AUTOMATED DESIGN OF GRAVITY-ASSIST TRAJECTORIES TO MARS AND THE OUTER PLANETS

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(Received 7 August, 1990; revised 25 March, 1991; accepted 5 August, 1991)

Abstract. In this paper, a new approach to planetary mission design is described which automates the search for gravity-assist trajectories. This method finds all conic solutions given a range of launch dates, a range of launch energies and a set of target planets. The new design tool is applied to the problems of finding multiple encounter trajectories to the outer planets and Venus gravity-assist trajectories to Mars. The last four-planet grand tour opportunity (until the year 2153) is identified. It requires an Earth launch in 1996 and encounters Jupiter, Uranus, Neptune, and Pluto. Venus gravity-assist trajectories to Mars for the 30 year period 1995–2024 are examined. It is shown that in many cases these trajectories require less launch energy to reach Mars than direct ballistic trajectories.

Key words: Gravity-assist trajectories, flyby missions, grand tour, Earth-Mars trajectories, orbit mechanics.

1. Introduction

Gravity-assist trajectories (Broucke 1988) are modeled at JPL (the Jet Propulsion Laboratory) for a wide variety of solar system missions. Among these are missions to the outer solar system such as the 1977 Voyager Grand Tour. Also of much current interest are missions to the outer solar system preceded by multiple flybys of some subset of the inner planets such as VEGA (Venus-Earth Gravity Assist) and VEEGA (Venus-Earth-Earth Gravity Assist) trajectories which are discussed by Diehl and Myers (1987). Another important application of gravity assist has been in planetary satellite tours such as those which will be flown by the Galileo spacecraft (Diehl *et al.*, 1983; Longuski and Wolf 1986) through the Jovian system and the Cassini spacecraft at Saturn. In this paper an improved algorithm (Williams 1990) is described which systematically searches for all conic solutions in such a way that the entire solution space (all permutations) for a desired class of multiple encounter missions may be generated. As examples the new design tool is applied here to the problems of searching for outer planet trajectories and Venus gravity-assist trajectories to Mars.

2. Vis Viva Matching

The central problem in 'patching' together trajectories between targets of a multiple encounter mission is that of matching incoming conditions (Lambert solutions in

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Celestial Mechanics and Dynamical Astronomy **52**: 207–220, 1991.

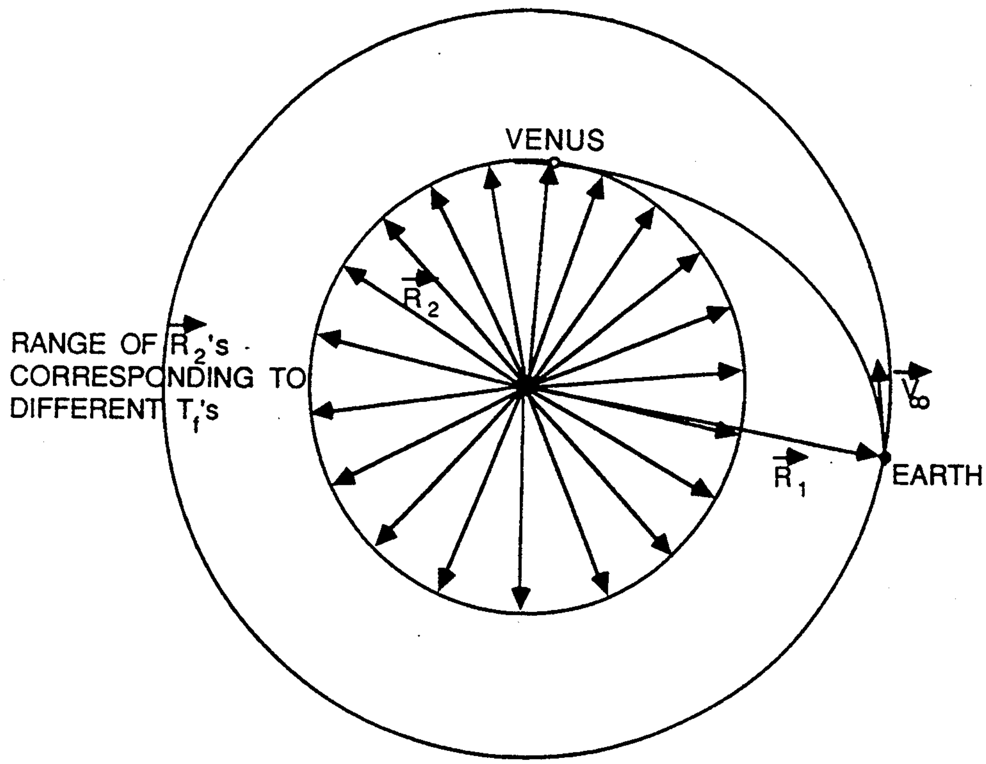

Fig. 1. Problem of finding an Earth to Venus trajectory.

this case) of a previously selected trajectory with departure conditions to the next target planet. We will refer to this procedure as *vis viva matching*. A discussion of Lambert's problem and patched-conic techniques is provided by Battin (1987).

