AAS 15-127

THE SMALL-BODY DYNAMICS TOOLKIT AND ASSOCIATED CLOSE-PROXIMITY NAVIGATION ANALYSIS TOOLS AT JPL

Stephen B. Broschart,* Matthew Abrahamson,* Shyam Bhaskaran,* Eugene G. Fahnestock,* Reza R. Karimi,* Gregory Lantoine,* Thomas A. Pavlak,* and Loic Chappaz†

Over the past several years, an ecosystem of MATLAB©-based tools has been developing at NASA's Jet Propulsion Laboratory (JPL) for early-mission analysis of encounter-phase navigation at primitive bodies. These tools increasingly draw from a common implementation of capabilities known as the Small-Body Dynamics Toolkit (SBDT). Fundamentally, the SBDT provides support for trajectory integration and geometric analysis of the environment, providing force models (gravity, solar pressure, comet outgassing), equations of motion, polyhedron shape utilities, altitude calculations, occultation checking, and more. Using the capabilities of the SBDT, analysis tools for mapping performance analysis (PB-CAGE), navigation performance analysis (AutoNAV), and trajectory design space characterization (SBMCT) have been developed. This paper provides a brief overview of the SBDT and these associated analysis tools.

INTRODUCTION

The environment near primitive bodies has several characteristics that present unique challenges when developing a mission design and navigation strategy for an extended spacecraft encounter. The highly-irregular shape and gravity of these bodies, the small relative magnitude of the gravitational acceleration, the lack of accurate *a priori* knowledge of key dynamic parameters, and the potential for an outgassing acceleration from the body are all challenges that require extensions to the analysis software used to support planetary missions.

The Mission Design and Navigation section at NASA's Jet Propulsion Laboratory (JPL) has participated in development and execution of deep-space missions to every planet in the solar system as well as several primitive-body rendezvous missions (including NEAR[1], Dawn[2], and Rosetta[3]). Nonetheless, the wide range of target sizes and paradigms that are considered for primitive-body missions continues to expose significant gaps in JPL's institutional analysis software related to prolonged encounter design. Further, the best algorithms for addressing some of these capability gaps have not been determined yet.

*Mission Design and Navigation Section, Jet Propulsion Laboratory, California Institute of Technology, M/S: 301-121, 4800 Oak Grove Dr., Pasadena, CA 91109

†Ph.D Candidate, School of Aeronautics and Astronautics, Purdue University, 701 W Stadium Ave., West Lafayette, IN 47906

In response, a suite of engineering tools have emerged at JPL to help design primitive-body encounters for proposals and to facilitate research of the needed trajectory and navigation strategies. To facilitate rapid capability development by engineers, a common library called the Small-Body Dynamics Toolkit (SBDT) has been created in MATLAB©. This paper gives an overview of the SBDT and describes several of the more mature tools that use the SBDT library. These tools include the Primitive-Body Coverage and Geometry Evaluator (PB-CAGE), AutoNAV, and the Small-Body Mission Characterization Tool (SBMCT). The idea is that these tools provide a nimble testbed where engineers can implement new capabilities very quickly to address their analysis needs. Ultimately, as the most useful algorithms are distilled, they will be implemented into the institutional software suite.

SMALL-BODY DYNAMICS TOOLKIT (SBDT)

The Small-Body Dynamics Toolkit (SBDT) is a collection of primitive-body-specific trajectory design and analysis tools written in MATLAB©. The SBDT gives the user the capabilities to propagate, analyze, and visualize spacecraft trajectories and the dynamical environment near realistic asteroid, comet, or small planetary-moon models. The various components can be put together through user-created functions or scripts to perform a wide-variety of trajectory analyses, including close-proximity encounter design, Monte-Carlo studies, stability analysis, geometric analyses, and fuel budget calculations.

The latest release of the SBDT, version 5.0[4], supports three types of gravity models (pointmass, constant density, or spherical harmonic), three types of primitive-body shapes (sphere, ellipsoid, or arbitrary polyhedron), a solar radiation pressure (SRP) model, and several comet outgassing models. The dynamic models available in version 5.0 include the two-body, Hill, circular-restricted three-body, elliptic-restricted three-body, and four-body problems (for modeling binary asteroid systems), all of which allow for irregular gravity and SRP acceleration. One of the most significant aspects of the SBDT design is that all of the functions work equally well with any of the shape, gravity, or force models described above. A set of data format standards have been applied across the toolkit such that all SBDT functions execute so long as the data structure is correctly populated.

History of the SBDT

The SBDT began at the University of Michigan as a collection of functionalities created to support a Ph.D. research project[5]. Development of version 1.0 continued from 2001 through 2006. From 2007 through 2009, the SBDT was informally developed further in support of various research projects and proposals at JPL, culminating in version 2.0 which supported development of Discovery program proposals in 2010. Additional developments through the summer of 2013, largely funded by NASA's Near-Earth Observations Program through the Primitive Body Navigation (PBN) task were informally referred to as version 3.0. Further focused development under the PBN task has resulted in significant advances; versions 4.0 (Sep. 2013), 4.1 (May 2014), and 5.0 (Sep. 2014). Requirements have been defined for Version 6, which is expected to be completed in March 2015.

