Trajectory Design in the Spatial Circular Restricted Three-Body Problem
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Abstract

With recent motivation for manned missions to asteroids or Mars, interest in demonstrating human exploration technologies within the Earth-Moon system has increased. The Circular Restricted Three-Body Problem (CR3BP) offers a unique solution space and serves as a useful framework for preliminary trajectory design studies such as for planar trajectories in this dynamical regime is often nontrivial. To determine trajectories that meet a variety of mission constraints, tools that provide insight into the available solution space are essential. The Poincaré map is a powerful tool that, in combination with a constraint on the energy level, allows a reduction in dimension such that, for the planar problem, the system is reduced to two and the phase space is completely represented by the projection onto a plane. In the spatial problem, however, Poincaré maps must represent at least four states and are therefore challenging to visualize. In this investigation, strategies from the field of data visualization are employed to represent the information contained in higher-dimensional Poincaré maps using a two-dimensional image. Four-dimensional map representations are demonstrated to compute free and low-cost transfers between periodic libration, halo (blue), and vertical (magenta) orbits in the Earth-Moon system. Alternative Poincaré maps, such as the periapse map, require visualization of the full six-dimensional state space. Sample six-dimensional periapse map representations are demonstrated to locate families of periodic orbits about the Moon, as well as transfers to these space.

The Circular Restricted Three-Body Problem

In the Earth-Moon CR3BP, the motion of a spacecraft, assumed massless, is examined as it moves in the vicinity the Earth and the Moon, each assumed to be point masses. A barycentric rotating frame is defined such that the rotating x-axis is directed from the Earth to the Moon, the y-axis is parallel to the direction of the angular momentum of the Earth-Moon system, and the z-axis completes the desired, orthogonal tripod. Five equilibrium points exist, and are depicted in Figure 1. The Jacobian constant, C, is the single known integral of the motion that represents a conservative body force environment. Several families of solution exist that prove useful in mission design, including families of periodic and quasi-periodic orbits that are present in a variety of the equilibria, including the planar and vertical Lyapunov orbits, as well as the families of halo orbits; sample orbits about the L2 and L3 points in the Earth-Moon system are plotted in Figure 2.

Poincaré maps in the Planar CR3BP

The Poincaré map is a valuable tool that provides insight into the complicated dynamics of the CR3BP. Defining a suitable solution space is part of the dynamics is achieved by propagating initial conditions and displaying intersections of the resulting trajectories with the map. The use of a Poincaré section, in addition to a constraint on the values of the Jacobi constant, reduces the dimension of the system by two. In the planar problem, the state space is, therefore, entirely represented by the projection onto a plane. As an example, consider the stable manifold of an L2 Lyapunov orbit for C = 3.15, plotted in Figure 4(a); the gray surface represents the boundary of the motion for the spacecraft for this value of C. The Poincaré map depicting the intersections of these manifolds with the surface defined by the x-location of the Moon appears in Figure 4(b); these intersections with ξ and η contours in the y = 0 phase space. For each point on the map, x is defined by the location of the Moon, y and η are plotted in the map, and z is computed from the specified value of Jacobi constant.

Application of Four-Dimensional Poincaré Maps

Many free transfers between planet Libration point orbits exist in the planar CR3BP, however, location of free transfers between periodic orbits in the spatial problem is challenging. To search for free and low cost transfers, invariant manifolds and higher-dimensional Poincaré maps are employed. As an example, consider the manifolds, plotted in Figure 7(a) associated with the L2 vertical orbit that exists for the energy level C = 2.849. The manifolds are propagated until intersection with the surface defined by ζ = 0. The corresponding map appears in Figure 7(b), where the states x, y, z are represented analogously to the states in Figure 5. The zoomed view of the map reveals a pair of unstable manifold (red) and stable manifold (blue) crossings, circled in black, for which the vectors possess nearly zero length, i.e. x = ξ = 0; this suggests that a free or low cost transfer may exist near the energy level C = 2.849. After application of a differential corrections process, a maneuver-free transfer is located for C = 2.98412, and appears in Figure 8.

Application of Six-Dimensional Poincaré Maps

The periapsis surface of section, initially demonstrated by Paskowitz, Scheres, and Scheeres, demonstrates high-fidelity characteristics of the periapsis space. A periapsis map is defined by the surface of section Θ = (θ, φ) = (0, 0), where θ is the elevation angle and φ is the longitude. Poincaré maps in the CR3BP record the closest lunar approaches along a trajectory as crossings of the map. To fully represent periapsis map crossings, at least five Cartesian states must be displayed; a simple example is computed of the grid of periapsis initial conditions (C = 3.15) plotted in Figure 11(a); the gray surface represents the boundary of the motion for the spacecraft for this value of C. These initial conditions are propagated in the CR3BP and subsequent crossings of the map are recorded. Crossings along those trajectories that remain in the vicinity of the Moon are demonstrated by the trend of the crossings in Figure 11(b). Alternatively, each map crossing is represented as in Figure 11(c), with each state colored by the elevation angle λ used to initiate this trajectory using this representation, and zooming in on the region of the right side the Moon, the map in Figure 11(c) is obtained; each crossing is colored by inclination relative to the plane. A large variety of low-cost, capture map crossings are present in the CR3BP, including periodic orbits. Because the crossings of the periapsis map belong to trajectories generated from randomly selected initial conditions in Figure 11(a), it is unlikely that a periodic orbit will appear directly on the map; however, stable periodic orbits may be located by observing the behavior of nearby crossings on the map. A group of blue crossings appears for which y = ξ = 0 = η, indicating a periodic orbit may exist nearby. The state circled in red corresponds to the cyan trajectory in Figure 11(d); this trajectory is corrected for periodicity, revealing the blue lunar orbit in Figure 11(d), with map crossing circled in blue in Figure 11(e).

Conclusions

The Circular Restricted Three-Body Problem (CR3BP) serves as a useful framework for preliminary trajectory design in a multi-body force environment, however mission design in this dynamical regime is nontrivial. To facilitate trajectory design in such an environment, it is desirable to develop tools that provide insight into the available solution space. Methods to display the information contained in higher-dimensional Poincaré maps are explored and prove useful for mission design applications. Maps for which four or more states are represented visually yield much insight into the available solution space. Free and low-cost transfers are quickly located that may otherwise remain difficult to detect. Structures on the map are identified that reveal the existence of nearby periodic orbits; a low-cost transfer requiring a maneuver of only 7.8 m/s is located from the map.

References