2.1. The Basic Problem

A standard method for searching for interplanetary trajectories is illustrated in Figure 1. The mission designer first selects a launch date and launch energy in the form of a V_∞. When performing a *flyby*, these initial conditions (time of encounter and $\mathbf{V}_\infty$) are fixed by the arrival conditions of the trajectory on the previous 'leg'. Using the launch date or flyby date, a planetary ephemeris provides the initial position vector ($\mathbf{R}_1$). In order to obtain a Lambert solution, the designer must provide two other constraints. One constraint is the number of revolutions (m) the spacecraft is allowed to make about the central body. The other constraint is the time of flight (T_f) between consecutive encounters. In practice a range of discrete flight times is considered. One first specifies the number of revolutions about the central body the target planet is allowed to make. This establishes the range of flight times to be considered. For example, 0 target revolutions defines a flight time range of 0 to 1 target periods, 1 target revolution defines a range of 1 to 2 target periods, etc. If the target planet is Venus ($P = 225$ days), and if 'target revolutions = 0' is chosen, only Lambert solutions with flight times of 0 days to 225 days will be sought. If 'target revolutions = 1' is selected, solutions with flight times of 225 days to 450 days will be sought, and so forth.

The next step is to divide the range of flight times into equal increments. The planetary ephemeris is accessed again to compute the position vector of the target planet ($\mathbf{R}_2$) at the end of each increment. We now have several different sets of ($\mathbf{R}_1, \mathbf{R}_2, m$, and T_f) where $\mathbf{R}_1$ and m are the same in each set. The Lambert problem may then be solved for each of the sets.

2.2. The Vis Viva Curve

Each Lambert solution will imply a specific velocity vector at $\mathbf{R}_1$ from which one may obtain a V-infinity vector ($\mathbf{V}_\infty$) and therefore a required departure vis viva. To visualize this information, we can plot vis viva as a function of flight time for each of the discrete flight times. (Note: the vis viva is, of course, twice the energy and it is sometimes given the variable name C_3 where $C_3 = \mathbf{V}_\infty \cdot \mathbf{V}_\infty$.)

For different sets of launch and arrival dates we can associate specific vis viva values. As an example, the vis viva curve for the first leg of the Mariner Venus-Mercury mission is shown in Figure 2.

2.3. The Vis Viva Matching Problem

Since V_∞ is one of our initial conditions, shown here as target vis viva ($\mathbf{V}_\infty \cdot \mathbf{V}_\infty$) in Figure 2, we want to find a match, if one exists, between this initial (target) condition and our set of vis viva values. In these figures the desired Lambert solutions are those corresponding to the *intersection of each vis viva-curve and the target vis viva*. The algorithms which perform vis viva matching are described in detail by Williams (1990).

2.4. Existence of a Trajectory

Once a vis viva match is found, the existence of a trajectory is determined by comparing the incoming $\mathbf{V}_\infty$ from the previous leg with the outgoing $\mathbf{V}_\infty$ determined from the match. The angle between these two vectors determines the required 'bending angle'. The amount of bending the planet is able to provide (δ) is a function of the incoming velocity or $\mathbf{V}_\infty$, the gravitational parameter (μ), and the minimum flyby radius (R_p). The bending angle is

$$\delta = 2 \sin^{-1}[1/(1 + R_p V_\infty^2/\mu)] \,.$$

If the required bending angle is less than δ, a solution exists.