Applications of the SBDT

The SBDT has been used in a wide variety of applications over its fourteen-year history. Figure 1 illustrates a few examples. Early use focused on analyzing and designing control laws for hovering spacecraft at asteroids[6, 7]. The capabilities were also frequently used early on to help characterize the dynamical environment near asteroids whose shape had been observed by Earth-based radar (e.g., [8, 9]). The SBDT was used to support an early study of the effectiveness of the gravity tractor for planetary defense in 2009[10] and more recently to support several studies of autonomous spacecraft navigation strategies. The SBDT has also been used to design and characterize new types of orbits[11] and for orbit stability studies[12].

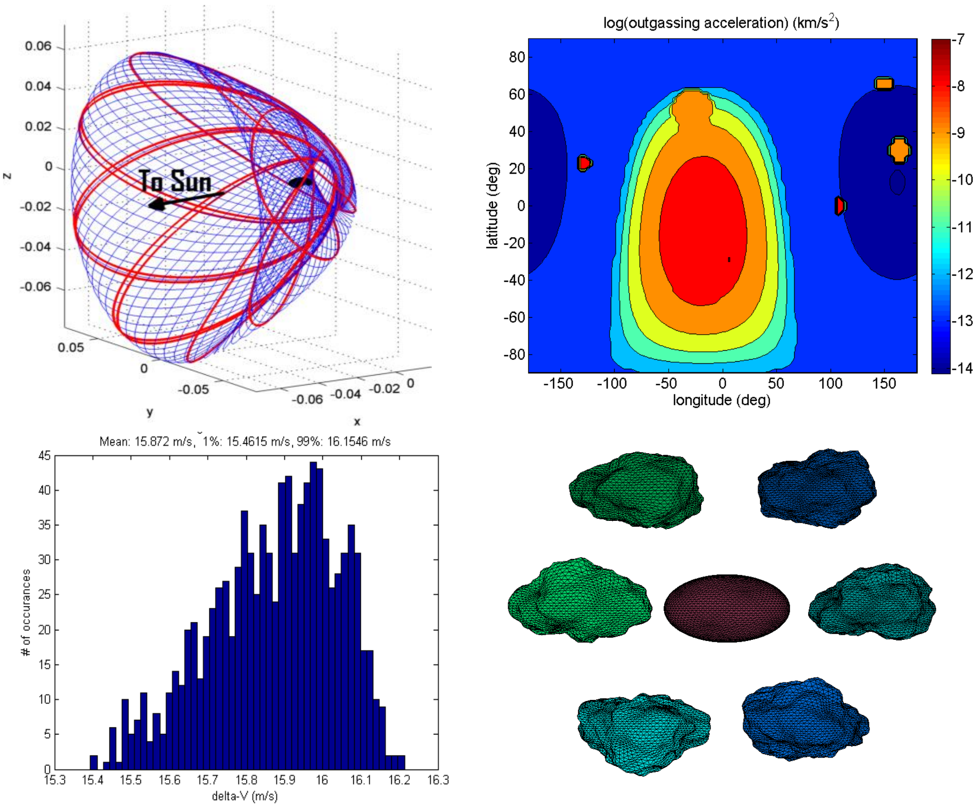


Figure 1. Examples of SBDT analyses: (top left) Periodic orbit calculation (a quasi-terminator orbit is shown); (top right) Accelerations resulting from comet outgassing as a function of latitude and longitude; (bottom left) Statistical delta-V analysis for an encounter campaign; (bottom right) Six-randomized shape models generated based on the ellipsoid at center.

The most extensive use of the SBDT to date has been in encounter design for various mission proposals. SBDT has supported roughly 8 Discovery and New Frontiers proposals at varying levels. Typical analyses provided include Monte-Carlo characterization of the potential range of body parameters, identification of regimes where orbiting is possible, development of end-to-end encounter timelines and trajectories, and creation of statistical delta-V budgets.

Shape Models

SBDT has a standardized shape model definition that can account for three different body shapes: spherical, ellipsoidal, and polyhedron. For all three body types, a set of initialization functions exist that create a MATLAB© structure variable populated with all the fields in Table 1. For spherical and ellipsoid shapes, a discretization algorithm is run to create an equivalent polyhedron shape.

The facets of this polyhedron can then be efficiently used for evaluating properties as a function of surface location, such as instrument coverage (as in the PB-CAGE tool) or surface escape speed (as in the SBMCT).

Several utility functions are also included to help the user manipulate the complex shapes associated with primitive bodies. There is a capability to create randomized fractal deformations of a given shape that is useful in Monte-Carlo studies and situations where the exact shape of the body is not known. There is also a utility for converting polyhedron shape specifications between VRML 1.0, PLY, OBJ/Wavefront, SPICE DSK, and face/vertex text file formats. There is also a line-of-sight range function that allows for computing altitude, occultations, or self-obstruction for any of the shape model types.

Acceleration Models

The SBDT provides a variety of acceleration models that are significant when integrating dynamics in the primitive-body environment. These include models for gravity potential (which must account for significant irregularity), solar radiation pressure (SRP) acceleration, and comet outgassing. It has been well-documented that any of these forces may be the most dominant acceleration on the spacecraft since the mass of the primitive body is often so small.

