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Stationkeeping of Lissajous Trajectories in the Earth-Moon System with Applications to ARTEMIS

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In the last few decades, several missions have successfully exploited trajectories near the Sun-Earth L_1 and L_2 libration points. Recently, the collinear libration points in the Earth-Moon system have emerged as locations with immediate application. Most libration point orbits, in any system, are inherently unstable and must be controlled. To this end, several stationkeeping strategies are considered for application to ARTEMIS. Two approaches are examined to investigate the stationkeeping problem in this regime and the specific options available for ARTEMIS given the mission and vehicle constraints. (1) A baseline orbit-targeting approach controls the vehicle to remain near a nominal trajectory; a related global optimum search method searches all possible maneuver angles to determine an optimal angle and magnitude; and (2) an orbit continuation method, with various formulations determines maneuver locations and minimizes costs. Initial results indicate that consistent stationkeeping costs can be achieved with both approaches and the costs are reasonable. These methods are then applied to Lissajous trajectories representing a baseline ARTEMIS libration orbit trajectory.

INTRODUCTION

In recent years, scientists and engineers have viewed orbits in the vicinity of the Earth-Moon collinear libration points as promising locations for scientific data collection and/or communications options. Since the late 1960s, numerous missions have successfully exploited trajectories in the vicinity of the Sun-Earth L_1 and L_2 libration points. While the Sun-Earth libration points are still of interest to future space missions, e.g., the James Webb Space Telescope, the collinear libration points in the Earth-Moon system have recently emerged as a location with immediate application. To date, no spacecraft trajectories have yet exploited the regions near the Earth-Moon libration points, but ARTEMIS (Acceleration Reconnection and Turbulence and Electrodynamics of the Moon's Interaction with the Sun) is scheduled to become the first such mission when the two ARTEMIS spacecraft enter the vicinity of the Earth-Moon L_1 and L_2 points in August and September 2010, respectively. To support a mission such as ARTEMIS, stationkeeping for orbit maintenance in libration point trajectories is required. Several Earth-Moon libration point orbit stationkeeping strategies are considered and two emerge that meet the requirements for this application. Cost comparisons in terms of impulsive delta-V (DV) requirements are presented between the different approaches. A traditional baseline orbit-targeting approach is considered. A global optimum search scheme was also examined for application. A balancing approach as part of an orbit continuation scheme is also investigated. Additionally, orbit maintenance costs are compared for select periodic and non-periodic orbits propagated in different dynamical models, that is, the Circular Restricted Three-Body (CR3B) as well as Moon-Earth-Sun ephemeris models using high-fidelity modeling that incorporates all perturbations. This preliminary investigation serves as a basis for the selection of processes for further development to use for operational support of the first Earth-Moon libration point mission, ARTEMIS.

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Background

Most useful libration point orbits near the collinear locations, including quasi-periodic Lissajous trajectories, are inherently unstable and must be controlled. A variety of stationkeeping strategies have previously been investigated, most notably for applications in the Sun-Earth system; fewer studies have considered trajectories near the Earth-Moon libration points.[1-8] For Earth-Moon applications, however, orbit maintenance is more challenging than in the Sun-Earth system, in part because of the shorter time scales, the larger orbital eccentricity of the secondary, and the fact that the Sun acts as a significant perturbing body both in terms of the gravitational force as well as solar radiation pressure. To accurately assess the impact of these significant differences, the analysis must be modeled as a true four-body problem. Besides these inherent issues associated with the Earth-Moon system, there are also aspects of this mission that are unique to ARTEMIS. Although a baseline trajectory is defined to design the mission, there is no true reference motion that is required. Since the two spacecraft were originally designed for a different mission — one in the vicinity of the Earth — and are already flying, fuel is now extremely limited. Thus, with the unique operational constraints, accomplishment of the maintenance goals with the minimum cost in terms of fuel is the highest priority.

There are several scenarios in this current analysis, all involve numerical integration and incorporate the third-body perturbations. First, two trajectory types are developed to represent an Earth-Moon libration point orbit. (1) Periodic halo orbits and non-periodic Lissajous trajectories are first integrated in a barycentric Earth-Moon rotating coordinate frame consistent with the circular restricted three-body problem (CR3B) for a desired number of revolutions. Higher-fidelity baseline orbits are then computed using operational numerical methods by discretizing the CR3BP solutions into a series of patch points and re-converging the solution in a Moon-centered Moon-Earth-Sun ephemeris model using multiple shooting. (2) The actual design orbit for one of the ARTEMIS spacecraft, specifically P1, also serves as the basis for some of the simulations. During this initial investigation, then, a number of notable stationkeeping methods are considered for application to any Earth-Moon libration point mission and specifically for the ARTEMIS mission with its specific operational constraints. It is noted that the ultimate goal is NOT a reference orbit; rather the focus is a method that minimizes fuel use, minimizes operations requirements in terms of the frequency of the maneuvers, and permits a navigation strategy to be set in place for support as well. This fact influences the strategies that are investigated and observations within this general framework are a key result of this preliminary investigation.

ARTEMIS Mission

The ARTEMIS mission is actually an extension to the Time History of Events and Macroscale Interactions during Substorms (THEMIS) mission. THEMIS encompasses five spacecraft in Earth orbit. The ARTEMIS mission involves moving the two spacecraft in the outer-most elliptical Earth orbits and, with lunar gravity assists, re-directing the spacecraft to both the L_1 and L_2 lunar libration point orbits via transfer trajectories that exploit the multi-body dynamical environment. The two spacecraft are denoted P1 and P2. Once the Earth-Moon libration point orbits are achieved and maintained for several months, both P1 and P2 will be inserted into elliptical lunar orbits. The current baseline is a two-year mission with departure maneuvers that began in June 2009, to target multiple lunar flybys in February 2010 that eventually place the spacecraft on the transfer trajectory. The P1 spacecraft will enter Earth-Moon Lissajous orbits in August 2010 and P2 will follow in September 2010. Artemis will provide comprehensive Earth-lunar environment analysis using particles and fields instruments. The post apoapsis raising transfer trajectory with two lunar flybys appears in Figure 2.

The Goddard Space Flight Center's Navigation and Mission Design Branch (NMDB), code 595, is currently supporting the ARTEMIS mission and will be the prime for Earth-Moon libration point orbit navigation, trajectory design, and maneuver planning and command information generation.[9] The ARTEMIS mission is a collaborative effort between NASA GSFC, the University of California at Berkeley (UCB), and the Jet Propulsion Laboratory (JPL). JPL provided the reference transfer trajectory from the elliptical orbit phase through libration orbit insertion. The University of California at Berkeley (UCB) provides daily monitoring and maintenance of all spacecraft operations and the generation of maneuver planning for uploads using GSFC software.

