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A TETHER SLING FOR LUNAR AND INTERPLANETARY EXPLORATION†

J. PUIG-SUARI, J. M. LONGUSKI and S. G. TRAGESSER

School of Aeronautics and Astronautics, Purdue University, West Lafayette, IN 47907-1282, U.S.A.

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Abstract—In this paper, we analyze the concept of a tether sling for lunar and planetary missions. By continuous application of torque (generated by a solar-powered electric motor), the spacecraft, attached to the end of the tether, achieves the required injection velocity. Since no propulsive maneuver is required, the tether supplies a virtually inexhaustible capacity to catapult deep space probes. It is shown that the engineering difficulties associated with this problem are surmountable and often have simple and elegant solutions. The tether must be tapered according to a simple formula, which allows the tether to support its own mass as well as the probe's. Although chemical propulsion provides a much better mass ratio for high energy transfer, the great advantage of the tether is its *simplicity* and *reusability*.

1. INTRODUCTION

Perhaps the earliest mention of using tethers for momentum transfer between orbiting spacecraft is presented by Colombo [1]. The concept of using a tether as a sling to catapult rocks off the surface of the moon was first mentioned by Carroll [2] who dealt with a uniform tether having a characteristic velocity of about 0.7–1.0 km/s, above which the tether breaks. Tillotson [3] realized that to increase the effective speed of a tether sling, the tether must be tapered.

Many other researchers have contributed to the ideas of momentum transfer and orbit pumping with tethers which capitalize on propulsive techniques that do not involve expendable propellant. These authors, too numerous to list here, are well referenced by Penzo and Ammann [4] and Beletsky and Levin [5].

In this paper, we derive an exact expression for the taper of a tether sling and we analyze the power requirements for the spin-up maneuver. Next, we consider design configurations which address the spinning and transverse torque problems associated with an orbiting tether sling facility. Finally, we demonstrate, by numerical examples, the potential efficacy of tether sling facilities stationed in Earth orbit, on the lunar surface and on the satellites of Mars.

2. ANALYSIS

2.1. Uniform tether

For a spinning tether sling the maximum tension occurs at the hub. If the tether has uniform diameter (Fig. 1), the tension at the hub is:

$$F = \rho A \frac{v^2}{2} \quad (1)$$

where ρ is the density of the tether, v is the speed of the tether end point and A is the cross-sectional area of the tether. Note that eqn (1) ignores tension due to the payload at the end of the tether. The maximum force that a tether can withstand is:

$$F = \sigma A \quad (2)$$

where σ is the ultimate strength of the tether material. Combining eqns (1) and (2), the maximum speed that can be obtained at the end of a uniform tether before it breaks is:

$$v_c = \sqrt{\frac{2\sigma}{\rho}}. \quad (3)$$

We call this value the characteristic velocity, and it represents the maximum speed that can be obtained by a spacecraft propelled by a uniform tether sling. Values of v_c for graphite tethers can be as high as 2.5 km/s. However, to obtain this theoretical maximum, the ratio between the mass of the tether and the mass of the spacecraft must be infinite, since the tether is only strong enough to support its own weight. Clearly, even if the value of v_c were high enough for interplanetary travel, the large tether mass would be unacceptable.

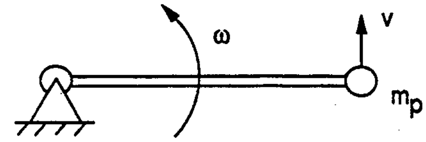

Fig. 1. Uniform tether.

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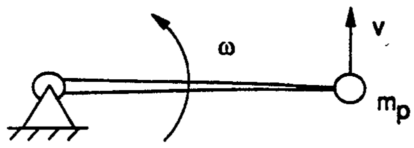

Fig. 2. Tapered tether.

2.2. Tapered tether

In the uniform tether system the strength of the tether is matched to the tension forces at the hub. The tension forces at other points along the tether are smaller, and the diameter of the tether could be reduced (Fig. 2). This would reduce the mass of the tether and, consequently, the tension at the hub. To find the taper equation, first we compute the tension force at a distance x from the hub for a tether of length l to be:

$$F_x = \int_x^l \frac{v^2}{l^2} y \, \mathrm{d}m_y + \frac{v^2}{l} m_p \tag{4}$$

where m_p is the mass of the payload and $\mathrm{d}m_y$ represents the mass of a differential tether element located at a distance y along the tether, which can be written as:

$$\mathrm{d}m_y = \rho A_y \, \mathrm{d}y \tag{5}$$

where A_y is the area of the tether at point y.