Gravity Models The SBDT supports three different gravity model definitions; pointmass, constant density, and spherical harmonic. Like with the shape model, a MATLAB© structure is populated by the initialization code for each different gravity model. The contents of this ***gravity*** structure are described in Table 2. Notably, an equivalent spherical harmonic model is created for all gravity types[13]. An SBDT function called ***potential*** can evaluate the gravitational potential, acceleration, Hessian, and Laplacian[14, 15] given any valid gravity model structure.

Solar Pressure For smaller asteroids and comets, SRP is often has the strongest influence of any on the spacecraft motion relative to the primitive body. SRP acceleration depends on the distance from the Sun and the geometry, orientation, and material properties of the spacecraft. Accordingly, acceleration models can be quite complex for high-fidelity modeling. Since the SBDT is meant for preliminary analysis, a simpler approach is used where the spacecraft geometry is represented by a flat-plate with defined reflectance properties that always points toward the Sun. As such, the ***solarPressure*** function computes the acceleration on the spacecraft as in Eq. (1),

$$\ddot{\tilde{\mathbf{r}}}_{SRP} = \frac{G1}{m_{s/c}/A_{s/c}} \frac{1 + 2\nu_{diff} + 2\nu_{spec}}{||\tilde{\mathbf{r}}_{s/c,Sun}||^3} \tilde{\mathbf{r}}_{s/c,Sun}, \tag{1}$$

where $G1$ is the solar flux constant ($\approx 1e14$ (kg/km^2)*(km^3/s^2)), $m_{s/c}$ and $A_{s/c}$ are the spacecraft mass and area, respectively, ν_{diff} and ν_{spec} are the diffuse and specular reflectance coefficients of the spacecraft, respectively, and $\tilde{\mathbf{r}}_{s/c,Sun}$ is the spacecraft position relative to the Sun.

Comet Outgassing Several comet outgassing models have recently been added to the SBDT. The tool supports the format provided by the COMA tool[16], which allows for arbitrarily complex gas flow fields, the axially-symmetric format of the ICES tool[17], a thermal-equilibrium model, and a simple "flashlight" outgassing jet model[18]. The parameters needed for each model are specified in a MATLAB© structure called ***outgas***. The details of the outgassing acceleration models are omitted here due to their complexity.

Table 1. Contents of the SBDT V5.0 *shape* Data Structure (M = # of vertices, N = # of faces, and $P = M+N-2$ = # of edges)

Field	Description
$.centerOfMass$	3×1, center-of-mass of the shape in km
$.circumscriber$	1×3, radii of a tri-axial ellipsoid that completely contains the body in km
$.dee$	1×3, dynamically-equivalent ellipsoid dimensions in km
$.edges$	$P \times 6$ matrix describing the edges of a polyhedron shape
$.ellipA$	Ellipsoid radius along longest axis in km
$.ellipB$	Ellipsoid radius along intermediate axis in km
$.ellipC$	Ellipsoid radius along shortest axis in km
$.extent$	1×6 matrix containing the maximum and minimum surface coordinate in the X, Y, and Z directions in km
$.faceCenters$	$N \times 3$ matrix containing the mean of the three constituent vertices for each polyhedron facet
$.faceMax$	$N \times 3$ matrix containing maximum for each coordinate for each face in km
$.faceMin$	$N \times 3$ matrix containing minimum for each coordinate for each face in km
$.faceNormals$	$N \times 3$ matrix containing the normal unit vector for each polyhedron facet
$.faces$	$N \times 3$ matrix containing the equivalent polyhedron face definition
$.facetArea$	$N \times 1$ containing the area of each polyhedron facet
$.gravCompE$	$P \times 3 \times 3$, precomputed quantities for gravity calculations
$.gravCompF$	$N \times 3 \times 3$, precomputed quantities for gravity calculations
$.gravCompJ$	$N \times 3 \times 3$, precomputed quantities for gravity calculations
$.inertiaTensor$	3×3, inertia tensor in the body-fixed frame in km^2
$.maxRadius$	maximum extent of the body in any direction in km
$.minRadius$	minimum extent of the body in any direction in km
$.meanRadius$	radius corresponding to a sphere of equal volume in km
$.moments$	1×3, moments of inertia of the shape in km^2
$.oblateCircumscriber$	1×3, radii of a tri-axial ellipsoid that completely contains the body throughout its rotation in km
$.shapeType$	integer representation of the gravity model type (1: Sphere, 2: Ellipsoid, or 3: Polyhedron)
$.surfaceArea$	surface area of the shape in km^2
$.typeText$	text description of the shape model type (Sphere, Ellipsoid, or Polyhedron)
$.vertices$	$M \times 3$ matrix containing the location of the polyhedron vertices in km
$.volume$	volume of the shape in km^3

Table 2. Contents of the SBDT V5.0 *gravity* Data Structure (n = degree and order of the spherical harmonic expansion)