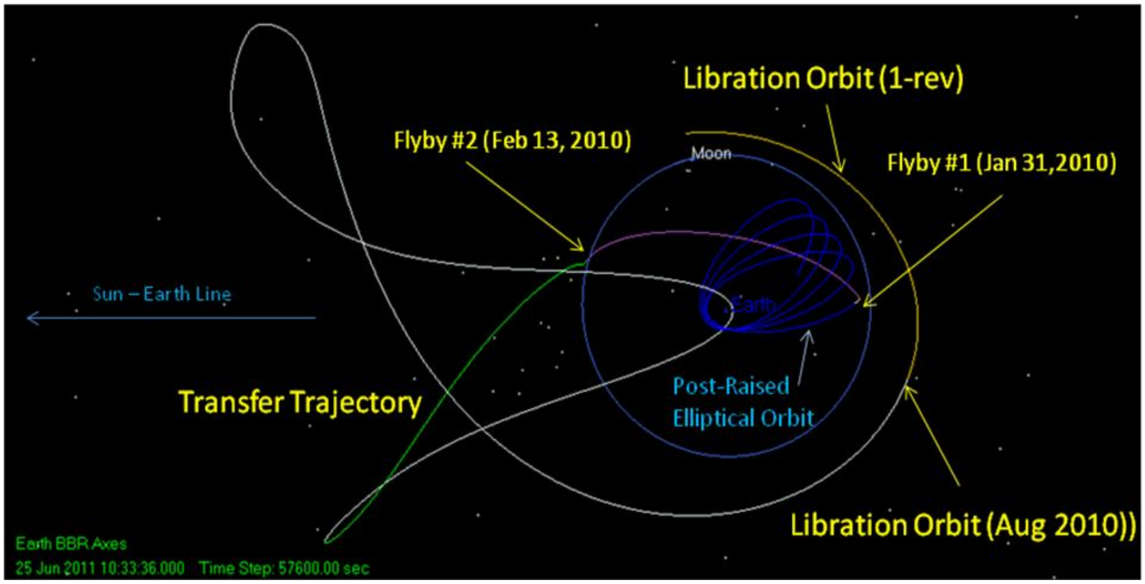

Figure 2: Complete ARTEMIS P1-Spacecraft Transfer from an Elliptical Earth Orbit to the Earth-Moon L2 Libration Point Orbit

The initial design of the P1 Earth-Moon L_2 orbit appears in Figure 3. There are no size or orientation requirements on these orbits other than to minimize the orbital maintenance requirements as both ARTEMIS spacecraft have limited libration orbit stationkeeping DV budgets of ~25m/s for deterministic and statistical maneuvers from the insertion to end-of-mission. This DV budget includes the final libration point orbit stationkeeping, the transfers between libration orbits, and the transfer into lunar orbit. The L_2 *y*-amplitude is approximately 60,000 km since the overall amplitudes are determined from the use of a ballistic transfer insertion. Consequently, at the end of the transfer, the final lunar libration point orbit is influenced heavily by the Moon since the transfer orbit passes relatively close to the Moon at each negative *x*-axis crossing with respect to the L_2 libration point.

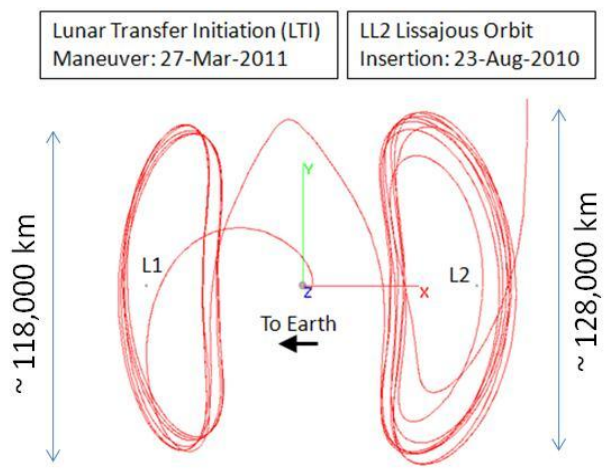

Figure 3. ARTEMIS P1 L_1 and L_2 orbits

ARTEMIS Initial Conditions and Modeling

A full ephemeris model (DE405 file) is developed incorporating gravity from the three point masses representing Earth, Moon, and Sun and a solar radiation pressure force based on a sample spacecraft area and mass. The simulations are based on a variable step Runge-Kutta 8/9 or PrinceDormand 8/9 integrator. The libration point locations are also calculated instantaneously at the same integration interval. The initial conditions correspond to the predicted incoming ballistic transfer trajectory for the baseline ARTEMIS mission as specified below in Earth Centered Cartesian (J2000) coordinates. The epoch corresponding to this state is only 3.5 days before the first Earth-Moon coordinate *x*-axis crossing.

- TT Mod Julian Epoch = '25431.500000'
- Coordinate System = Earth MJ2000Eq, Cartesian;

- $X = 352040.228712154$; $VX = 0.6796631890749434$;
- $Y = -318477.515825368$; $VY = 0.6758361947047898$;
- $Z = -131405.837508665$; $VZ = 0.2069195932200096$;
- DryMass = 87.474kg; Cd = 2.2; Cr = 1.17; SRPArea = 0.95m^2.

To compute maneuver requirements in terms of DV, different strategies involve various numerical methods: traditional Differential Correction (DC) targeting with central or forward differencing, optimization using the VF13AD algorithm from the Harwell library, as well as grid search algorithms. For the corrections scheme, equality constraints are incorporated, while for the optimization scheme, nonlinear equality and inequality constraints are employed.

ARTEMIS Spacecraft Overview

Each ARTEMIS spacecraft is spin-stabilized with a nominal spin rate of roughly 20 RPM. Spacecraft attitude and rate are determined using telemetry from a Sun sensor (SS), a three-axis magnetometer (TAM), and two single-axis inertial rate units (IRUs). The propulsion system on each spacecraft is a simple monopropellant hydrazine blow-down system. The propellant is stored in two equally-sized tanks and either tank can provide propellant to any of the thrusters through a series of latch valves. Each observatory was launched with a dry mass of 77 kg and 49 kg of propellant, supplying a wet mass of 126 kg at beginning of life.

Each spacecraft has four 4.4 Newton (N) thrusters – two axial thrusters and two tangential thrusters. The two tangential thrusters are mounted on one side of the spacecraft and the two axial thrusters are mounted on the lower deck, as seen in Figure 4. The thrusters fire singly or in pairs – in continuous or pulsed mode – to provide orbit, attitude, and spin rate control. Orbit maneuvers can be implemented by firing the axial thrusters in continuous mode, the tangential thrusters in pulsed mode, or a combination of the two (beta mode). Since there are no thrusters on the upper deck, the combined thrust vector is constrained to the lower hemisphere of the spacecraft.