In order to minimize the mass, the strength (area) of the tether at any point x should be matched to the tension force at that point (F_x). Thus, from eqns (2) and (4), the tether area at point x becomes:

$$A_x = \frac{F_x}{\sigma} = \frac{v^2}{\sigma l}\left(\frac{\rho}{l}\int_x^l yA_y \, \mathrm{d}y + m_p\right). \tag{6}$$

Differentiating eqn (6) with respect to x yields:

$$\frac{\mathrm{d}A_x}{\mathrm{d}x} = \frac{v^2\rho}{\sigma l^2}(-xA_x). \tag{7}$$

Integrating eqn (7) we obtain:

$$A_x = A_l \exp\left[\frac{v^2\rho}{l^2\sigma}\left(\frac{l^2}{2} - \frac{x^2}{2}\right)\right] \tag{8}$$

where A_l is the area at the end of the tether, which is given by the strength required to withstand the

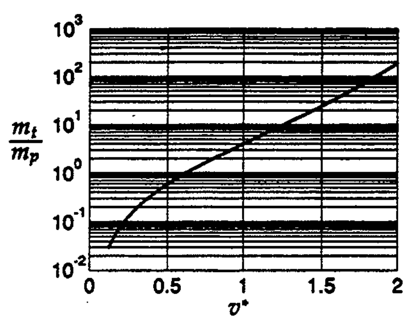

Fig. 3. Tether mass ratios.

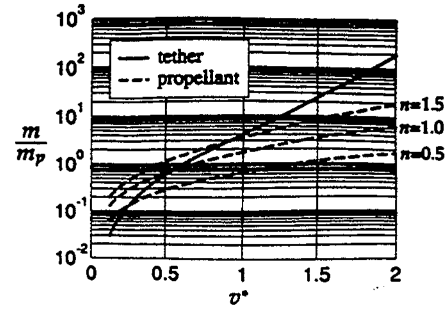

Fig. 4. Mass ratio comparisons.

force from the payload:

$$A_l = m_p \frac{v^2}{\sigma l}. \tag{9}$$

Combining eqns (8) and (9), the area of the tapered tether at a point x along the length becomes:

$$A_x = m_p \frac{v^2}{\sigma l} \exp\left[\frac{v^2\rho}{2\sigma}\left(1 - \frac{x^2}{l^2}\right)\right]. \tag{10}$$

The total mass of the tether is:

$$m_t = \int_0^l \rho A_x \, \mathrm{d}x. \tag{11}$$

Substituting eqn (10) into eqn (11) yields:

$$m_t = m_p \frac{\rho v^2}{\sigma l} \exp\left(\frac{\rho v^2}{2\sigma}\right) \int_0^l \exp\left(\frac{-\rho v^2}{2\sigma}\frac{x^2}{l^2}\right) \mathrm{d}x. \tag{12}$$

A change of variables in the integral reduces eqn (12) to:

$$m_t = m_p v \sqrt{\frac{2\rho}{\sigma}} \exp\left(\frac{\rho v^2}{2\sigma}\right) \int_0^{v\sqrt{\rho/2\sigma}} \exp(-t^2) \, \mathrm{d}t \tag{13}$$

which can be expressed in terms of the error function (erf) [6]:

$$m_t = m_p v \sqrt{\frac{\rho}{2\sigma}} \sqrt{\pi} \exp\left(\frac{\rho v^2}{2\sigma}\right) \mathrm{erf}\left(v\sqrt{\frac{\rho}{2\sigma}}\right). \tag{14}$$

Finally, the mass ratio of the tapered tether system can be written as:

$$\frac{m_t}{m_p} = \sqrt{\pi} v^* \exp(v^{*2}) \mathrm{erf}(v^*). \tag{15}$$

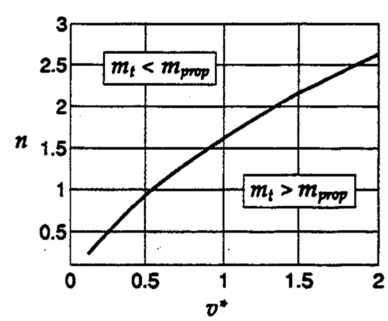

Fig. 5. Performance index vs. break-even speed.

where v^* is the non-dimensional velocity:

$$v^* = v\sqrt{\frac{\rho}{2\sigma}} = \frac{v}{v_c}. \quad (16)$$

This allows the relation between the mass ratio and any combination of tether material and payload speed to be represented by a single graph (Fig. 3). Equation (15) clearly indicates that the tapered tether sling has no speed limitation. The mass ratio increases with non-dimensional velocity, which, for a given tether material, is proportional to payload speed.