Field	Description
$.Cnm$	$(n+1) \times (n+1)$ matrix containing the $C_{0,0}$ through $C_{n,n}$ spherical harmonic expansion coefficients
$.c22$	Normalized $C_{2,2}$ of the spherical harmonic gravity expansion
$.density$	bulk density of the body in kg/km^3
$.gm$	gravitational parameter of the body in km^3/s^2
$.gravType$	integer representation of the gravity model type (1: Pointmass, 2: Constant Density, or 3: Spherical Harmonic)
$.j2$	Normalized J_2 of the spherical harmonic gravity expansion (=$-C_{2,0}$)
$.mass$	mass of the body in kg
$.normalized$	1 if harmonic expansion is normalized, 0 if not
$.Snm$	$(n+1) \times (n+1)$ matrix containing the $S_{0,0}$ through $S_{n,n}$ spherical harmonic expansion coefficients
$.sphereOfInfluence$	Pointmass gravity model is used outside of this radius, km
$.sphHarmOrder$	Degree and order (n) of the spherical harmonic expansion used.
$.sphHarmRefRadius$	reference radius for spherical harmonic expansion in km
$.typeText$	text description of the gravity model type (Pointmass, Constant Density, or Spherical Harmonic)

Table 3. Contents of the SBDT V5.0 *bodyFrame* Data Structure

Field	Description
$.pm.period$	sidereal rotation period in hours
$.pm.wdot$	rotation rate of the body frame, deg/day
$.pm.w0$	orientation of the prime meridian at J2000, deg
$.pole.decdot$	linear rate-of-change in declination of the pole, deg/century
$.pole.dec0$	declination of the pole at J2000, deg
$.pole.radot$	linear rate-of-change in right ascension of the pole, deg/century
$.pole.ra0$	right ascension of the pole at J2000, deg
$.refFrame$	inertial reference frame for body frame definition

System Geometry

The geometry of the spacecraft with respect to the primitive body and the rest of the solar system must be known to SBDT to compute the above accelerations. The models used to orient the spacecraft in the solar system are discussed here briefly.

Orbit Models As of version 5.0, the SBDT supports only conic orbit definitions for the primitive body around its central body (i.e., the Sun or a planet).

Rotation Models and Coordinate Frames The body-fixed rotating frame associated with a given primitive body is defined using the IAU/IAG conventions for primitive bodies[19]. The parameters are defined in another MATLAB© structure called ***bodyFrame*** (Table 3). As of version 5.0, most SBDT functions only support a uniform rotation model, though some capabilities support linear variation in the pole orientation over time. Non-principal axis rotation support (which applies to many asteroids and comets[20]) is planned for a future release.

Dynamic Models

The SBDT provides integration routines that use the standard model definitions given above to integrate spacecraft trajectories in several classic dynamical models. The SBDT is primarily aimed toward early mission analysis and research, so the provided models are of a more analytical flavor instead of geared toward highly-accurate trajectory integration. The provided equations of motion include the two-body, circular-restricted three-body, Hill three-body, elliptic-restricted three-body, circular-restricted four-body, and elliptic-restricted four-body problems. It should be noted however that the modular nature of the SBDT allows the user to pick-and-choose dynamical models to include in their own integration routine, as has been done in the AutoNAV and C-PROX (a 6-DOF GN&C simulation) tools.

Other Capabilities

Besides the core capabilities of SBDT described above, the SBDT version 5.0 offers a number of other capabilities that can be helpful when performing mission and navigation analysis for a primitive-body mission. In the interest of brevity, these will only be mentioned here. Additional capabilities not described above include: plotting routines, functions to compute Jacobi constant and other conserved quantities, utilities for converting between calendar dates and seconds past J2000, functions to convert states between Cartesian coordinates and conic orbit elements, utilities to strip extraneous data from polyhedron shapes, and a function to rotate states between coordinate frames.

PRIMITIVE BODY COVERAGE AND GEOMETRY EVALUATOR (PB-CAGE)

Global coverage of the surface with one or more instruments is a typical science objective for primitive-body missions. The Primitive-Body Coverage and Geometry Evaluator (PB-CAGE) provides a capability for mission design engineers to compute surface coverage statistics as part of the trajectory design process[21]. This allows for close-proximity trajectory design strategies to be assessed against coverage-related science objectives during the design process, which increases the efficiency of the design cycle both during development and operations.

For planetary orbiters, ground "swath" calculations are important for selecting an appropriate orbit geometry; PB-CAGE provides a similar, but generalized capability that is appropriate for the irregular surface geometries and highly-perturbed spacecraft dynamics associated with primitive-body missions. The core functionality of PB-CAGE is to compute which surface elements of a triangular-faceted polyhedron lie within a circular or rectangular field-of-view, for a given spacecraft location, pointing direction, and instrument properties. The instrument properties can be chosen to simulate optical or thermal infrared mapping, laser or LiDAR altimetry, or radar investigations. The rest of the inputs can be generated through use of the SBDT. Using these inputs, PB-CAGE executes its core functionality along a given trajectory to accumulate appropriate coverage related statistics as a function of time. PB-CAGE accounts for self-shadowing and/or self-obstruction issues that can be significant for the irregular shapes typical of primitive-bodies.