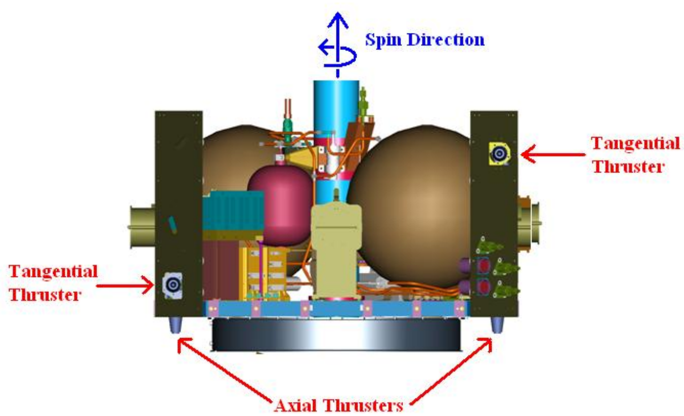

Figure 4. ARTEMIS Spacecraft Design

ARTEMIS Spacecraft Maneuvers Constraints

The ARTEMIS spacecraft are spinning vehicles with the spin axis pointed within 5 degrees of the south ecliptic pole. These spacecraft can implement a DV (thrust direction) along the spin axis towards the south ecliptic pole direction or in the spin plane, but cannot produce a DV in the northern hemisphere relative to the ecliptic. Thus, all maneuvers are currently planned using only the radial thrusters. While the axial can be used if necessary, the axial thrusters are not calibrated as well as the radial thrusters. This constraint limits the location of many maneuvers. For the lunar gravity assist targeting and in the manifold trajectory design, the trajectory was optimized incorporating a nonlinear constraint that placed the DV in the spin plane and the epoch corresponding to the maneuver is varied to yield a radial maneuver direction. This method will also be employed during the stationkeeping process.

STATIONKEEPING STRATEGIES

The stationkeeping strategies considered herein include available methods that satisfy the following self-imposed conditions: full ephemeris with high-fidelity models, globally optimized solutions, and methods that can be applied for any Earth-Moon orbital requirements at L_1 or L_2. Other strategies were investigated but many of the standard approaches cannot be employed for various reasons, e.g., because a reference orbit is required which is not necessarily available nor desired, the strategy is based on the CRTB model only, or because a proposed approach cannot accommodate the ARTEMIS spacecraft constraints.[2,3,7,9-11] Numerous references in the literature offer discussion of stability and control for vehicles at both collinear and triangular libration point locations. Hoffman[1] and Farquhar[10] both provide analysis and discussion of stability and control in the Earth-Moon collinear L_1 and L_2 locations, respectively, within the context of classical control theory or linear approximations while, more recently, Scheeres offers a statistical analysis approach.[11] Howell and Keeter[2] address the use of selected maneuvers to eliminate the unstable modes associated with a reference orbit; Gomez et al.[3] developed and applied the approach specifically to translunar libration point orbits. Marchand and Howell[12] discuss stability including the eigenstructures near the Sun-Earth locations. Folta et al.[13] present an analysis of stationkeeping options and transfers between the Earth-Moon locations and the use of numerical models that include discrete linear quadratic regulators and differential correctors.

From this wide variety of control analyses, two types are investigated in detail for this application; it is likely that they will eventually converge into a single strategy. First, following a basic control design process, a baseline orbit control-point targeting approach employs a differential corrections (DC) algorithm to maintain the vehicle near a nominal trajectory which is determined a priori. This initial step supplies insight into the investigation of a global optimum search method, a strategy that can be applied to any trajectory designed within a higher-fidelity environment. An alternative method that is promising for application to ARTEMIS is to balance the orbit by meeting goals several revolutions downstream, thereby ensuring a continuous orbit without near-term requirements or the reliance on specific orbit specifications. Additional strategies also exist as options and, for completeness, are expected to be examined for the ARTEMIS mission. While the list in Table 1 is not all-inclusive, it does reflect previously investigated methods, many of which have been successfully applied. The table includes insights and comments on the advantages and disadvantages relevant to ARTEMIS. For consideration in determining the applicability of any strategy, a unique feature of the ARTEMIS Lissajous orbit is the changing inclination. Over the roughly 4 months from insertion into the Lissajous orbit until the next phase, the Lissajous trajectory evolves from a highly inclined motion to one that is nearly planar. The impact of the inclination change on the stationkeeping is one aspect that is to be assessed. The table includes comments concerning the following strategies:

1) X-axis Crossing with X-axis Velocity Constraint

This method was first used in Sun-Earth libration point orbit stationkeeping strategies. While it yields a useful baseline method that places maneuvers at set locations and constraints that can be easily visualized, it may not be appropriate for Earth-Moon orbits for several reasons including: the fact that the DVs are not minimized, navigation or maneuver errors may result in a trajectory that does not return to the next x-axis crossing, and operations may limit where the maneuver can occur due to coverage issues as well as the fact that the impulse may not be achievable due to spacecraft constraints. The sensitivities are not well-defined with this approach and the recovery may lead to higher DVs.

2) Unstable Mode Cancellation

Cancellation of the unstable mode via a DV in the unstable direction can also be used for stationkeeping. This method is investigated for potential delivery of the minimal DV; however, the location of the maneuver might also be insensitive to small variations. The difficulty in the implementation is that the unstable mode is usually determined via Floquet modes from the state transition matrix information generated from a predetermined trajectory, or reference trajectory. For application to ARTEMIS, the trajectory information is not expected to be pre-generated to compute a minimal DV.

Table 1. Control Strategies and Selection Criteria Examined for Application to ARTEMIS.

	Strategy	Goal(s)	Advantage	Disadvantage	Selection Criteria
1	**Target x-axis crossing with x-axis velocity constraint** [1,5-8]	Velocity at crossing, parallel to *x*-axis, is zero	-Validated in Sun-Earth operations - Can use a DC or optimization process to target a single parameter	- Overly constraining - Can lead to increased DVs - Dynamics may not result in reaching subsequent *x*-axis crossings - May not meet operational constraints	- Operationally constraining and larger DV budget - Sensitivity issues in computing maneuvers to achieve targets can require high recovery DVs - Not selected for this application
2	**Unstable mode cancellation** [2,3,12,15]	Cancel unstable component of the error	- Simple design based on dynamical properties of the libration point orbit - Multiple algorithms available to apply DV	- Requires the use of STMs based on reference orbit or the EM libration point - Intensive and iterative calculations	- Reference orbit not available - Sensitivity to mode calculation due to lunar eccentricity + solar gravity - Not selected for this application
3	**Continuous Controllers** [13]	Converge onto a reference orbit	- Possibly less sensitive to navigation and execution errors - Maintains orbit within user- defined small torus	- Requires a reference orbit -Uses near continuous thrusting (may be discretized) -Requires computation of gain from an STM that is based on the libration point or actual orbit -Linear approximations for control feedback	- Requires a reference orbit - Does not apply to ARTEMIS spacecraft operationally (near continuous control) - Not selected for this application
4	**Baseline Orbit Control-Point Targeting** [2,3,4,12,14,16]	Target multiple points along orbit	- Rigorous method with guaranteed results - Based in ephemeris model	-Selection of control points results in the computation of a reference orbit - Possibly larger DVs	- Reference points computed as a first guess from an available baseline - Used only as first guess utility for ARTEMIS
5	**Boxed Environment** [1,2,3]	Define constraints in terms of distance from *x*-axis and *y*-axis	- Always converges if box size and targeting scheme combined properly	- Current implementation algorithms are limited - Logic required in s/w to check for trajectories that depart the system	- Used in this application only to identify and re-direct solutions that are not converging
6	**Orbit Continuation** [1,2,3]	Velocity (or energy) is determined to deliver s/c several revs downstream (e.g., *x*-axis velocities all slightly negative)	- Guarantees a minimal DV to achieve orbit continuation - Several control constraints (see 1 above) can be applied - 3-D application	- Needs accurate integration and full ephemeris modeling - Logic required in s/w to check for departure trajectories - Optimization requires monitoring of process	- Analyzed in this paper with intent to apply to ARTEMIS
7	**Global Optimum Search** [8]	Search over orbital parameters at specified orbital locations to seek minimum DV	- Guaranteed minimal DV magnitude and direction for several orbits	- Optimization requires monitoring of process - Requires accurate integration and full ephemeris modeling	- Analyzed in this paper with intent to apply to ARTEMIS