The mass requirements of a tether sling can easily be compared with those of a traditional propulsive system. From the rocket equation, the ratio of propellant mass to payload mass can be written as:

$$\frac{m_{\text{prop}}}{m_p} = \exp(nv^*) - 1 \quad (17)$$

with $n = v_c/(I_{sp}g)$, where I_{sp} is the specific impulse of the propellant and g is the standard gravitational acceleration on the Earth's surface. Using this equation, the propellant mass ratio for various values of n can be combined with the tether sling results in a single graph (Fig. 4). From the plot it is clear that the tether sling is superior to traditional rocket systems when low speeds are required, while rocket systems have better performance at higher speeds. The point where the two systems have equal mass ratio changes with n. The performance index is plotted versus this "break-even speed" in Fig. 5. It should be noted that the results in Figs 4 and 5 do not include the fact that the tether sling has multilaunch capabilities which dramatically reduce the mass ratio (when many payloads are launched).

2.3. Energy requirements

In order to determine the energy required to spin up a tapered tether sling we must first determine its moment of inertia about the hub, I_t:

$$I_t = \int_0^l x^2 \rho A_x \, dx. \quad (18)$$

Equation (18) can be solved in terms of error functions using integration by parts and the same

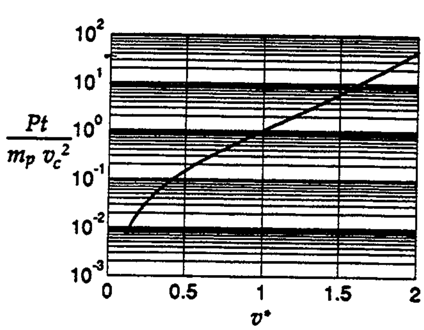

Fig. 6. Solar cell sizing.

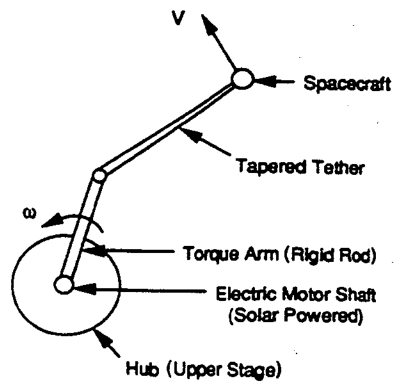

Fig. 7. Orbiting tether facility.

change of variables used to solve eqn (12):

$$I_t = m_p l^2\left[-1 + \frac{\sqrt{\pi}}{2v^*}\exp(v^{*2})\text{erf}(v^*)\right]. \quad (19)$$

If the inertia of the payload is included, the total inertia of the system is:

$$I = m_p l^2 \frac{\sqrt{\pi}}{2v^*}\exp(v^{*2})\text{erf}(v^*). \quad (20)$$

Thus, the rotational kinetic energy required for the velocity, v, at the end body is

$$E_{\text{req}} = \frac{\sqrt{\pi}}{4} m_p \exp(v^{*2})\text{erf}(v^*)v^* v_c^2. \quad (21)$$

The energy supplied by a solar array with area, A, and power per area output, P/A, is

$$E_{\text{solar}} = \frac{P}{A}At = Pt. \quad (22)$$

Equating expressions (21) and (22) yields the governing non-dimensional energy equation for the tether sling

$$\frac{E}{E_c} = \frac{Pt}{v_c^2 m_p} = \frac{t}{t_c} = \frac{\sqrt{\pi}}{4}\exp(v^{*2})\text{erf}(v^*)v^* \quad (23)$$

where E_c and t_c are the characteristic energy and time, respectively. Note that the ls cancel, which leaves us free to choose the tether length to meet any acceleration constraint. The form of eqn (23) is convenient for plotting the relationship among the design variables (P/A of the solar cell, area of the solar array, time necessary to spin-up, tether material, mass of the payload and v). Using Fig. 6, the required energy that is to be generated by the solar array per kilogram of payload can be determined for a given tether material and spacecraft velocity.

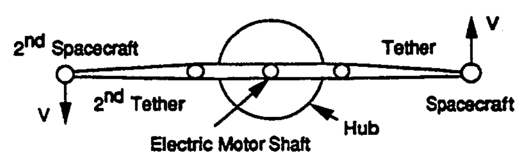

Fig. 8. Transverse torque balancing.