Example PB-CAGE results appear in Figure 2. In this case, coverage of a model of the asteroid Itokawa[22] from a spacecraft in a quasi-terminator orbit has been assessed. The left image shows the trajectory and the footprints from a sequence of images on the surface. The right image shows the accumulated variety of solar incidence and emission angles achieved during the orbit (as described in [23]).

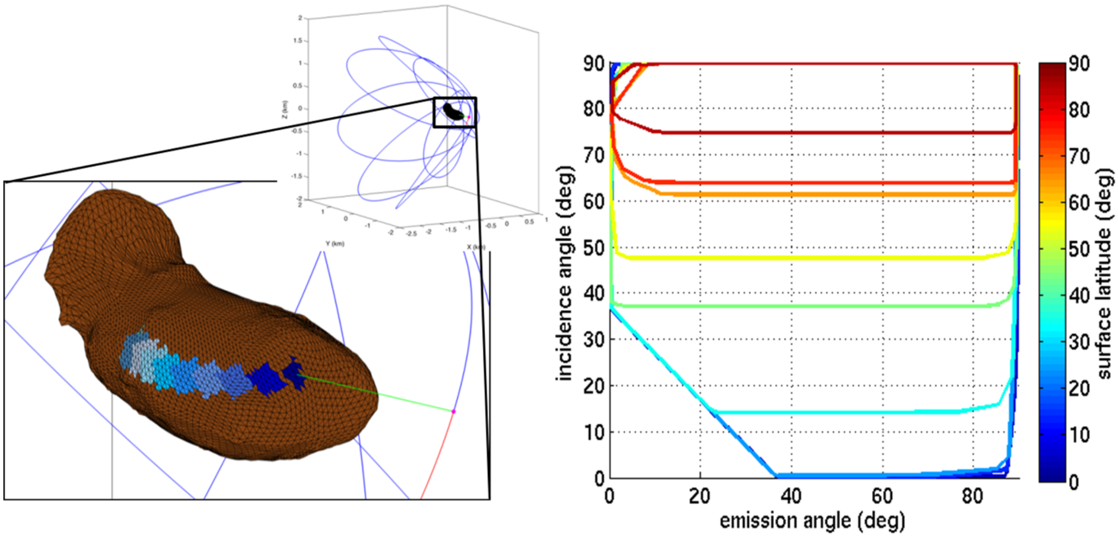


Figure 2. Example PB-CAGE analysis: (left) Quasi-terminator spacecraft orbit with a few instrument footprints indicated in shades of blue; (right) The range of solar incidence and emission angles achieved as a function of latitude.

PB-CAGE is implemented in MATLAB© and is currently on version 1.3, which requires an installation of SBDT v5.0 or later.

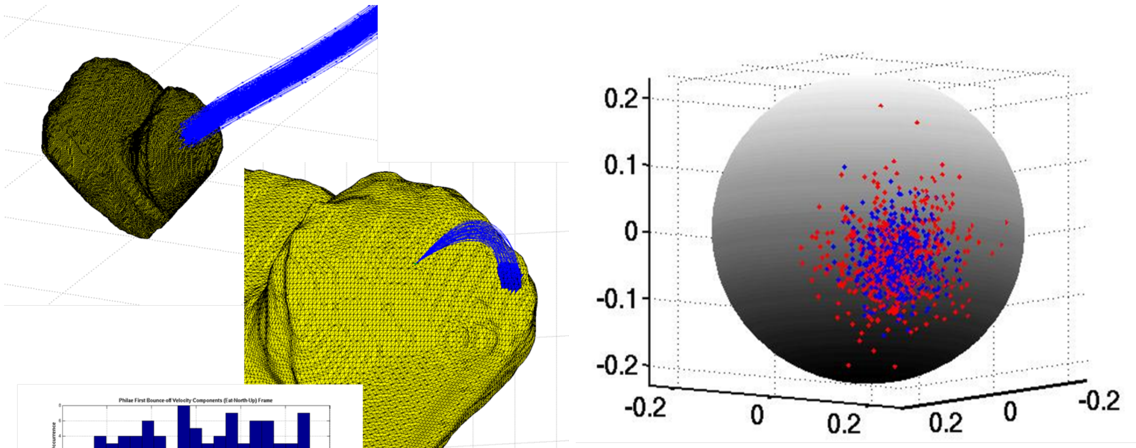
AUTONAV

The current version of the AutoNAV software (v3.0) is a Monte-Carlo navigation simulation tool based in MATLAB©*. AutoNAV allows for a full dynamic simulation of the navigation problem for each simulation instance, integrating a truth model for the trajectory (separate from the one used in the filter), generating realistic measurement observables, then running a least-squares orbit determination process to get an estimated spacecraft state.

*The ground simulation software discussed here should not be confused with the on-board flight software of the same name.