3) Continuous Controllers

The use of a continuous (or near-continuous via discretized maneuvers) control strategy requires a predetermined reference trajectory and some form of ancillary data to compute the accelerations for control. This method also requires a form of feedback as well as the computation of gains based on the reference STM. The controller is designed to be linear in nature, thereby assuming small deviations from the reference.

4) Baseline Orbit Control-Point Targeting

The orbit control-point targeting strategy is a straightforward approach in which the spacecraft is maintained near a predefined baseline orbit. Impulsive maneuvers are implemented at regular intervals to target future points along a nominal reference trajectory. In practice, the targeting is accomplished by first selecting a set of control points along the baseline orbit *a priori*. A more sophisticated strategy simultaneously incorporates multiple control points downstream to compute a maneuver; the location of the future control points can also be optimized.[14] For purposes of this study, a single control point is employed and the control points are generally placed along or near the *x*-axis in the Earth-Moon rotating reference frame; an ephemeris model is employed. It is assumed that navigation, burn, and modeling errors are incorporated and then, as the spacecraft passes close to each control point, a single- or a multiple-shooting Newton method is used to compute an impulsive DV maneuver such that the spacecraft reaches the next control point along the nominal trajectory (assuming no errors). After each maneuver has been calculated, simulated errors are added and the trajectory is propagated forward. This procedure is repeated for the desired number of control points.

5) Boxed Environment

The goal with this scheme is the maintenance of the vehicle within a pre-determined 'box' defined in terms of some specified distances from the *x*- and *y*-axis. The trajectory can be propagated forward using a simple bi-section method or DC to determine a DV, which permits the orbit to evolve for several revolutions. An initial guess is computed based on a targeting scheme similar to an *x*-axis crossing process or an acceptable alternative. The orbit is propagated until it violates a parameter value related to the size of the box about the libration point. A violation indicates that the orbit is departing from the vicinity of the libration point orbit and is either escaping along a manifold towards the Moon, towards a general Sun-Earth direction, or towards the stable libration point locations at L_4 and L_5. A maneuver can be bisected until a satisfactory end condition is met and a "minimal" DV is achieved to allow a longer-term propagation. This method is valuable since it guarantees a solution.

6) Orbit Continuation

The continuation method uses maneuvers performed at optimal locations to minimize the DV requirements while ensuring the continuation of the orbit for several revolutions downstream. This method uses goals in the form of energy achieved, velocities, or time at any location along the orbit. For example, a goal might be defined in terms of the *x*-axis velocity component at the *x*-axis crossings. In this analysis, a velocity is selected that can be related to the energy at any particular time. To initialize the analysis, a DC scheme is used, based on the construction of an invertible sensitivity matrix by numerical sampling orbital parameters downstream as a consequence of specific initial velocity perturbations.[13] The orbit is continued over several revolutions by checking the conditions at each successive goal then continued to the next goal. This allows perturbations to be modeled over multiple revolutions. The targeting algorithm uses several variables and target goals are specified uniquely for each orbit class that is controlled. Targeting is implemented with parameters assigned at the *x*-*z* plane crossing such that the orbit is balanced and another revolution is achieved; the maneuver supplies velocity in the *x*-axis direction that subsequently continues the libration point orbit. Additionally, the VF13AD1 optimizer is used to minimize the stationkeeping DV by optimizing the direction of the DV and the location (or time) of the maneuver. Included in the DC and optimization process are the constraints required to maintain the ARTEMIS maneuvers in the spin plane.

7) Global Optimum Search

Another alternative stationkeeping strategy utilizes a global search scheme in an effort to determine the smallest DV maneuver that maintains the spacecraft in the vicinity of the libration point for 1-2 additional revolutions. The control-point method may be appropriate when mission requirements necessitate a strict adherence to a baseline trajectory or as part of a larger scheme, i.e., when control points are used as a utility to seed another strategy. In contrast, the global search strategy is more appropriate for missions in which the primary goal is spacecraft maintenance in the general vicinity of the libration point using as little fuel as possible. It has been applied very successfully for Sun-Earth libration point mission analysis.[8] A major advantage of the global search method is that it does not require a baseline solution. It may be ideal for missions like ARTEMIS when the time interval during which the spacecraft remains in libration point orbits may vary greatly if propellant is consumed faster or slower than anticipated. However, because of the fast time-scales in the Earth-Moon system, careful formulation of the global search method is critical and some assumptions typical with previous implementations will not be successful. For example, it is not clear that a planar DV can be assumed a priori.

Navigation and Maneuver Errors

The computation of the stationkeeping DV for an Earth-Moon libration point orbit is influenced greatly by the inclusion of both navigation and maneuver implementation errors. In this analysis, a simple spherical navigation error of 1-km position and 1-cm/s velocity 1-sigma, was generated by the use of a covariance matrix. The maneuver errors are modeled by multiplying the computed DV times the desired error, e.g., DV * 1.01 for a 1% hot maneuver. Here, only maneuver errors of 1 percent are employed since such errors are consistent with the observed ARTEMIS maneuver errors and multiple other operations (once the thruster calibration is incorporated). The navigation error is computed as follows where a position and velocity error is then added to the current state. The covariance matrix is the diagonal of the estimated errors in the ARTEMIS navigation solutions that have been seen in Earth-Moon operations. Then, a random error is computed by the Matlab 'eig' function that produces a diagonal matrix D of eigenvalues and a full matrix, V, whose columns are the corresponding eigenvectors such that X*V = V*D. The square root of the column vector of D is then used to compute the standard deviation. The 3-sigma random error is then computed from multiplying a random error by the standard deviation times the full matrix V. These errors are generated after each maneuver and then added to the post maneuver state. The maneuver error is then also applied at the same time. This state with errors is then propagated to the next maneuver location and the process is repeated. The errors are applied by generating the error for each maneuver in the sequence of maneuvers to cover 126 or 129 days. This method is then repeated multiple times for each method to generate statistically sound results.