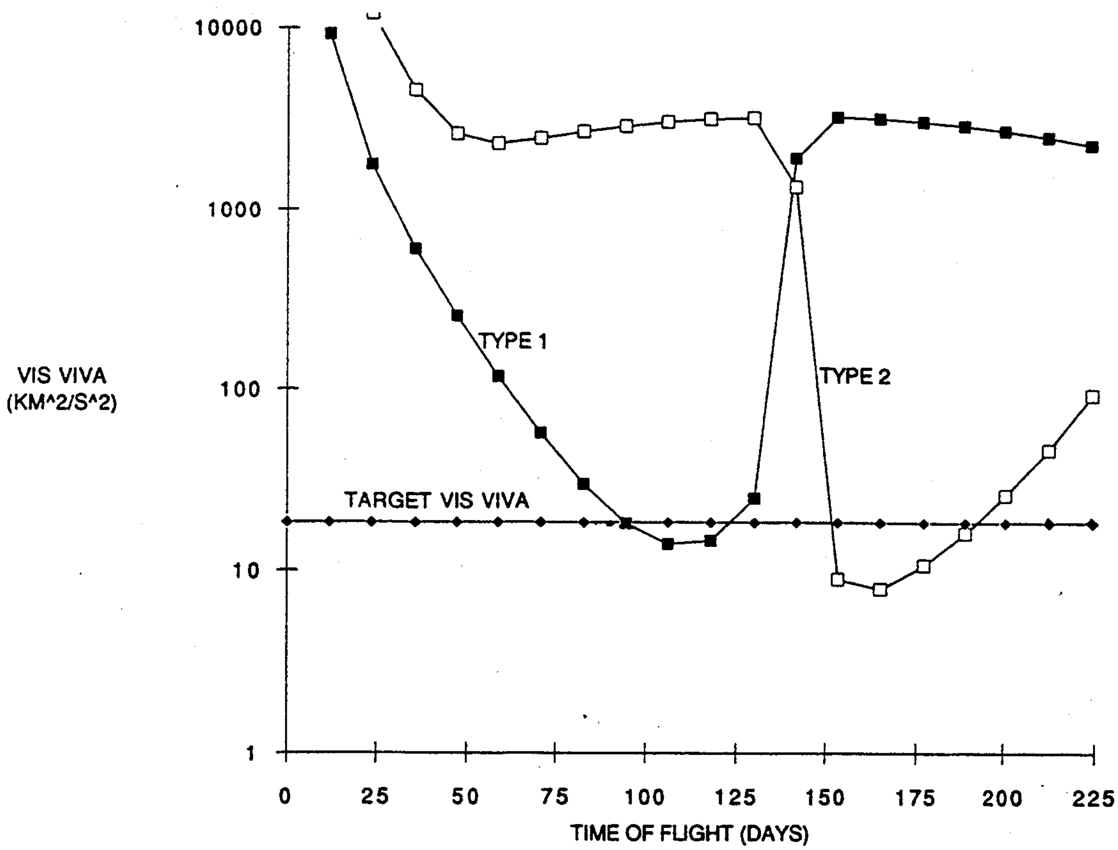

Fig. 2. Vis viva curves.

3. Automatic Search Routine

Some appreciation for the scope of the problem of designing multiple encounter trajectories can be obtained by considering Figure 3. This shows all the Lambert solutions for grand tour (Earth-Jupiter-Saturn-Uranus-Neptune-Pluto) opportunities having an Earth launch date of 20 August 1977 (the Voyager II launch date) and a launch V_∞ of 10.2 km/s. The four digit code (e.g. 5 0 0 1) indicates the target planet (5 = Jupiter), the number of spacecraft revolutions about the sun, the number of target revolutions about the sun, and the orbit number (selected from the available solutions). Most of the solutions have unreasonably long flight times and therefore do not represent interesting mission choices. In the figure, solid lines indicate missions with cumulative flight times less than 20 years, and the broken lines indicate those missions with longer than 20 year flight times. Note that only one of the two possible Earth to Jupiter trajectories is shown here. The other (5 0 0 2) contains an equally large 'solution tree'.

The best way to understand our new search algorithm would be to examine Table I which demonstrates the results obtained in searching for the 1977 Voyager Grand Tour. Table I summarizes the solutions found by the automated search

TABLE I
Results for variations of the 1977 Voyager Grand Tour.

EVENT	CODE	PLANET	h_p	θ	V_∞	PERIOD	r_p	r_a	C_3	T_f	ΣT_f
LAUNCH DATE = 8/20/1977 at 0: 0: 0 LAUNCH V-INFINITY = 10.20											
1	5 0 0 1	Earth	200	-18.26	10.20	2760.42	1.00	6.70		660.0	660.0
2	6 0 0 1	Jupiter	491547	4.96	8.39		5.07		87.8	737.3	1397.4
3	7 0 0 1	Saturn	73491	-27.80	11.81		9.57		272.7	1513.1	2910.6
4	8 0 0 1	Uranus	59080	135.43	15.99		14.66		352.2	1232.0	4142.6
5	9 0 0 0	Neptune			18.09	*Comment: This gets all the way to Neptune*					
1	5 0 0 1	Earth	200	-18.26	10.20	2760.42	1.00	6.70		660.0	660.0
2	6 0 0 2	Jupiter	1012867	-85.38	8.39	7985.44	5.17	10.46		4796.7	5456.8
		Saturn			5.55						
1	5 0 0 1	Earth	200	-18.26	10.20	2760.42	1.00	6.70		660.0	660.0
2	6 0 0 3	Jupiter	783550	90.18	8.39	7992.19	5.18	10.46		4812.7	5472.8
		Saturn			5.23						
1	5 0 0 2	Earth	200	-15.67	10.20	2615.17	1.00	6.42		684.9	684.9
2	6 0 0 1	Jupiter	565507	4.93	7.94		5.03		63.8	766.7	1451.6
3	7 0 0 1	Saturn	92058	-27.78	11.10		9.58		242.8	1567.1	3018.8
4	8 0 0 1	Uranus	73163	134.90	15.20		14.47		315.8	1278.6	4297.4
5	9 0 0 0	Neptune			17.22	*Comment: This gets all the way to Neptune*					