AutoNAV allows for a user to perform relatively quick assessments of expected navigation performance for a given concept while accounting for the significant non-linearities in the dynamics and non-Gaussian error distributions (through the Monte-Carlo approach). This is often critical to getting accurate results in the primitive-body environment. AutoNAV runs a simple robust filter that ensures quick and reliable convergence (to support fast and automated Monte-Carlo analysis). It also relies only on *in-situ* measurement types (simulated landmark tracking and LiDAR range measurements), which provide the majority of the relative navigation information during encounters. When integrated with other SBDT tools, AutoNAV can provide estimates of orbit determination accuracy, delivery performance, statistical delta-V budgets, and image overlap requirements for a given encounter strategy relatively quickly.

As with the other tools, a key characteristic of AutoNAV is the flexibility the engineering user has to introduce new functionality. This is demonstrated by the breadth of applications the tool has supported to date. AutoNAV was originally designed as the basis for the Deep Space on-board Autonomous Navigation system[24], then used for ground simulations of the Stardust and Deep Impact on-board AutoNav systems[25, 26, 27]. The software was then modified to evaluate the performance of on-board and ground navigation strategies in the early stages of various primitive-body mission concept developments, including autonomous descent to the surface of a comet[28, 29] and performance analysis of a kinetic impactor concept[30]. Figure 3 shows some representative analysis from (left) a reconstruction of the descent and bounce of Rosetta's Philae lander and (right) a kinetic impactor study done to support the Impactor for Surface and Interior Science (ISIS) mission concept.



	Mean (m/s)	99% (m/s)
TCM 1	1.8 / 1.8	4.7 / 4.7
TCM 2	1.4 / 1.3	3.6 / 3.2
TCM 3	8.3 / 4.7	21.2 / 12.0
Total	11.5 / 7.8	25.5 / 16.0

Figure 3. Example AutoNAV analyses: (left) A Monte-Carlo "tube" of statistical descents for Philae, a set of potential bounce trajectories, and the resulting histogram of the East-North-Up components of the bounce velocity; (right) A set of impact points for a kinetic impactor simulation and the corresponding statistical delta-V budget.

SMALL-BODY MISSION CHARACTERIZATION TOOL (SBMCT)

The Small-Body Mission Characterization Tool is a monolithic code that generates a report on the dynamical environment for a spacecraft around a given small body. Based on a set of inputs (i.e., body model, spacecraft model, and mission parameters), it computes equilibrium points, boundaries between dynamical regions where different forces dominate, semi-major axis ranges for stable orbits, fuel costs for hovering, and a variety of surface properties (e.g., slopes, escape speeds, etc.). The report generated by the SBMCT serves as a useful brief that can be used by a mission designer to quickly understand the trajectory possibilities around a particular body. This report can also be used as a quick reference during mission planning meetings. Figure 4 shows an example of the types of analyses that can be done with SBMCT.

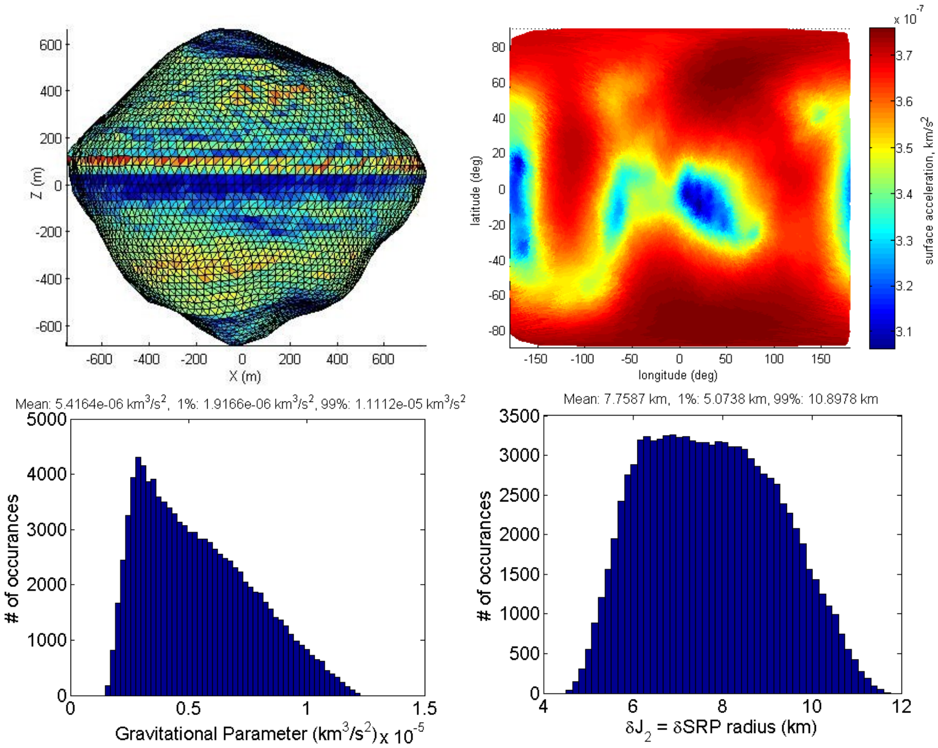


Figure 4. Example SBMCT analyses: (top left) Color indicates surface slopes on the primary of binary asteroid KW4; (top right) Surface acceleration plot as a function of surface location; (bottom left) Expected range of distribution in the gravitation parameter; (bottom right) Variation in the orbit size where the J_2 perturbation equals the SRP perturbation.