Maneuver Locations and Orbital Revolutions

A consideration in the analysis is the number of revolutions to be employed both for the 'targeting' as well as the placement of the maneuvers. Both one-revolution and a one-and-a-half-revolution are used for the targeting goals and a half- revolution or a full-revolution for the maneuver locations are incorporated. For example, if the maneuver is performed on the *x*-axis crossing in an Earth-Moon rotating coordinate system, then maneuvers are implemented either at every *x*-axis crossing or at every other crossing. The investigation explores the following locations for the maneuvers; *x*-axis crossing; maximum *y*-amplitude; and at an interval of ~3.8 days which yields 4 maneuvers per orbit. The effect of multiple maneuvers per revolution was modeled to coincide with the anticipated ARTEMIS tracking schedule.

RESULTS

The results are decomposed into two sections, one which addresses the general question of Earth-Moon libration point orbit stationkeeping and the other which considers the direct applications to the ARTEMIS mission design. Both approaches are very similar and use similar error generation, maneuver locations, orbital goals, and initial conditions. The methods selected from the Table are influenced by 4-body dynamics. As is apparent in Figure 5, any change in energy from an unstable Earth-Moon libration point orbit will result in a departure from the orbit, either towards the Moon or in an escape direction. The DV required to effect these changes are very small, since natural perturbations will also result in these escape trajectories. To continue the orbit downstream and maintain the path in the vicinity of the libration point, this information can be exploited to selectively choose the goals that must be achieved to continue the orbit from one side of L_2 to the other. For the method applied directly to ARTEMIS the goals include the energy (velocity) at the *x*-axis crossing to simple wrap the orbit in the proper direction, always inward and towards the libration point.

The first set of results from the stationkeeping analysis (labeled 'general' for simplicity) uses the ARTEMIS orbital parameters for initial conditions, but is not tied to the ARTEMIS constraints. Recall that ARTEMIS is not constrained by any specific orbital requirements on orientation or size. A design trajectory has, of course, been computed for planning purposes, but a specific reference orbit for ARTEMIS is not generated. Therefore, the use of a reference trajectory and control points is an option for the general analysis without regard to constraints, and the results are useful for comparison and the techniques can still be applied in the overall design process.

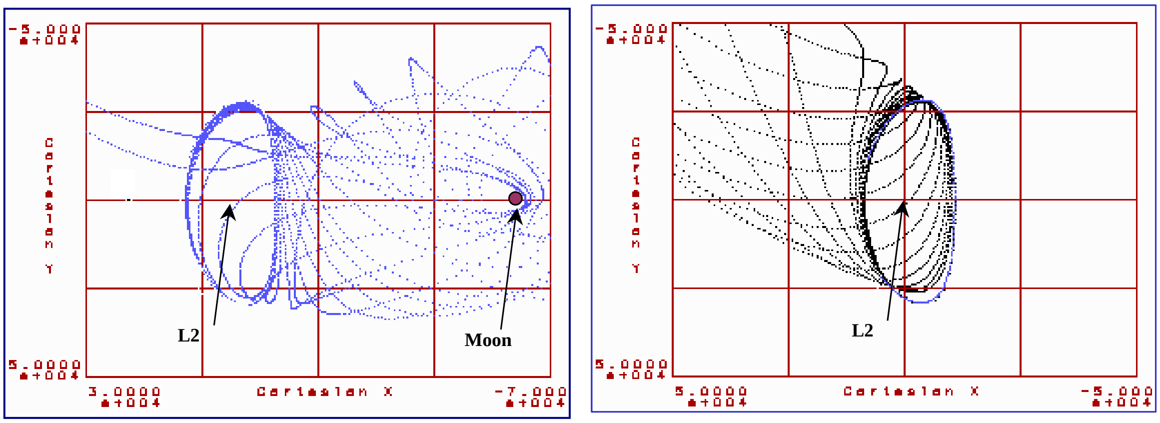

Figure 5. L_2 Trajectories and Arcs along the Unstable Modes

Results are also generated specifically for ARTEMIS and, thus, the second set of results as part of the stationkeeping analysis includes ARTEMIS constraints as described above with all maneuvers being applied in the spin plane of the spacecraft. With these conditions noted, the maneuvers analyzed here can still serve as a starting point for discussions of stationkeeping costs in any other mission since the dynamics are not significantly influenced by the maneuver direction. This constraint can be considered as an error if desired.

Results from a 'General' Stationkeeping Analysis

Reference Trajectories. To initiate an investigation of stationkeeping strategies in this problem, it is important to develop adequate models. As noted previously, in the Earth-Moon system, lunar eccentricity, and solar gravity can significantly influence the libration point orbits and both effects must be included in this analysis. Thus, models that incorporate these effects are developed. A periodic halo orbit, planar Lyapunov orbit, and non-periodic Lissajous trajectory are first integrated in a barycentered Earth-Moon rotating coordinate frame for a desired number of revolutions. Higher-fidelity baseline orbits are computed by discretizing the CR3B solutions into a series of patch points and iterating to yield the solution in a lunar-centered Moon-Earth-Sun ephemeris model using multiple shooting. A baseline L_2 near-planar Lyapunov orbit, a quasi-halo orbit, and a representative Lissajous trajectory appear in Figure 6. The Lissajous trajectory possesses characteristics similar to the ARTEMIS design trajectory; particularly note the changing inclination. The near-halo reflects the initial revolution of the Lissajous trajectory; the near-planar Lyapunov orbit represents the size and inclination of the final revolution of the Lissajous trajectory. Recall that all three are computed in an ephemeris model and serve as references for the initial stationkeeping investigation.

Baseline Orbit Control-Point Targeting. As a first step, a comparison between the stationkeeeping costs for the L_2 halo and Lissajous trajectories for the orbits as computed in different models is completed to assess the impact of the perturbations. Some representative results appear in Table 2. Consider the results for the control-point targeting strategy. Note that the error was introduced only in the *x*-direction (which typically produces the largest impact); only one trial is represented. The cost does increase for a trajectory computed in the ephemeris model but the cost is still of the same order. It is notable; however, that use of the ephemeris model does result in an increasing number of trajectories that depart from the region of the libration points if more trajectories are sampled. This potential can drive up the average cost over a number of trials. Any algorithm must include a strategy to identify and re-direct such escaping trajectories.