algorithm for a multiple encounter trajectory having the sequence 'Earth-Jupiter-Saturn-Uranus-Neptune-Pluto' with flight times less than 20 years. In the table heliocentric and planetary flyby conditions for each leg of each trajectory are provided in columns. Each line represents one 'leg' of the mission and includes the *event* number, the four digit *code* (see Figure 3), launch or flyby *planet*, planetary *flyby conditions* (altitude, h_p (km), θ (deg), V_∞ (km/s)), central body *orbit parameters* (period (days), periapsis, r_p (AU), and apoapsis, r_a (AU), or vis viva, C_3 (km^2/s^2), as appropriate), and *flight time*, T_f, and *total flight time*, ΣT_f (days). Note that the results are identical with Figure 3 and that the Voyager Grand Tour does not admit an encounter with Pluto within a flight time of 20 years.

4. Future Outer Planet Grand Tours

Wallace *et al.* (1981) describes a JPL study which catalogues different classes of multiple encounter outer planet mission opportunities between 1989 and 1995. Here we extend the search out to 2050. For the time period 1985–2050 the following trajectory paths were searched for solutions: E56789, E5679, E5689, E56879, E5789, E579, E589, and E59. Note that the nomenclature E56789 refers to a launch from Earth (E) and sequential encounters with Jupiter (5), Saturn (6), Uranus (7), Neptune (8) and Pluto (9). In the initial search, an Earth launch V_∞ of 12 km/s was used (Voyager II launched at 10.2 km/s). Also, launch dates were examined at 20 day intervals over the entire period for each path. Where regions of particular interest appeared, smaller intervals were examined.

A planetary alignment similar to that in 1977, allowing a repeat of the Voyager II trajectory, does not occur again until the year 2153. This can be verified by considering the planetary positions in Figures 4 and 5. These figures show the motion of the outer planets during two ten-year periods, which are separated by 176 years. During the latter period Pluto is unfortunately not in a favorable location. In fact, as will be shown, *the last opportunity to include Pluto in a four-planet grand tour mission occurs in 1996*. There are, however, periodic opportunities for fast (flight times less than a Hohmann trajectory) trips to Pluto as well as Uranus and Neptune which encounter fewer than four planets.

A search for additional opportunities of the classical (Voyager) grand tour, represented by the path E56789, was conducted for the period from 01/01/1965 to 01/01/2180. The simulations revealed that this opportunity does not occur again until the year 2153. Within the constraints of this analysis (launch V_∞ of 12 km/s, and the launch date intervals of 20 days), the actual date is 9 June 2153, which is 175.8 years after the Voyager II launch, and 186.0 years after the first possible opportunity in 1967. Clearly this family of trajectories is governed by the synodic period of Uranus and Neptune which is 172 years.

Having exhausted the possibility that a *five-planet grand tour* exists in the near future, we began searching for *four-planet* tours, involving only the outer planets. The only successful result was achieved with the path E5789 (Earth-