The original code that would evolve into the current SBMCT (v1.4) was written in C++ at the University of Michigan in collaboration with JPL in 2004 and 2005. Additional development was done on the tool in 2007 at JPL using internal funding and the current name was adopted. No further development was done until 2014, when the capabilities of the tool were implemented in MATLAB© using the SBDT capabilities under funding from the PBN task.

CONCLUSIONS

There is an ecosystem of engineering-driven analysis tools that has been developing at JPL over the last several years in response to the unique challenges associated with encounter-phase mission design at primitive bodies. This collection of tools is being built around a common capability core called the Small-Body Dynamics Toolkit, which provides frequently-needed capabilities like gravity calculation, geometric analysis of complex shapes, and trajectory integration. The existence of this core library provides for rapid development of tools for more complex analyses that inherit the reliability of the core algorithms.

Several tools that make use of SBDT have been described here, including: the Primitive-Body Coverage and Geometry Evaluator (a coverage analysis tool also known as PB-CAGE), AutoNAV (a Monte-Carlo navigation simulation tool), and the Small-Body Mission Characterization Tool (a tool also known as SBMCT that generates a summary report about the dynamical environment near a primitive body). Between these a wide range of different mission paradigms have been supported over the years.

This ensemble of tools has given JPL the ability to provide uniquely-advanced proposal support in the area of close-proximity mission design and navigation over the past two Discovery program announcements of opportunity. The fact that these tools are in MATLAB© and programmed by engineers gives these tools the flexibility to be very responsive to additional new paradigms arising in this evolving area for mission concepts.

ACKNOWLEDGMENTS

The work described in this paper was carried out at the Jet Propulsion Laboratory, California Institute of Technology, under a contract with the National Aeronautics and Space Administration.

REFERENCES

[1] J. K. Miller, A. S. Konopliv, P. G. Antreasian, J. J. Bordi, S. Chesley, C. E. Helfrich, W. M. Owen, T. C. Wang, B. G. Williams, and D. K. Yeomans, "Determination of Shape, Gravity, and Rotational State of Asteroid 433 Eros," *Icarus*, Vol. 155, No. 1, 2002, pp. 3–17.

[2] D. Han, "Orbit transfers for Dawns Vesta operations: navigation and mission design experience," *23rd International Symposium on Space Flight Dynamics, Pasadena, California, October 29-November 2, 2012*, 2012.

[3] S. Bhaskaran, S. Broschart, D. Han, B. Owen, N. Mastrodemos, I. Roundhill, B. Rush, and D. Surovik, "Rosetta Navigation at Comet Churyumov-Gerasimenko," presented at the AAS Guidance, Navigation, and Control Conference, Breckenridge, CO, Jan. 30-Feb. 5, 2015. AAS Paper 15-122, 2015.

[4] S. Broschart, G. Lantoine, T. Pavlak, E. Fahnestock, and L. Chappaz, "Small-Body Mission Dynamics Toolkit, version 5.0," New Technology Report (NTR) 49707, California Institute of Technology (Jet Propulsion Laboratory), 2014.

[5] S. B. Broschart, *Close Proximity Spacecraft Maneuvers near Irregularly Shaped Small-bodies; Hovering, Translation, and Descent*. PhD thesis, University of Michigan, 2006.

[6] S. B. Broschart and D. J. Scheeres, "Control of Hovering Spacecraft near Small-Bodies: Application to Asteroid 25143 Itokawa," *Journal of Guidance, Control, and Dynamics*, Vol. 28, No. 2, 2005, pp. 343–354.

[7] S. B. Broschart and D. J. Scheeres, "Boundedness of Spacecraft Hovering Under Dead-Band Control in Time-Invariant Systems," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 2, 2007, pp. 601–610.

[8] D. Scheeres, S. Broschart, S. Ostro, and L. Benner, "The Dynamical Environment About Asteroid 25143 Itokawa, Target of the Hayabusa Mission," presented at the AIAA/AAS Astrodynamics Specialist Conference and Exhibit, Providence, Rhode Island, Aug. 16-19, AIAA-2004-4864, 2004.

[9] S. J. Ostro, J.-L. Margot, L. A. Benner, J. D. Giorgini, D. J. Scheeres, E. G. Fahnestock, S. B. Broschart, J. Bellerose, M. C. Nolan, C. Magri, *et al.*, "Radar imaging of binary near-Earth asteroid (66391) 1999 KW4," *Science*, Vol. 314, No. 5803, 2006, pp. 1276–1280.

[10] D. Yeomans, S. Bhaskaran, S. Broschart, S. Chesley, P. Chodas, T. Sweetser, and R. Schweickart, "Deflecting a Hazardous Near-Earth Object," presented at the 1st IAA Planetary Defense Conference: Protecting Earth from Asteroids, Granada, Spain, 2009.

[11] S. B. Broschart, G. Lantoine, and D. J. Grebow, "Quasi-terminator orbits near primitive bodies," *Celestial Mechanics and Dynamical Astronomy*, Vol. 120, No. 2, 2014, pp. 195–215.

