An Accurate Method of Neglecting Dynamic Saliency of Synchronous Machines in Power Electronic Based Systems

S. D. Pekarek, Member
School of Electrical Engineering
University of Missouri-Rolla
Rolla, Missouri

E. A. Walters, Student Member
School of Electrical and Computer Engineering
Purdue University
West Lafayette, Indiana

Abstract - In this paper, motivations for neglecting dynamic saliency in the detailed models of power electronic based systems are presented. A new method of neglecting dynamic saliency is then introduced in which approximate operational impedances are derived from original dynamic salient models. New machine parameters are obtained by fitting to the approximate impedance curves, yielding model parameters in which dynamic saliency is eliminated. An example synchronous machine/ converter system is provided that demonstrates the accuracy and increased simulation efficiency resulting from this technique. Therein, errors resulting from neglecting dynamic saliency are reduced from greater than 25% using traditional approximations to less than 0.6%. In addition, simulations using the new approximate model are greater than 48% faster than simulations based upon the original dynamic salient model.

I. INTRODUCTION

There are several approximations, models, and model structures that are used to represent the dynamics of the synchronous machine. The complexity of the model chosen for a particular analysis is a function of the size of the system being studied and the detail required. In several applications, including the development of reduced-order models of large-scale power systems [1] and average-value models of machine/converter systems [2], an approximation is made in which the dynamic saliency of the synchronous machine is neglected. In large-scale transient analysis programs, neglecting the dynamic saliency provides a means to reduce computational requirements by eliminating time-dependent matrices [1]. In average-value analysis of machine/converter systems, neglecting dynamic saliency greatly reduces analytical derivations by eliminating time-varying commuting inductances [2].

Although several researchers have documented the use of the approximation, the actual methods used by researchers vary. In particular, some equipment manufacturers, particularly in North America, assume equal subtransient reactances and calculate q-axis machine parameters based upon this assumption [3]. In other cases, the system analyst neglects any differences, or uses a weighted average of the q- and d-axis dynamic reactances within the respective machine and network equations [1,2,4].

II. BACKGROUND

Several researchers have expressed concern over the use of these approximations, particularly in machines in which the ratio of the q- and d-axes dynamic reactances is large [2,5]. In the derivation of average-value models of synchronous machine/converter systems [2], errors of greater than 20% were documented when the dynamic saliency of an example synchronous machine was neglected.

In this paper, it is first shown that neglecting dynamic saliency can substantially reduce the computational overhead required in the detailed analysis of power electronic based generation systems. Such systems are typically used in shipboard and aircraft power and propulsion systems and consist of one or more generator/power electronic converters which supply power to a network consisting of highly-regulated loads. A new method of neglecting dynamic saliency is then derived. Using this method, the original parameters of the machine, which can be determined using any technique, are used to calculate operational impedances. From the original impedances, new approximate operational impedances are derived. The new curves match the original impedances over a user-specified frequency range, while having high-frequency asymptotes that are equal. A refitting of the machine parameters to the new approximate curves is performed in order to establish parameters that yield equal dynamic reactances.

This technique requires little work from the system analyst, and greatly reduces the errors resulting from neglecting dynamic saliency. Example simulations using a machine with an original dynamic reactance ratio of 1.9 are provided in which the errors resulting from neglecting dynamic saliency are reduced from over 25%, using traditional methods, to less than 0.6%. In addition, simulations using the new approximate model are greater than 48% faster than simulations based upon the original dynamic salient model.
Herein, the dynamic inductances are defined as

\[ L_{d}'' = L_{d} + L_{md}'' \]

\[ L_{q}'' = L_{q} + L_{mq}'' \]

where

\[ L_{mq}'' = L_{mq} \parallel L_{lkq1} \parallel \ldots \parallel L_{lkqM} \]

\[ L_{md}'' = L_{md} \parallel L_{fjd} \parallel L_{lkd1} \parallel \ldots \parallel L_{lkdN} \]