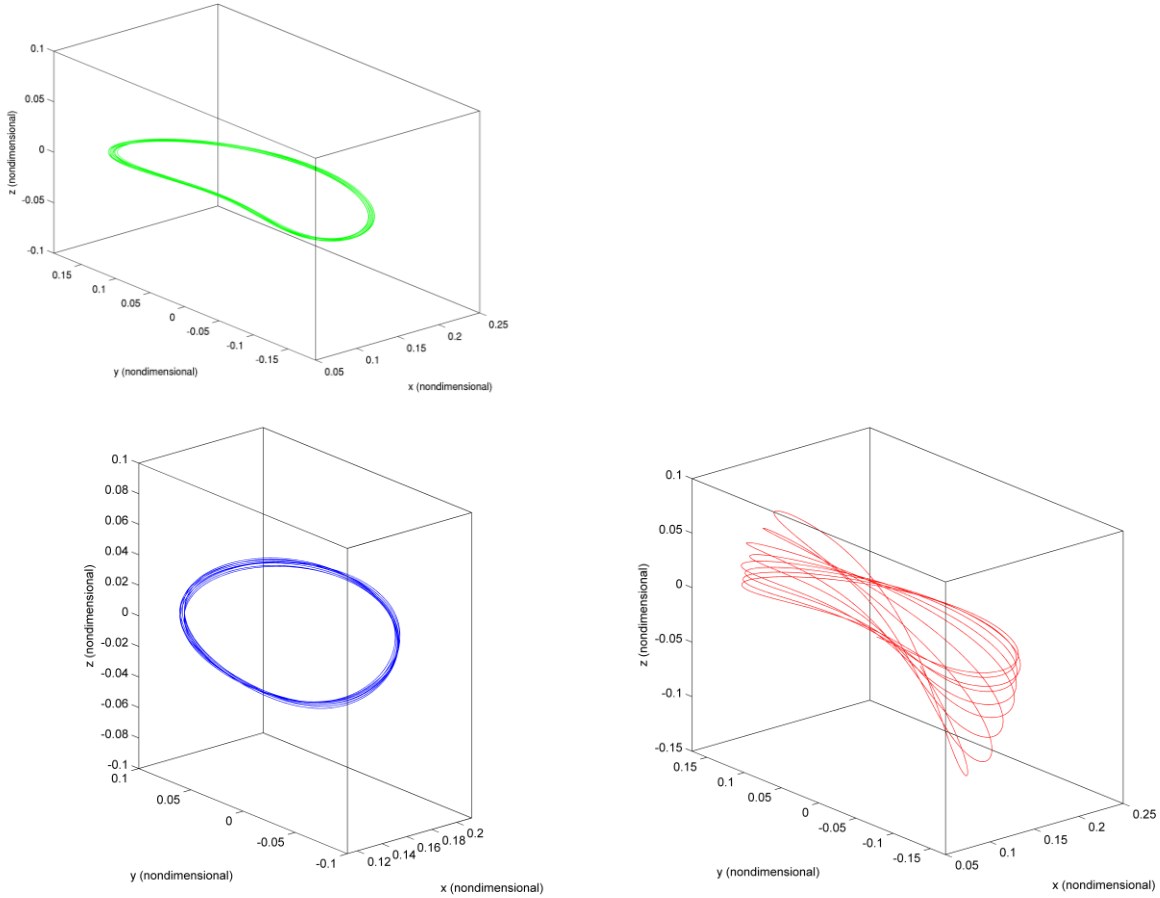

Figure 6. Baseline Moon-Earth-Sun Ephemeris L_2 Orbits: Near-Planar Lyapunov Orbit (top-left), Quasi-Halo Orbit (bottom left), and Lissajous Trajectory (right); Moon-Centered Rotating Frame

Table 2. Sample Stationkeeping Costs for the Control-Point Method (Error in +x Direction Only; One Trial)

Dynamic Model	Orbit	No. of Man	Total DV (m/s)	Avg. DV (m/s)	DV per year (m/s)	DV per month (m/s)	Avg. Time Between Man (days)
CR3B	Halo	17	15.16	0.8919	41.62	3.468	7.42
	Lissajous	16	12.26	0.766	34.46	2.872	7.64
Ephem	Halo	17	20.19	1.188	55.15	4.596	7.42
	Lissajous	16	13.25	0.8282	37.26	3.105	7.64

The errors are introduced in all three directions for the more complete stationkeeping analysis in Table 3. The numbers in the table reflect the results for 300 trials. The process is automated which is significant as noted later. The first row represents using two maneuvers per revolution for stationkeeping. Once reaching an x-axis crossing, control points at the next x-axis crossing are targeted. In the second row, also employing two maneuvers per revolution, the target points occur at the maximum y-axis location. This

Table 3. Baseline Control–Point Targeting Strategy, Lissajous Trajectory (Statistical Errors in All Directions; 300 Trials)

	No. of Maneuvers	Avg Total DV (m/s) Per 129 days	Avg DV per Maneuver (m/s)	Std Dev (m/s)	Avg DV per Year (m/s)	Avg Time Between Man (days)
X-axis, every crossing	16	9.45	0.59	2.45	26.59	7.63
Max Y-Amp every crossing	17	14.54	0.86	7.26	40.92	7.21
4 Pts/Rev (~3.8 days)	33	2.50	0.08	0.32	7.04	3.82

approach increases the cost. But the process is actually 'less reliable' in general. Although all 300 trials converge, the process was modified. An initial guess is incorporated into the process. Without the initial guess, the process can easily diverge. With the initial guess, the numerical computations are stabilized. Clearly, with this strategy, the most robust and low cost option is to incorporate more maneuvers per revolution.

The third option targets x-axis crossings and includes four maneuvers per revolution. There are no problems with convergence and the cost is relatively low. From the results of the specific analyses for the ARTEMIS trajectory, the control-point scheme will be modified to consider incorporating target locations that are at least 1.5 revolutions into the future to reduce the cost.

Global Search Method. A global optimum search method is also tested, with a planar constraint. In practice, at a desired maneuver location (a crossing of the x-z plane, for example), a maneuver plane is defined parallel to the x-y plane and through the current spacecraft location. A maneuver angle, α, is measured from the $+x$-axis and is varied from 0° to 180° in the plane. At each maneuver angle, a DV magnitude is computed that results in a trajectory with zero x-velocity at an x-z plane crossing that is several crossings in the future. For this scheme, seeking a trajectory with zero x-velocity at an x-z plane crossing is only one option for formulating the target state. The ultimate goal is to ensure that the spacecraft will remain in orbit near the libration point for the immediate future. Once all angles have been explored, the smallest maneuver is performed. As in the control-point method, simulated errors are then added and the trajectory is propagated forward. Consistent with the control-point strategy, a comparison between the stationkeeping costs for the L_2 halo and Lissajous trajectories for the orbits in different models is completed first. Some representative results appear in Table 4. Although only one trial appears in the table, there is again a slight increase in cost when the ephemeris model is incorporated. In this case, employing the Lissajous trajectory noticeably increases the cost. This is likely due to the shape of this Lissajous orbit and the changing inclination.