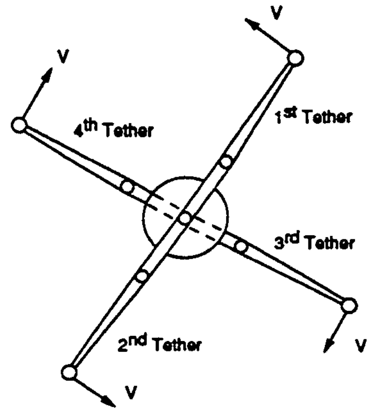

Fig. 9. Spin torque balancing.

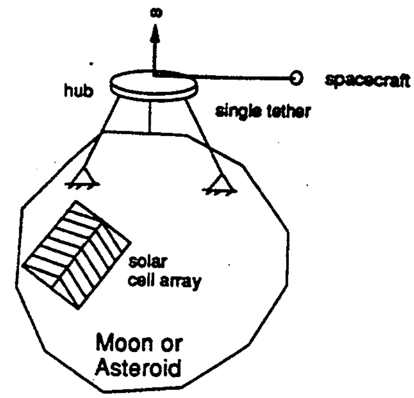

Fig. 10. Moon/asteroid facility.

Interestingly, from eqn (15), it is apparent that the non-dimensional energy (23) is directly proportional to the tether mass ratio:

$$\frac{E}{E_c} = \frac{Pt}{v_c^2 m_p} = \frac{1}{4}\frac{m_t}{m_p}. \tag{24}$$

3. DESIGN CONFIGURATIONS

The initial idea for a tether sling facility in Earth orbit involves the use of an upper stage as the hub for a single tether sling (Fig. 7). This configuration is very easily implemented but it presents two serious problems. First, if the connection between the tether and the hub is not located at the center of mass of the system, the resultant transverse torque causes the spin axis to wobble. Second, the spin-up torque acts on the hub as well as the tether, and, given the large dimensions (and inertia) of the tether, produces extremely high spin rates on the hub. The transverse torque can be eliminated by adding a second tether opposite the first one (Fig. 8). Note that this doubles the total mass of the facility, but the tether and energy requirements per unit mass of payload remain constant since the two-tether facility can simultaneously launch two spacecraft. However, in this configuration the hub spin-up problem remains. To balance the spin torque, a counter-rotating tether pair can be added on the opposite side of the hub (Fig. 9). Again, despite the increase in the mass of the sling facility, the mass ratio and energy requirements remain constant.

The problems mentioned above disappear when a very massive hub (with a large moment of inertia) is employed. This is not a practical solution if the hub must be launched into orbit; however, moons, planets and asteroids provide natural hubs with extremely high (extended) masses (Fig. 10).

For a host celestial body having an appreciable gravitational field, some additional engineering design issues may arise. The total force on the tether may be larger than that calculated above, requiring a more massive tether. Also, the tether will sag under the force of gravity, creating a clearance problem with the ground. On small moons and on asteroids, these effects will probably be minimal. However, on a large body, such as the Earth's moon, the effect of the gravity field should (in the final analysis) be considered in the modeling of the tether sling in order to obtain a more realistic design.

4. REPRESENTATIVE TETHER SLING FACILITIES

The following examples are included to illustrate some possible tether sling applications. The characteristics are calculated for a single-sling tether with a stationary hub, but, in general, the design parameters are equivalent if the number of tethers is equal to the number of spacecraft, and are easily scaled if only one spacecraft is ejected from a multiple tether facility. Note that while the designs are mission specific, a given system is capable of infinitely many launch scenarios, as long as the required velocity is less than the design value.

The first example is designed for a Hohmann transfer from LEO (low Earth orbit, $R = 6878$ km) to Mars ($v = 3.55$ km/s). The tether is assumed to be made of Kevlar with material properties

$$\sigma = 2.80\ \text{GN/m}^2 \tag{25}$$

$$\rho = 1450\ \text{kg/m}^3. \tag{26}$$

The characteristic velocity of this material is 1.97 km/s, which results in a non-dimensional velocity of $v^* = 1.81$. Using this value, the mass ratio of propellant to payload is found from eqn (17) for an assumed I_{sp} of 300 s:

$$\frac{m_{prop}}{m_p} = 2.34 \tag{27}$$

The mass ratio for the tether sling can be calculated from eqn (15) to be

$$\frac{m_t}{m_p} = 82.8 \tag{28}$$

This mass ratio corresponds to a non-dimensional energy of

$$\frac{E}{E_c} = \frac{1}{4}\frac{m_t}{m_p} = \frac{Pt}{v_c^2 m_p} = 20.7. \tag{29}$$