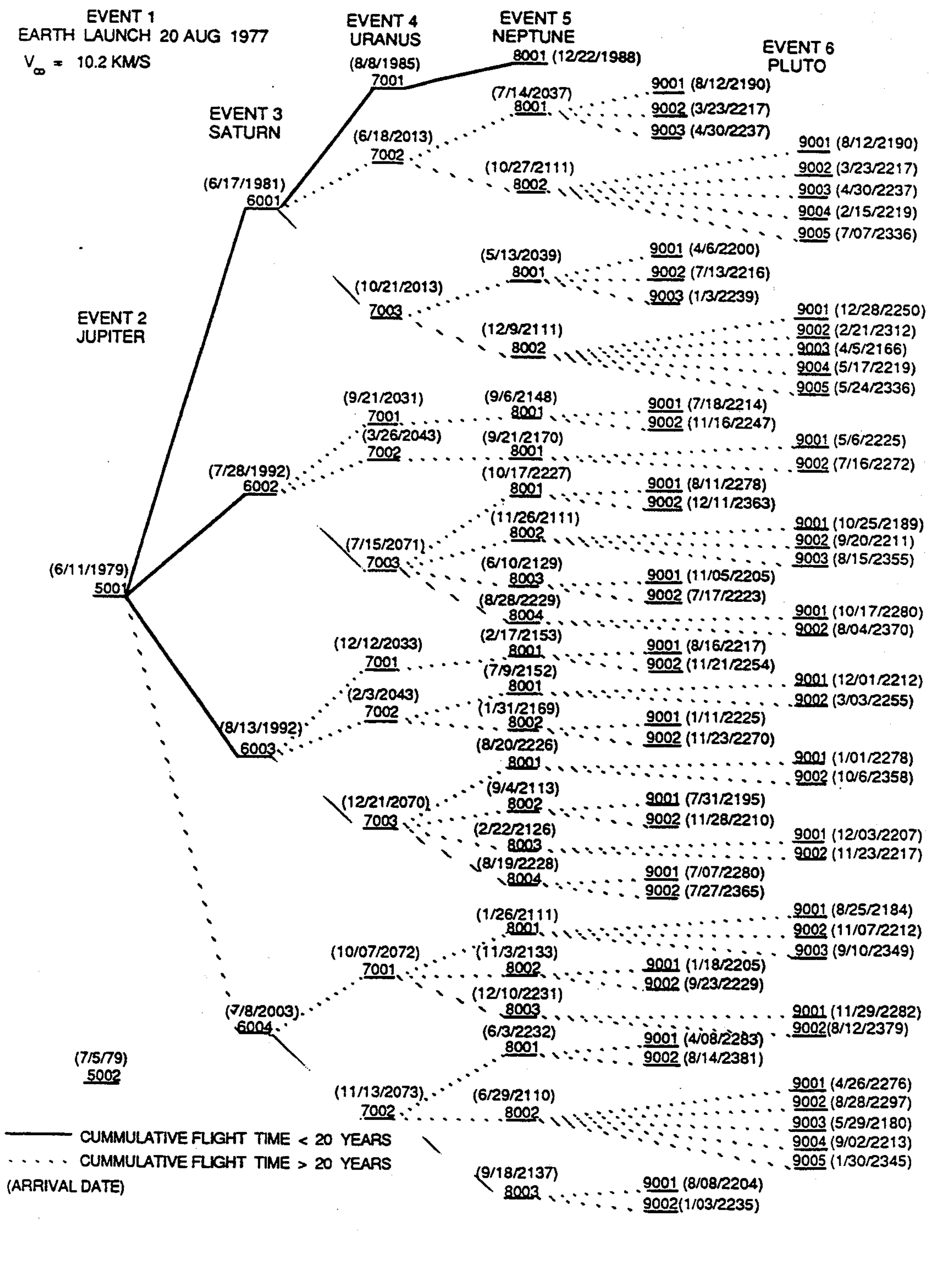

Fig. 3. All the conic solutions to the Classical Grand Tour (20 Aug 1977).

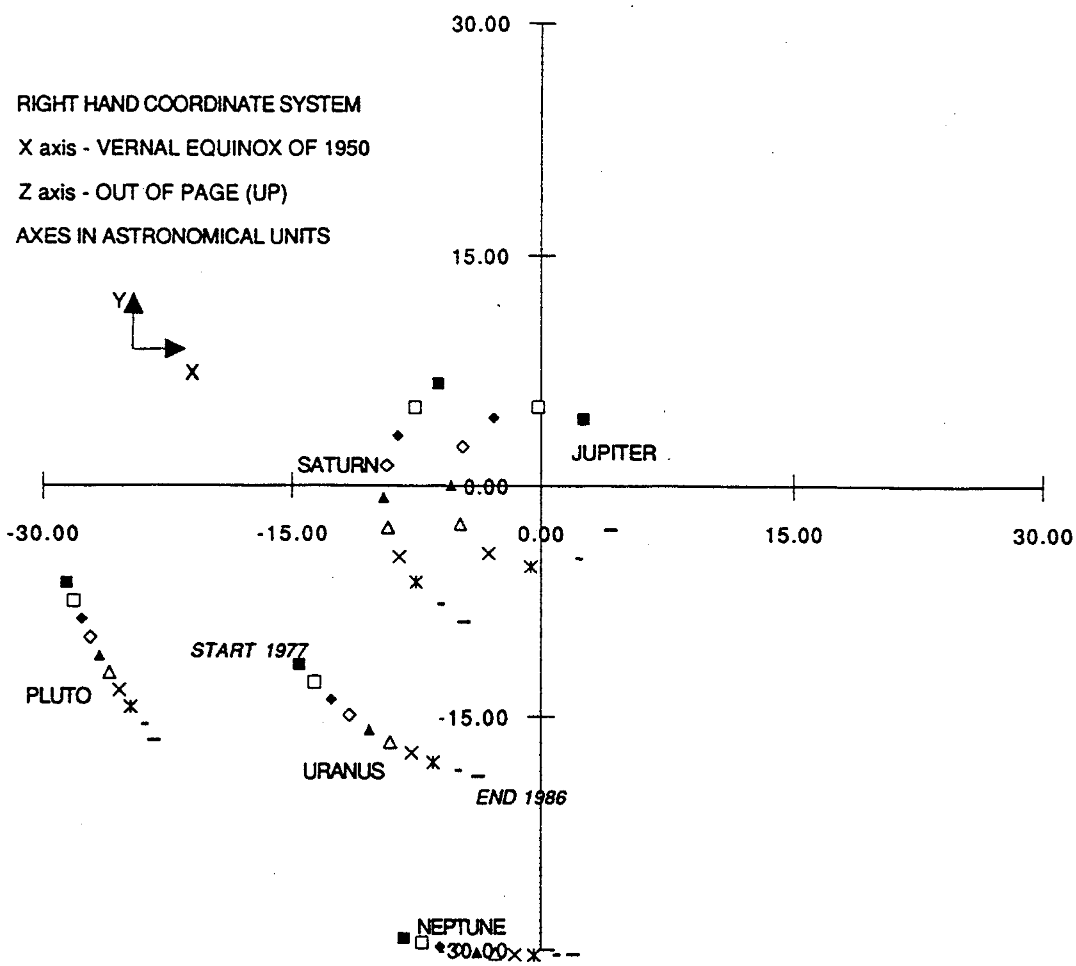

Fig. 4. Positions of the outer planets (1977–1986).