[12] L. Chappaz, S. B. Broschart, G. Lantoine, and K. Howell, "Orbital Pertubation Analysis near Binary Asteroid Systems," presented at the AAS/AIAA Spaceflight Mechanics Conference, Williamsburg, VA. AAS Paper 15-330., 2015.

[13] R. A. Werner, "Spherical Harmonic Coefficients for the Potential of a Constant-density Polyhedron," *Computers and Geosciences*, Vol. 23, No. 10, 1997, pp. 1071–1077.

[14] D. J. Scheeres, "Dynamics About Uniformly Rotating Tri-Axial Ellipsoids. Applications to Asteroids," *Icarus*, Vol. 110, No. 2, 1994, pp. 225–238.

[15] R. A. Werner and D. J. Scheeres, "Exterior Gravitation of a Polyhedron Derived and Compared with Harmonic and Mascon Gravitation Representations of Asteroid 4769 Castalia," *Celestial Mechanics and Dynamical Astronomy*, Vol. 65, No. 3, 1996, pp. 313–344.

[16] M. Mller and E. Grn, "An Engineering Model of the Dust Environment of the Inner Coma of Comet P Wirtanen, Part 2.," Tech. Rep. RO-ESC-TA-5501, European Space Agency, 1999.

[17] "ICES - Home," Website, Jan. 2015. http://ices.engin.umich.edu/.

[18] S. M. Byram, D. J. Scheeres, and M. R. Combi, "Models for the comet dynamical environment," *Journal of Guidance, Control, and Dynamics*, Vol. 30, No. 5, 2007, pp. 1445–1454.

[19] P. K. Seidelmann, B. Archinal, M. Ahearn, A. Conrad, G. Consolmagno, D. Hestroffer, J. Hilton, G. Krasinsky, G. Neumann, J. Oberst, *et al.*, "Report of the IAU/IAG Working Group on cartographic coordinates and rotational elements: 2006," *Celestial Mechanics and Dynamical Astronomy*, Vol. 98, No. 3, 2007, pp. 155–180.

[20] P. Pravec, A. Harris, P. Scheirich, P. Kušnirák, L. Šarounová, C. Hergenrother, S. Mottola, M. Hicks, G. Masi, Y. N. Krugly, *et al.*, "Tumbling asteroids," *Icarus*, Vol. 173, No. 1, 2005, pp. 108–131.

[21] T. Pavlak, E. Fahnestock, S. Broschart, R. Park, and L. Cangahuala, "Primitive-Body Coverage and Geometry Evaluator (PB-CAGE), v1.0," New Technology Report (NTR) 49366, California Institute of Technology (Jet Propulsion Laboratory), 2013.

[22] R. Gaskell, J. Saito, M. Ishiguro, T. Kubota, T. Hashimoto, N. Hirata, S. Abe, O. Barnouin-Jha, and D. Scheeres, "Gaskell Itokawa Shape Model V1.0," 2008. HAY-A-AMICA-5-ITOKAWASHAPE-V1.0, NASA Planetary Data System.

[23] T. A. Pavlak, S. B. Broschart, and G. Lantoine, "Quantifying Mapping Orbit Performance in the Vicinity of Primitive Bodies," presented at the AAS/AIAA Spaceflight Mechanics Conference, Williamsburg, VA. AAS Paper 15-389., 2015.

[24] S. Bhaskaran, S. Desai, P. Dumont, B. Kennedy, G. Null, W. Owen Jr, J. Riedel, S. Synnott, and R. Werner, "Orbit determination performance evaluation of the deep space 1 autonomous navigation system," *Advances in the Astronautical Sciences: Spaceflight Mechanics 1998*, 1998.

[25] S. Bhaskaran, J. E. Riedel, and S. Synnott, "Autonomous nucleus tracking for comet/asteroid encounters: the stardust example," *Aerospace Conference*, Vol. 2, IEEE, 1998, pp. 353–365.

[26] N. Mastrodemos, D. G. Kubitschek, and S. P. Synnott, "Autonomous navigation for the deep impact mission encounter with comet tempel 1," *Space Science Reviews*, Vol. 117, No. 1-2, 2005, pp. 95–121.

[27] M. J. Abrahamson, B. A. Kennedy, and S. Bhaskaran, "Autonomous Navigation Performance During The Hartley 2 Comet Flyby," presented at the AIAA SpaceOps Conference, Stockholm, Sweden, 2012.

[28] S. Bhaskaran, S. Nandi, S. Broschart, M. Wallace, L. Cangahuala, and C. Olson, "Small Body Landings Using Autonomous Onboard Optical Navigation," *Journal of the Astronautical Sciences*, Vol. 58, No. 3, 2011, pp. 409–427.

[29] M. J. Abrahamson and S. Bhaskaran, "Lidar and Optical-Based Autonomous Navigation for Small Body Proximity Operations," presented at the AAS/AIAA Space Flight Mechanics Meeting, Santa Fe, NM. AAS 14-420., 2014.

[30] S. Bhaskaran and B. Kennedy, "Closed loop terminal guidance navigation for a kinetic impactor spacecraft," *Acta Astronautica*, Vol. 103, 2014, pp. 322–332.