Table 1. Tether sling examples

Transfer	v (km/s)	v^*	m_{prop}/m_p	m_t/m_p	A/m_p (m^2/kg)	l_t (km)
LEO to Mars	3.55	1.81 (1.42)†	2.34	82.8 (18.3)	1.52 (0.540)	257
LEO to GEO	2.37	1.21 (0.950)	1.24	8.35 (3.41)	0.153 (0.101)	114
Moon to Mars	3.28	1.67 (1.32)	2.05	47.1 (12.3)	0.862 (0.364)	220
Moon to LEO	2.52	1.28 (1.01)	1.35	10.9 (4.21)	0.201 (0.124)	130
Phobos to Earth	1.88	0.957 (0.754)	0.894	3.49 (1.68)	0.151 (0.116)	72.1
Deimos to Earth	1.91	0.972 (0.766)	0.914	3.68 (1.76)	0.159 (0.121)	74.5

†Parentheses denote a graphite tether.

Assuming the power per area of a typical silicon solar cell (in LEO) to be 61 W/m^2 [7] we find from eqn (22)

$$\frac{At}{m_p} = 1.31 \times 10^6 \text{ m}^2\text{s/kg}. \tag{30}$$

Thus, if 10 days are allowed to spin the system up, a solar array area of approximately 1.52 m^2 is required for each kilogram of payload that is propelled. The final design parameter, the length of the tether, is determined from the maximum acceleration, a_{max}, by

$$l_t = \frac{v^2}{a_{max}}. \tag{31}$$

So, for a maximum acceleration of 5gs, the resulting length of the tether is 257 km.

If, on the other hand, a transfer from LEO to GEO is desired ($v = 2.37$ km/s), the non-dimensional velocity is 1.21, yielding a mass ratio of 8.35 and a non-dimensional energy of 2.09. This results in

$$\frac{At}{m_p} = 1.32 \times 10^5 \text{ m}^2\text{ s/kg}. \tag{32}$$

So a solar array area of approximately 0.153 m^2 is needed per kilogram of payload for a 10 day spin-up. The tether length resulting in a 5g acceleration is 114 km.

These two examples are summarized in Table 1 along with sling facility designs for other selected transfers. Note that the solar cell power per area output at the Phobos and Deimos facilities is 26 W/m^2 due to the increased distance from the sun. Also note that all of the cases discussed so far assume a Kevlar tether, a 10 day spin-up time and a 5g maximum acceleration. However, variations on these parameters are easily calculated from the non-dimensional energy and eqn (32).

Let us now examine variations on the first example, the LEO to Mars transfer. First, consider a tether made with Hercules IM7 graphite which has an ultimate strength $\sigma = 4.82$ GN/m^2 and density $\rho = 1.55$ g/m^3. This has a characteristic velocity of 2.49 km/s and a mass ratio of 18.3. The required solar array area becomes 0.540 m^2/kg (for a 10 day spin-up). The energy requirements are calculated for all the transfers using a graphite tether and are listed in parentheses in Table 1. Next, changing the spin-up time to 1 day results in a solar array area of 15.2 m^2/kg. Finally, a maximum acceleration of 10gs results in a tether length of 129 km.

We note that a modest improvement in the tether characteristic velocity results in a great improvement in mass ratio. The surprisingly small mass ratios at Phobos and Deimos suggest an important transportation alternative for the colonization of Mars.

5. CONCLUSION

Using the taper relation developed here, the tether sling can be designed to launch payloads at any speed. This makes space tethers an attractive alternative to chemical rockets for any type of mission, from low energy orbit transfers to very fast interplanetary trajectories. The orbiting tether facility is extremely efficient in missions where many spacecraft launches are involved. Such missions are being proposed to eliminate the possibility of single-point failures. Tether facilities on moons and asteroids could form a launch network for the exploration of the solar system. In particular, tether slings on the Moon and Phobos (or Deimos) could provide a conveyor belt for Mars colonization.

The great advantages of the solar-powered tether sling over chemical rockets are simplicitly and reusability. The performance of the sling is dramatically improved by modest improvements in the strength of density ratio to tether materials. In recent years we have witnessed tremendous advances in the strength of materials, and if this trend continues the tether sling will become an extremely competitive option in space transportation.

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