The back emfs are defined as

\[ v_q'' = \omega \lambda_q'' + \sum_{j=1}^{M} \left( \frac{L_{mq}'' L_{kqj}}{L_{lkqj}} \lambda_{kqj} - \frac{L_{md}'' L_{fjd}''}{L_{ld}''} \lambda_{fjd} \right) \]

\[ v_d'' = -\omega \lambda_q'' + \sum_{j=1}^{N} \left( \frac{L_{md}'' L_{kqj}''}{L_{lkqj}''} \lambda_{kqj} + \frac{L_{md}'' L_{fjd}''}{L_{ld}''} \lambda_{fjd} \right) \]

The back emf is given by

\[ v_{abc}'' = r_s f_{abc}'' + L_{abc}'' \dot{I}_{abc} \]

where \( r_s \) is a diagonal matrix of stator resistances, and \( L_{abc}'' \) is given by

\[ L_{abc}'' = \begin{bmatrix} L_{d} & 0 & 0 \\ 0 & L_{q} & 0 \\ 0 & 0 & L_{m} \end{bmatrix} \]
The stator voltage equations given by (16), along with the rotor voltage equations
\[ p\lambda_j = \frac{rL_j}{L_j}(\lambda_j - \lambda_{mq}) + v_j; j = kq1, ... kQM \]  
\[ p\lambda_j = -\frac{rL_j}{L_j}(\lambda_j - \lambda_{md}) + v_j; j = fd, kd1, ... kdN \]
where $\lambda_{mq}$ and $\lambda_{md}$ are the magnetizing flux linkages, define the so-called physical-variable voltage-behind-reactance (PVBR) model of the synchronous machine. It is important to note that no approximations are made in its derivation. Neglecting numerical error, the solution of the corresponding equations yields the same time-domain response of a full-order Park's circuit model upon which it is based.

III. NUMERICAL ADVANTAGES OF NEGLECTING DYNAMIC SALIENCY

Traditionally, a significant challenge in the development of power electronic-based systems has been the derivation of detailed models from which a system can be analyzed and designed. In particular, the use of power electronic devices makes it difficult for an individual to analytically derive the differential equations for all potential switching topologies of a system. In order to alleviate the task of analytically deriving the equations of complex power systems, several algorithmic methods have been developed [10, 11, 12]. The syntax of the algorithms, as well as the equations that are formed and solved, vary. However, regardless of the method, the complexity and model structure of the components are critical factors in establishing computationally efficient system models.

The PVBR model described in Section II has been shown to be an efficient model structure for detailed analysis of power electronic-based systems. Comparisons between model structures of synchronous machines used in algorithmic languages have demonstrated that the PVBR is significantly more efficient than standard physical-variable models [9] and models in which Park's circuit is used with associated coupling equations [13].

Although efficient, there are significant incentives to eliminate the time-varying inductances present in the model. The incentives are two-fold.

First, significant numerical gains can be achieved if time-varying inductances are eliminated. In particular, in differential equation-based algorithms, a major computational step is the manipulation of the system equations into an explicit state model form. This manipulation, in which currents are state variables, requires the solution of a system of equations of the form
\[ L_p L_x = -(r_x + pL_x)L_x + v_x \]
for the derivative of the state vector, $pL_x$. In (25), $r_x$, $L_x$, and $pL_x$ are algorithmically generated matrices consisting of branch resistances, inductances, and time-rates of change of inductances, respectively. The size of these matrices is directly proportional to the size of the system. If the matrix $L_x$ contains time-varying parameters, the solution of (25) must be performed at each integration time-step. In contrast, if $L_x$ is time-invariant, the inverse of $L_x$ may be precomputed and the solution involves only a matrix-vector multiply at each time-step. Thus, the disparity in the number of operations required at each time-step between the time-varying and time-invariant models, increase on the order of $n^2$, where $n$ repesents the number of states in the system. Additionally, the time-varying inductances can produce time-varying positive eigenvalues [9] that can reduce the efficiency when implementing a variable step-size integration algorithm [14].