Jupiter-Uranus-Neptune-Pluto). As shown in Figure 6, *such a trajectory exists* in the year 1995.

In order to find the last possible launch opportunity to Pluto, a search was conducted for the years 1995, 1996 and 1997 for launch V_∞'s of 11, 12, 13, and 14 km/s. No trajectories were found to Pluto in 1997. The last launch date for a grand tour to Pluto occurs in 1996 as shown in Figure 7. As illustrated in the figure, the minimum time of flight to reach Pluto is about 7400 days (20 years). Since Pluto is the only planet which has not been explored by spacecraft, it is interesting to consider gravity-assist trajectories which provide shorter flight times. By considering the two planet gravity-assist path, E59, it was found that *fast trajectories to Pluto* exist every 13 years. The use of Jupiter as the main gravity-assist planet provides short flight times of about 2500 days (7 years) at regular intervals, as shown in Figure 8.

Several other multiple-encounter opportunities with the far outer planets were discovered. The results are summarized in Table II. The last four-planet grand tour

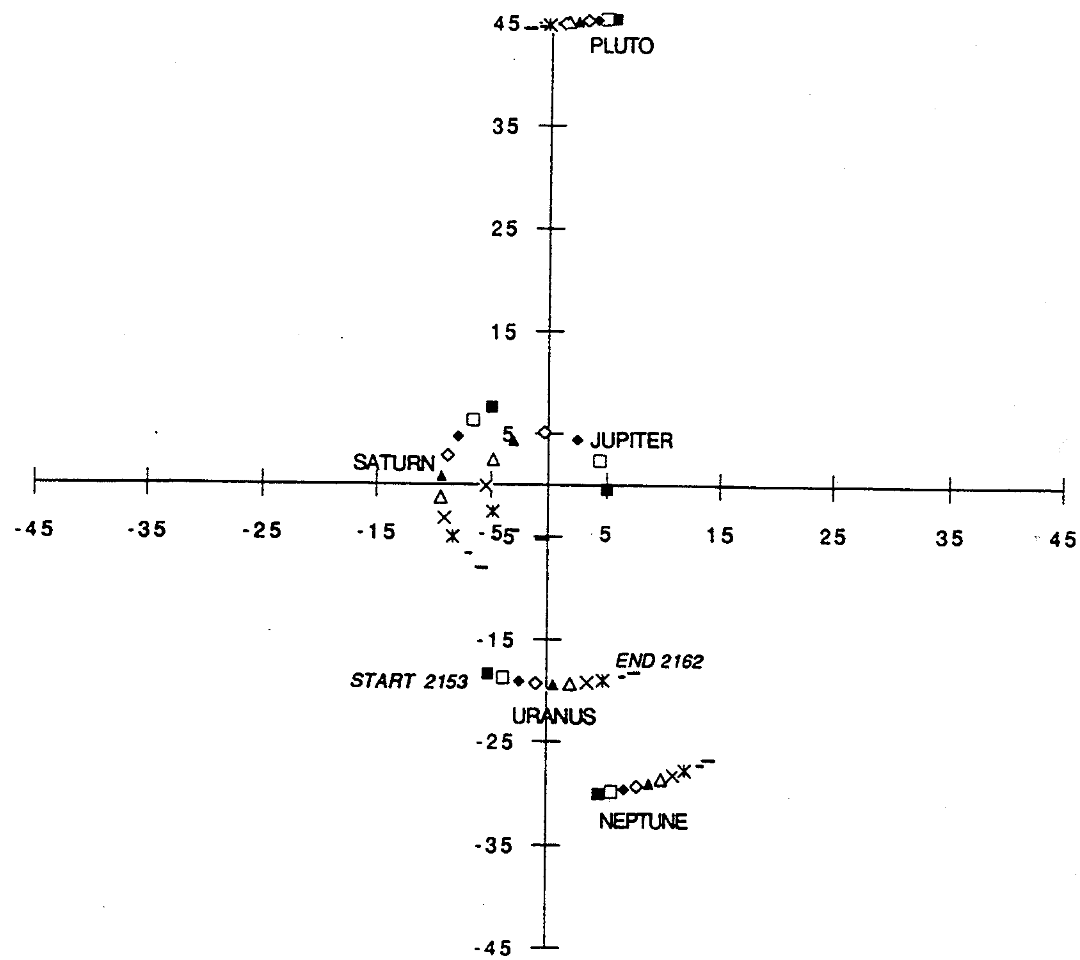

Fig. 5. Positions of the outer planets (2153–2162).

opportunity (until the year 2153) occurs in 1996. There are several three-planet opportunities involving the giant planets during the next half century. Fast trajectories to Uranus, Neptune and Pluto using Jupiter for gravity assist are available every 13–14 years.