For the sample Lissajous case computed in the ephemeris model in Table 4, a polar plot demonstrates the DV magnitude as a function of maneuver angle, α, for a representative maneuver location and appears

Table 4. Sample Stationkeeping Costs for the Global Optimum Search Method (Error in +x direction Only; One Trial)

Dynamic Model	Orbit	No. of Man	Total DV (m/s)	Avg. Δv (m/s)	Δv per year (m/s)	Δv per month (m/s)	Avg. Time Between Man (days)
CR3B	Halo	17	8.983	0.5284	24.65	2.054	7.39
Ephemeris	Lissajous	16	10.09	0.6305	28.18	2.348	7.69
	Halo	17	12.54	0.7374	34.37	2.86	7.4
	Lissajous	15	26.33	1.7552	78.13	6.51	7.69

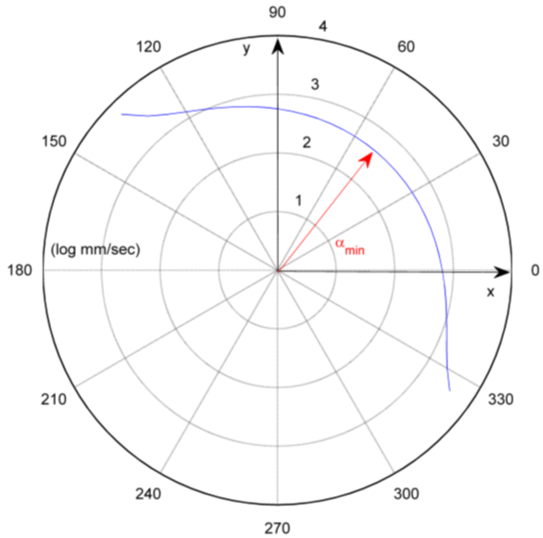

Figure 7. Optimal Maneuver Angle and Magnitude; Global Optimum Search Method

in Figure 7. The DV magnitudes are expressed in units of log(mm/sec) in the radial direction. The optimal DV angle and magnitude are indicated in red as well. At each maneuver location such a plot is generated and the optimal maneuver is implemented; errors are added and the simulation proceeds to the next location. It is noted that the process to obtain this result was computationally slow and unpredictable. Searching over all the potential angles to identify the best maneuver does not always yield an acceptable result with this strategy as currently implemented. One contributing factor is the type of Lissajous trajectory. As implemented, the maneuvers occur at *x*-axis crossings. Returning to the control-point strategy for a moment, it is observed that the required maneuvers for application to the halo orbit or to the Lyapunov trajectory from Figure 6 do not have a significant out-of-plane *z*-component; in contrast, the Lissajous with the large inclination change over the evolution of the trajectory includes maneuvers with significantly greater out-of-plane (*z*) components. To implement this strategy for many trials will require a modification to allow for out-of-plane components in the DVs. Although such an option can yield improved results, the computational costs prohibit a large number of trials. For the purposes of this initial investigation, an augmented strategy is not pursued at this time.

Stationkeeping Influenced by ARTEMIS Constraints

Beginning with the ARTEMIS initial conditions, a 126-day stationkeeping profile is generated for three maneuver locations for the aforementioned number of revolutions. A 126-day duration is used to map the results to the planned duration of ARTEMIS in the L_2 orbit. Each profile varied the maneuver location and then the number of revolutions for the conditions to achieve a continuation of the trajectory further downstream for more revolutions. Each simulation uses the statistically generated navigation errors and a constant maneuver error of +1%. Also, the constraint to maintain the DV within the spin plane of the ARTEMIS spacecraft is also met. Tables 5 and 6 summarize the average DV results for cases that applied a 1.5-revolution and a 1-revolution continuation, respectively. These results include only 10 trials. Several obvious results emerge. First, maneuvers that are applied only once per revolution are several times larger than those applied at least twice along an orbit. Also consistent with the preliminary results from the general stationkeeping analysis, the maneuvers applied at the maximum *y*-axis amplitude are also larger than those at the *x*-axis crossings. To compare the results to a strategy that employs more frequent maneuvers, a scenario was simulated that applied maneuvers once every 3.8 days (a four-maneuvers-per-revolution sequence). This is significant in that the planned ARTEMIS tracking coverage and navigation solutions are based on a three-day arc.

The overall results demonstrate that maneuvers at a frequency such that maneuvers occur at least once every seven days are desired to both minimize the DV budget and to align with the navigation solution deliveries. A more frequent maneuver plan (3.8-day updates) is only slightly better. The magnitudes of the individual maneuvers and the angle between the DV vector and the Earth-Moon rotating +*x*-axis appear in Figures 8 through 11. Note in these figures that the magnitude of the DV remains relatively constant using this stationkeeping scheme as the inclination of the Lissajous orbit decreases. Figures 8 and 9 are from a scenario with maneuvers that occur at each *x*-axis crossing. The radial data in Figure 9 are expressed

Table 5. DV Comparisons for Continuous Optimal Method using 1.5-rev (10 Trials)

Maneuver Location	No. of Maneuvers	Avg Total DV (m/s) Per 126 days	Avg DV per Maneuver (m/s)	Std Dev (m/s)	Avg DV per Year (m/s)	Avg Time Between Maneuver (days)
X-axis, every crossing	15	4.23	0.28	0.78	12.27	7.3
X-axis, once per orbit	7	34.14	4.88	7.07	106.51	15.2
Max Y-Amp Every crossing	15	6.26	0.42	.95	18.13	7.3
Max Y-Amp Once per orbit	7	38.29	5.46	6.98	110.91	14.9
4 Pts/Rev (~3.8 days)	33	4.74	0.15	0.33	13.72	3.8

Table 6. DV Comparisons for Continuous Optimal Method using 1-rev (10 Trials)

Maneuver Location	No. of Maneuvers	Avg Total DV (m/s) Per 126 days	Avg DV per Maneuver (m/s)	Std Dev (m/s)	Avg DV per Year (m/s)	Avg Time Between Maneuver (days)
X-axis, every crossing	15	10.95	0.73	0.77	31.71	7.3
X-axis, once per orbit	7	98.60	14.09	25.06	285.62	15.2
Max Y-Amp Every crossing	15	50.44	3.36	3.45	50.4	7.3
Max Y-Amp Once per orbit	7	217.52	31.08	31.44	630.13	14.9
4 Pts/Rev (~3.8 days)	33	10.94	0.33	0.59	31.70	3.8