Second, several commercial algorithmic languages, such as Spice [10] and Saber [11], do not include a component representation of time-varying inductors. Therefore, the elimination of time-varying inductances in the PVBR model provides a means to easily implement an efficient physical-variable model of the synchronous machine in such languages.

These incentives provide a motivation to develop an accurate physical-variable synchronous machine model in which the inductances are time-invariant.

IV. TRADITIONAL METHODS OF NEGLECTING DYNAMIC SALIENCY

For machines in which at least one $d$-axis amortisseur winding is represented, equipment manufacturers often provide machine parameters that have equal dynamic reactances [3]. Historically, deriving models that have equal dynamic reactances has been facilitated by the calculation procedures used to determine machine parameters. In particular, using traditional methods, parameters of the $d$-axis are calculated based upon open-circuit and short-circuit tests. Similar tests to determine $q$-axis dynamic values are impractical; therefore, approximations are often used. In particular, each axis is modeled with an identical number of rotor windings and the $q$-axis dynamic reactance is assumed to be equal to that of the $d$-axis, $X''_q = X''_d$. Values of the $q$-axis rotor circuit parameters are then determined using these approximations [3]. To the authors' knowledge, the accuracy of neglecting dynamic saliency using this technique has never been analytically established, although it has been questioned [5]. Furthermore, the test procedures used to obtain the $q$-axis parameters from these methods has been opened to concern [3]. This technique will not be explored in depth in this paper for two reasons. First, from an analyst's perspective, if parameters are provided in this form, the accuracy of neglecting dynamic saliency only becomes an issue if the responses predicted by the model are deemed inaccurate. In such instances, the analyst will most likely need to perform tests in which the $q$-axis machine parameters can be measured, such as a Standstill Frequency Response (SSFR) test. Results of these tests will most likely yield a model that does not have equal dynamic reactances, in which case the technique described in this paper can be applied. Second, with advances in test procedures to measure $q$-axis parameters, a more common approximation applied in power electronic-based system analysis is to neglect dynamic saliency in the machine and system equations [2]. The accuracy of neglecting dynamic saliency using this technique has been analytically examined for machine/converter systems, and serious concerns regarding the accuracy of the approximation have been raised [2].

Considering the PVBR model of the machine given in (16)-
(24), the time-varying inductances are eliminated if the following assumption is made, namely

\[ L''_b = \frac{L''_d - L''_q}{3} = 0 \]  \hspace{1cm} (26)

Determining mathematical bounds for errors resulting from this approximation is a tedious, if not intractable, task. Therefore, to better understand this approximation, it is helpful to transform the stator equations (16) in which (26) is applied, to the rotor reference frame. Application of Park's transformation to the approximate stator equations yields

\[ v'_{qs} = r'_{s} i'_q + \alpha (\frac{L''_d + L''_q}{2}) i'_d + \beta (\frac{L''_d + L''_q}{2}) i'_q + v''_q \]  \hspace{1cm} (27)

\[ v'_{ds} = r'_{s} i'_d - \alpha (\frac{L''_d + L''_q}{2}) i'_q + \beta (\frac{L''_d + L''_q}{2}) i'_d + v''_d \]  \hspace{1cm} (28)

Comparing the original and approximate models, it is seen that the \( q \)- and \( d \)-axis dynamic reactances in (5)-(6) are replaced with average values of the \( q \)- and \( d \)-axis dynamic reactances in (27)-(28). This is interesting, since (27)-(28) are similar to representations of the stator voltage equations which raised concerns regarding the accuracy of neglecting dynamic saliency in [2].