5. A Comparison of Mars Trajectory Mission Modes

In recent years there has been much discussion within the international aerospace community concerning future goals in space. One of the most ambitious goals which has been proposed is the establishment of a permanent human presence on Mars. Should this be pursued it will require a substantial supporting infrastructure in low Earth orbit and a 'continuous pipeline' of material between Earth and Mars. This will initially include scientific precursor missions to map the Martian surface, study weather patterns, and identify prospective landing sites. Later, in addition to transporting people, the pipeline will include other types of cargo such as base elements, consumables, and scientific equipment.

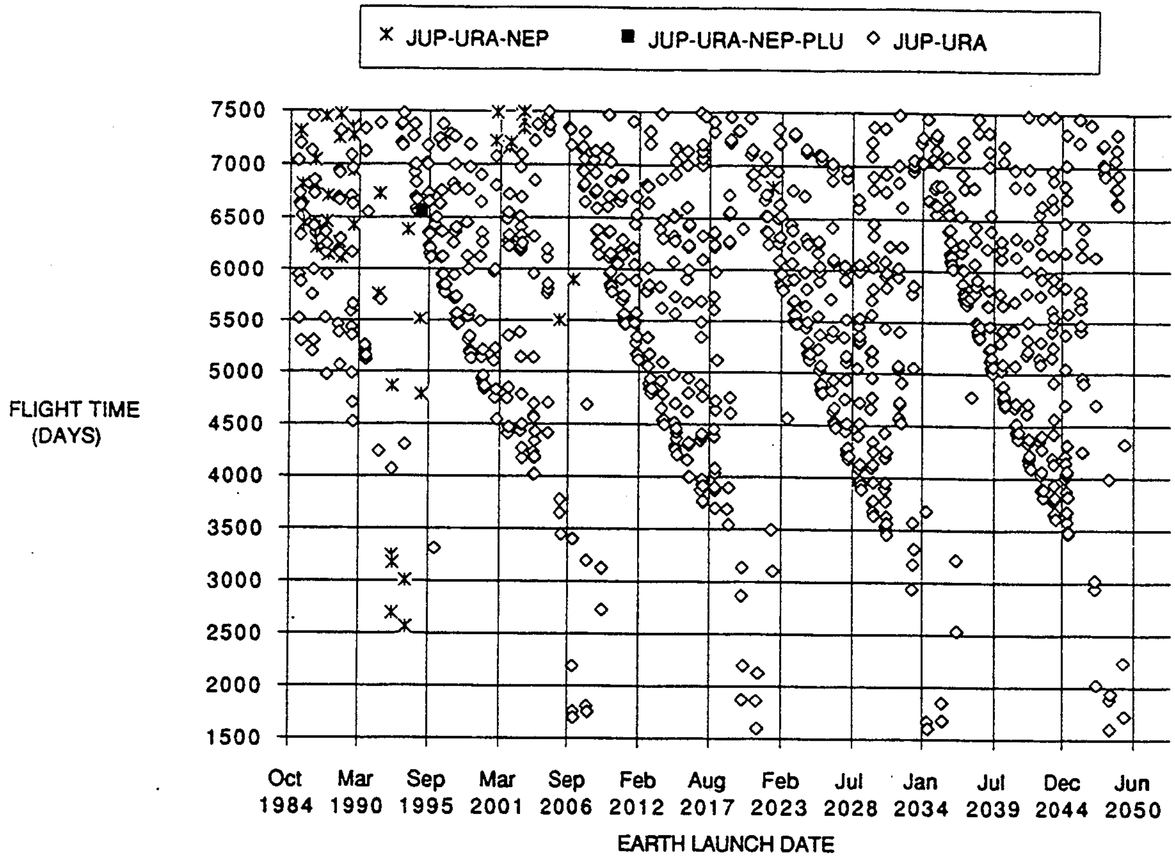

Fig. 6. Jupiter-Uranus-(Neptune)-(Pluto) trajectories (1985–2050).

TABLE II
Multiple encounter opportunities with far outer planets.