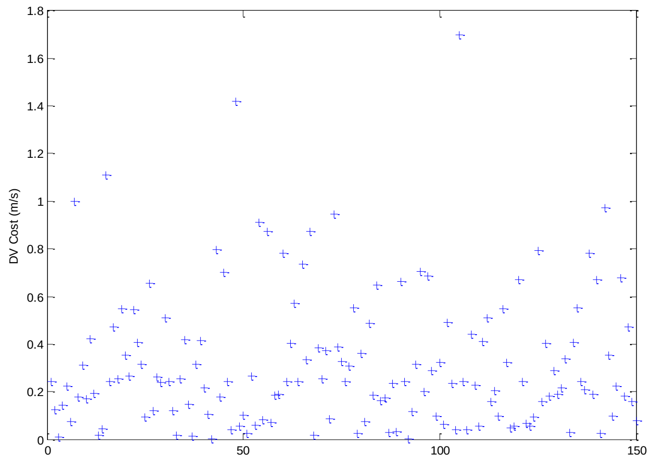

Figure 8. DVs for Maneuvers at each X-axis Crossing

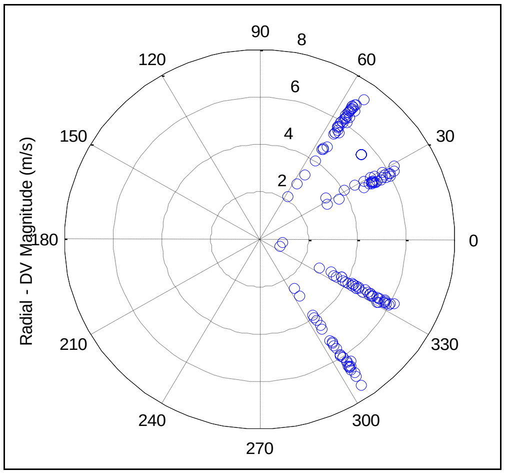

Figure 9. Angle Between X-axis and DV Vector for Maneuvers each X-axis Crossing

in units of log(mm/sec) in the radial direction. The DV angle with respect to the *x*-axis is dependent upon the navigation and maneuver errors introduced and varies between the angles shown. The optimal direction as demonstrated in figure 7 needs to be recomputed for each maneuver. This result indicates that there may be a relationship between the direction of a maneuver performed at regularly located positions along the orbit and the environmental dynamics, especially if the direction of the uncertainties is also repeatable. Recall that the number of maneuvers in figures 8 and 9 are from a 126-day simulation (ARTEMIS L_2 orbit duration), that includes 15 maneuvers per 126 days, with 10 trials to generate a minimum set of statistical data for DV analysis. Figures 10 and 11 present a similar set of plots for maneuvers that occur 4 times per orbit (3.8-day intervals). Again we see a similar pattern to the angle information. The simulations used to generate figures 8 through 11 applied the 1.5-revolution duration for continuing the orbit and use navigation uncertainties of 1-km position and 1-cm/s velocity.

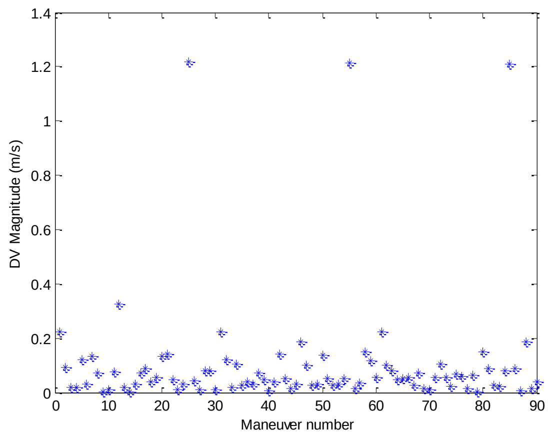

Figure 10. DVs for Maneuvers at 3.8-Day Intervals

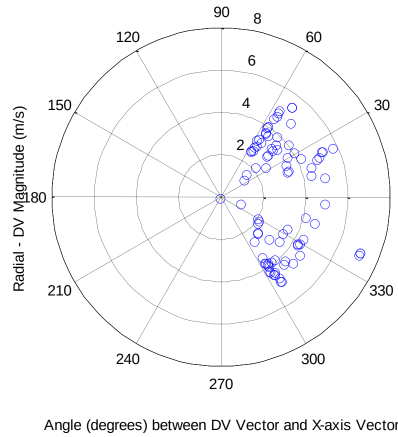

Figure 11. Angle Between X-axis and DV Vector For 3.8-Day Maneuvers

Observations

The analysis has provided us with some unique and obvious observations for stationkeeping.

- Strategies investigated yield DV budgets less than 25 m/s per year.
- The overall results demonstrate that maneuver frequency of at least once every seven days is desired to minimize the DV requirements and to align with the navigation solution deliveries. A more frequent maneuver plan (3.8-day updates) is only slightly better.
- The Earth-Moon dynamics dictate that orbital conditions to be satisfied should include at least 1.5 revolutions. This strategy will include major perturbations and the effects of the lunar orbit eccentricity. The number of revolutions used in determining the orbit conditions to continue the orbit is key in minimizing the DV requirements.
- The inclination of the orbit (at least for the ARTEMIS scenarios analyzed) does not affect the DV magnitude and the magnitude remains constant. (This observation assumes out-of-plane components in the DVs are available.)
- As the ARTEMIS orbit's inclination decreases, the *y*-amplitude increases and affect the selection of the *x*-axis crossing conditions that are selected as targets to successfully continue the path downstream.
- The DV direction with respect to the *x*-axis may be repeatable for a given maneuver scenario.
- Libration point orbit stationkeeping DV requirements are reasonable and similar in comparison to other low lunar orbit DV maintenance requirements.[13] The Lunar Prospector and Lunar Reconnaissance Orbiter require approximately 11 m/s per month.

A Proposed ARTEMIS Strategy

Given the constraints of the ARTEMIS mission orbit, spacecraft maneuvers are currently planned at a frequency of seven days to ensure a stable navigation solution while minimizing the DVs and staying within the ARTEMIS DV budget. The maneuvers are also planned to occur at or near the *x*-axis crossings and use a continuation method to maintain the orbit. Orbital conditions will be set to permit the energy or velocity at the crossings to continue the orbit for at least 1 and ½ revolutions. This strategy also benefits the operations by permitting a routine schedule.

SUMMARY

Two methods have been demonstrated that result in low stationkeeping DV requirements and that meet the ARTEMIS mission requirements. It has been demonstrated that a full ephemeris model and the associated errors from navigation and maneuvers are required to accurately model the accelerations that affect the DV. The dynamics of the Earth-Moon environment also must be modeled over a sufficient duration. This duration should be equal to or greater than 21 days to account for the lunar eccentricity and to a lesser, but still important degree, the perturbation from the Sun. An increase in the frequency of the maneuvers tends to reduce the overall DV requirements as does the placement of the maneuvers near the *x*-axis crossing. In our analysis, stationkeeping cost with realistically modeled navigation errors has a floor of about 15 m/s per year, less than the DVs from previous studies that approached 60 m/s per year.

CONCLUSIONS

While there are a number of strategies available that incorporate the Earth-Moon dynamics, the actual mission applications and mission constraints must also be considered. The methods developed here allow a general application whether there is a reference orbit, spacecraft constraints on DV direction, or orbital parameters requirements. The required stationkeeping DV can be minimized and is comparable to a lunar mission. With the ARTEMIS P1 and P2 spacecraft on-track for Earth-Moon libration orbit insertion, investigation of robust strategies and options to improve the DV required for stationkeeping are continuing.

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