To further compare the original and approximate models, it is interesting to consider the models in the frequency domain. In particular, by simulating a machine to a steady state and setting the rotor angular velocity and stator resistance to zero, perturbations to \( v'_{qs} \) and \( v'_{ds} \) can be made and the resulting perturbations in \( i'_{qs} \) and \( i'_{ds} \) measured. The resulting input impedance of the \( q \)- and \( d \)-axis equivalent circuits, \( Z_q(s) \) and \( Z_d(s) \), can then be obtained numerically and used to establish the operational impedances \( X_q(s) \) and \( X_d(s) \) [15]. The numerically calculated \( q \)- and \( d \)-axes operational impedances of the synchronous machine described in Section VI are shown in Fig. 2. Comparing the impedances of the original and traditional approximate model shows the frequency domain errors associated with the traditional method of neglecting dynamic saliency. In particular, in the traditional approximate model, the curves are both shifted by \[ \frac{X''_q - X''_d}{2} \] over the entire frequency range.

V. NEW METHOD OF NEGLECTING DYNAMIC SALIENCY

Observing that the traditional method of neglecting dynamic saliency introduces errors over the entire frequency range, a new method is derived that only introduces errors at and beyond a user specified frequency, \( f_r \). Independent of how the synchronous machine parameters are determined, the operational impedances for the \( q \)- and \( d \)-axes can be expressed as

\[ X_q(s) = \frac{1 + \tau q_1 s}{(1 + \tau q_1 s)(1 + \tau q_2 s) \cdots (1 + \tau_q M s)} \]  \hspace{1cm} (29)

\[ X_d(s) = \frac{1 + \tau d_1 s}{(1 + \tau d_1 s)(1 + \tau d_2 s) \cdots (1 + \tau_d N s)} \]  \hspace{1cm} (30)

where \( \tau q_1, \tau q_2, \ldots, \tau_q M \) and \( \tau d_1, \tau d_2, \ldots, \tau_d N \) are determined from the machine parameters as shown in [15]. From these expressions, the high-frequency asymptotes, which correspond to the dynamic reactances, are defined as

\[ X_q'' = X_q(\frac{\tau q_1 \tau q_2 \cdots \tau_q M}{\tau^2 q_1 \tau^2 q_2 \cdots \tau^2_M}) \]  \hspace{1cm} (31)

\[ X_d'' = X_d(\frac{\tau d_1 \tau d_2 \cdots \tau_d N}{\tau^2 d_1 \tau^2 d_2 \cdots \tau^2_N}) \]  \hspace{1cm} (32)

In general, \( X'_q \) and \( X'_d \) will not be equal. Therefore, from (21), the PVBR model will contain time-varying inductances. As shown in the Section III, significant advantages can be realized if time-varying inductances are eliminated. Thus, the goal of this technique is to fit machine parameters to approximate impedance curves that have equal dynamic reactances while matching the original operational impedances for frequencies less than a user specified rotor fit frequency, \( f_r \). This goal can be accomplished by altering the rotor windings in the original model or by adding a winding in either or both axes. From experience with this technique, either or both \( q \)- and \( d \)- axes may be manipulated with identical results. Therefore, for simplicity, only the \( q \)-axis parameters will be modified herein.

Once the rotor fit frequency, \( f_r \), is set by the user, the refitting of the impedances is a two step procedure. The first step is to consider whether an additional winding is required. A winding will need to be added if

\[ \tau_{QM} > \frac{1}{20\pi f_r} \]  \hspace{1cm} (33)

Failure to add a winding if (33) is satisfied will produce a \( q \)-axis impedance that will not match the original \( q \)-axis impedance to the specified frequency \( f_r \).

The second step is to determine the \( q \)-axis operational impedance time constants. If a rotor winding is added, numerator and denominator time constants, \( \tau_{q(M+1)} \) and \( \tau_{Q(M+1)} \), must be added to (29) to fully describe the higher order system. If a winding is not added, \( \tau_{qb} \) and \( \tau_{QM} \) must be modified to provide equal dynamic reactances. In either case, only the numerator and denominator time constants associated with the highest order
result of the equal dynamic reactances. To determine these changed time constants, two constraint equations must be specified.

