derivatives of Bessel functions, the right hand side is always zero, except when $k = l$. When L'Hospital's rule is applied, we obtain:

$$\int_0^z tJ_\mu(kt)J_\mu(lt)dt \\ = \frac{z^2}{2}\left[J'_\mu(kz)\right]^2 \quad \text{when } k = l \text{ and } J_\mu(kz) = 0 \\ = \frac{1}{2k^2}\left[(kz)^2 - \mu^2\right]\left[J_\mu(kz)\right]^2 \quad \text{when } k = l \text{ and } J'_\mu(kz) = 0 \tag{4}$$