The first constraint equation is set from the rotor fit frequency \( f_r \). This frequency approximates the frequency below which the original and approximate model's operational impedances match. The value chosen is arbitrary; however, errors resulting from the approximate model will decrease as \( f_r \) increases. For small values of \( f_r \), the accuracy will approach that of traditional approximate models. As \( f_r \) increases, the accuracy of the new approximate model will approach that of the original dynamic salient model. From experience, to obtain unobservable errors in system responses, a rotor fit frequency of over 120 Hz is required. From the rotor fit frequency, the denominator constraint equation is specified as

\[
\tau Y = \frac{1}{2\pi 10^{-f_r}}
\]

where \( Y \) represents \( (M+1) \) if a rotor winding is added and \( M \) otherwise. To provide identical operational impedances at and below the rotor fit frequency \( f_r \), a factor of 10 is introduced in the denominator to shift the operational impedance pole by one decade [17].

The second (numerator) constraint equation is specified in order to produce equal dynamic reactances and is defined as

\[
\tau QY = \frac{X_d(1+X_d)(1+X_{q1}X_{d1}\cdots X_{q1}X_{d1})(1+X_{q2}X_{d2})(1+X_{q3}X_{d3})(1+X_{q4}X_{d4})}{X_q(1+X_q)(1+X_{q1}X_{d1}\cdots X_{q1}X_{d1})(1+X_{q2}X_{d2})(1+X_{q3}X_{d3})(1+X_{q4}X_{d4})}
\]

\[ (34) \]

where \( Y \) is defined as in (34). Upon determining the \( q \)-axis time constants, new \( q \)-axis parameters are determined as specified in [15]. To demonstrate this new technique, the numerically calculated \( q \)- and \( d \)-axis operational impedances of the machine described in Section VI with \( f_r = 160 \text{ Hz} \) are shown in Fig. 3.

From these plots, it is shown that this technique only affects the operational impedances for frequencies above the specified frequency \( f_r \). Therefore, this new technique eliminates low frequency errors associated with the traditional approximate method, while eliminating the time-varying inductances as a result of the equal dynamic reactances.

If \( f_r \) is large, a high frequency rotor time constant is introduced. However, the rotor terms required to produce accurate results are generally slow compared to many stator time constants in power electronic based systems. Therefore, the effect on simulation performance is minimal.

VI. COMPUTER STUDY

In order to illustrate the accuracy and numerical efficiency resulting from the new approximation technique, the results of two computer studies are described. The synchronous machine chosen for the studies is a 125-kW gas-turbine generator with parameters summarized in Table 1. This machine has been studied by other researchers [2], and is used herein because of its large dynamic saliency (\( X_d''/X_d''' = 1.90 \)). Although the machine chosen for the studies is designed for use in finite-inertia power electronic based generation, it is insightful to consider the machine in a more traditional power system study. In the first study, it is assumed that the synchronous machine is initially connected to an infinite bus delivering rated KVA at rated power factor. The input torque is held fixed at 1.0 pu and the excitation \( e_{kd} \) is fixed at 2.5 pu. With the machine operating in steady-state, a balanced three-phase fault is applied to the stator terminals at \( t = 6.1 \text{ s} \). The fault is then removed and the stator voltages are reapplied at 6.15 s. It is understood that a three-phase fault at the stator terminals rarely occurs in practice, and the circuit breakers would most likely remove the machine from a system. However, this somewhat academic example has historically been used to explain the dynamics of synchronous machines [15]. Therefore, it is well known and understood by a wide research community.

The responses predicted by three models are compared in Fig. 4. The three models correspond to 1) the original model in which no approximations are made (OM), 2) the traditional approximate model (TAM) in which (26) is assumed, and 3) the new approximate model (NAM) in which dynamic saliency is neglected using the new refitting technique. The new machine parameters were calculated by using a rotor fit frequency of \( f_r = 160 \text{ Hz} \). A list of the original and recalculated machine parameters is given in Table 1.

Table 1: Parameters of 125-kW synchronous machine (per unit).

| \( r_s \) | 5.15 \( \cdot \) 10\(^{-3} \) | \( X_{ds} \) | 0.08 | \( X_{msd} \) | 1.77 |
| \( X_{mq} \) | 1.00 | \( r_{kd} \) | 0.024 | \( r_{fd} \) | 1.11 \( \cdot \) 10\(^{-3} \) |
| \( X_{ikd} \) | 0.334 | \( X_{fd} \) | 0.137 | \( r_{kq2} \) | 0.061 |
| \( X_{ikq2} \) | 0.330 | \( r_{kq1}^* \) | 10.44 | \( X_{*ikq1} \) | 0.146 |
| \( r_{kq2}^* \) | 0.062 | \( X_{*ikq2} \) | 0.331 | * indicate refitted parameter. |

Comparing the responses, it is seen the NAM is substantially more accurate than the TAM. In fact, differences between the NAM and the OM are unobservable. In contrast, the TAM produces a significantly inaccurate response in all the system variables. A list of the maximum errors of the two approximate models, relative to the original model, is shown in Table 2.

In the second study, a system consisting of the synchronous machine described above connected to a line-commutated con-
In this example, the TAM is slightly more efficient than the NAM, which is expected since the state model of the system is relatively small in order. Therefore, the addition of a rotor state has a more significant impact than would occur for larger-scale systems. Although the CPU time required by the OM are not prohibitive for this study, as the size of the system increases, the computations required to reform the state model at each time-step increases. Therefore, the difference in efficiency between the NAM and OM will significantly increase with the size of the system.

---

Table 2: Maximum error relative to original model (study I).

<table>
<thead>
<tr>
<th>variable</th>
<th>traditional approximate model</th>
<th>new approximate model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$i_{qs}$</td>
<td>29.7%</td>
<td>1.3%</td>
</tr>
<tr>
<td>$i_{ds}$</td>
<td>27.5%</td>
<td>0.1%</td>
</tr>
<tr>
<td>$T_e$</td>
<td>31.1%</td>
<td>0.7%</td>
</tr>
<tr>
<td>$i_{fd}$</td>
<td>24.4%</td>
<td>0.2%</td>
</tr>
</tbody>
</table>

In the computer studies, it is assumed that the system is initially operating in the steady state with a base load resistance of 1.8 pu connected to the dc output terminals. At $t = 1.25$ s, the load is increased to a value of 0.05 pu. This study reflects a realistic condition in which a high-impedance fault is applied to the dc terminals. This can occur in combat-based power systems wherein sharpel and artillery produce relatively high-impedance paths from positive to negative rail. The simulated responses of the three models are shown in Fig. 6. In calculating the responses, Gear's integration algorithm was used with a maximum and minimum time step of $1 \cdot 10^{-4}$ and $1 \cdot 10^{-8}$ s, respectively. The local truncation error, which is used to determine the actual time step, was set to $1 \cdot 10^{-4}$ for all state variables.

Comparing the responses, it is again seen that differences between the NAM and OM are unobservably small. A list of the maximum errors of the two approximate models, relative to the original model, is shown in Table 3. The maximum errors between the NAM and OM are less than 0.6% for all system variables. In contrast, the TAM response is significantly less accurate. Errors for the TAM system variables are greater than 25%, which correspond to results published in [2]. Close inspection of the responses also reveals that the TAM does not accurately predict the switching dynamics in either the steady-state or transient response.

The solution of the NAM and TAM responses was accomplished by pre-computing the matrix $L_k$ at each change in topology. Thereby, the solution of (25) only required a matrix-vector multiply at each integration time-step. In contrast, because of the time-varying inductances of the OM model, a linear solve routine was required to form the OM state model at each integration time-step. The CPU times required to calculate the 0.2 s studies on a 133-MHz Pentium PC, are given in Table 4. Therein, it is seen that the time-invariant form of the PVBR is significantly more efficient than the time-varying form. The NAM is on the order of 48% faster, while the TAM is on the order of 58% faster.


VII. ACKNOWLEDGEMENTS

This work was supported in part by University of South Carolina under Grant No. N00014-96-1-0926 with the Office of Naval Research and in part by the Missouri Research Board under Grant No. R3-42420.

IX. REFERENCES


[3] IEEF Joint Working Group on Determination and Applica-