Launch	Planets	Flight Time (days)	Comments
1996	Jupiter-Uranus-Neptune-Pluto	7400	Last opportunity for four planets
1993–1995	Jupiter-Uranus-Neptune	2500	Three-planet opportunity
1998	Jupiter-Saturn-Uranus	5000	" " "
2005–2007	Jupiter-Uranus-Neptune	4700	" " "
2016–2019	Jupiter-Saturn-Neptune	2500	" " "
2021–2022	Jupiter-Uranus-Neptune	6500	Last opportunity for E578
2036	Jupiter-Saturn-Uranus	3800	Three-planet opportunity
2037	Jupiter-Saturn-Uranus	4200	" " "
Every 14 years	Jupiter-Uranus	1500	Two-planet opportunity
Every 13 years	Jupiter-Neptune	2500	" " "
Every 13 years	Jupiter-Pluto	2500	" " "

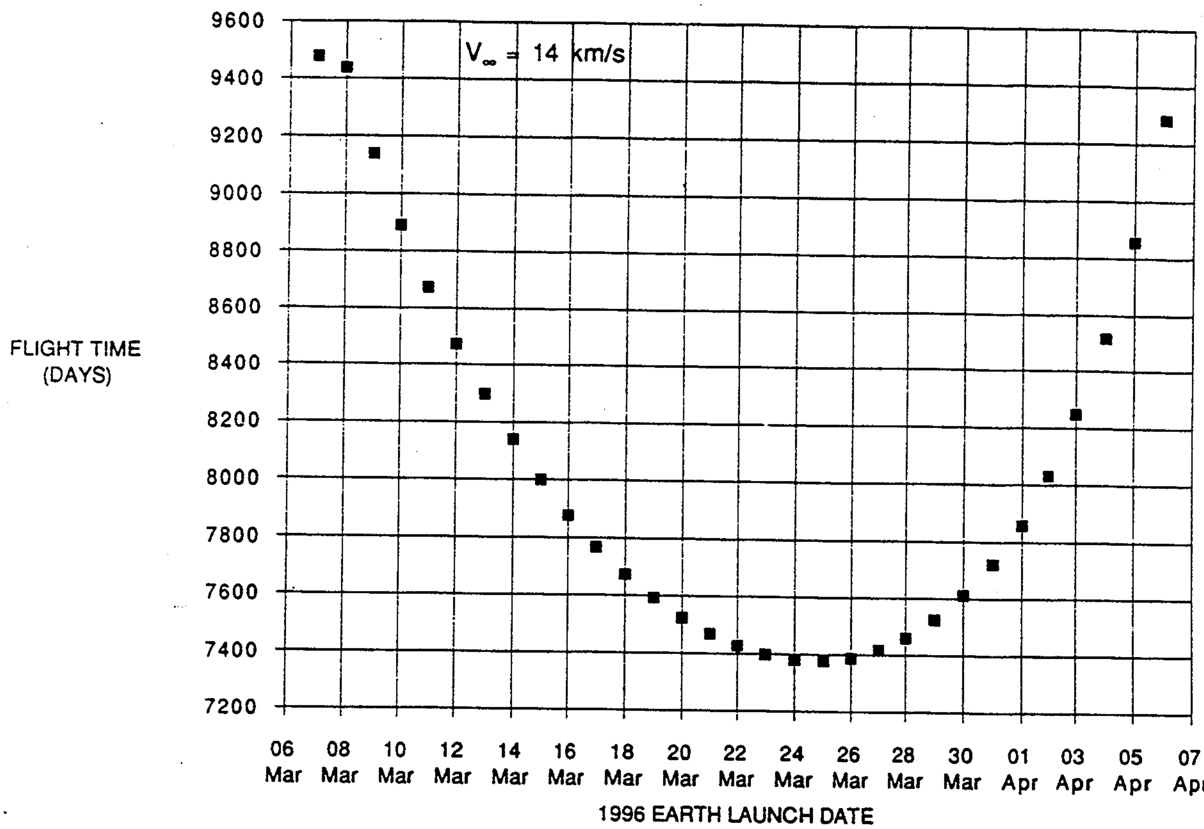

Fig. 7. Jupiter-Uranus-Neptune-Pluto trajectories (1996).

Because of the harmful physiological effects of zero-g and extended exposure to cosmic radiation, it is likely that the only acceptable mode for transporting humans will be the fastest practical trajectory. This restriction does not, however, apply to unmanned scientific or cargo missions. For this reason, it is prudent to examine other mission modes which may require the expenditure of less propellant and therefore reduce expensive earth-to-orbit launch requirements.

Under ideal conditions the V_∞ required at Earth to get to Mars is about 3 km/s while that required to go from Earth to Venus is only about 2.5 km/s. This difference motivated a search for Venus gravity- assist trajectories to Mars. Both ballistic and VEGA (Venus-Earth Gravity-Assist) trajectories to Mars (E234) were computed for a 30 year period beginning in 1995. Only trajectories requiring a launch vis viva less than 12.25 km^2/s^2 are plotted. Venus swingby trajectories (with one Venus flyby and no Earth flyby, E24) were also computed for the entire 30 year period. Under these conditions Venus swingby solutions were found only in the year 2015. Also, the minimum launch vis viva requirement for the Venus swingby solution is somewhat larger (11 vs 9 km^2/s^2) than a VEGA in the same time frame. As an example, results of the ten year period, 2005–2015, are shown in Figure 9.

Several conclusions can be drawn. First, in many cases VEGA trajectories offer a smaller Earth launch energy requirement to reach Mars than a direct ballis-

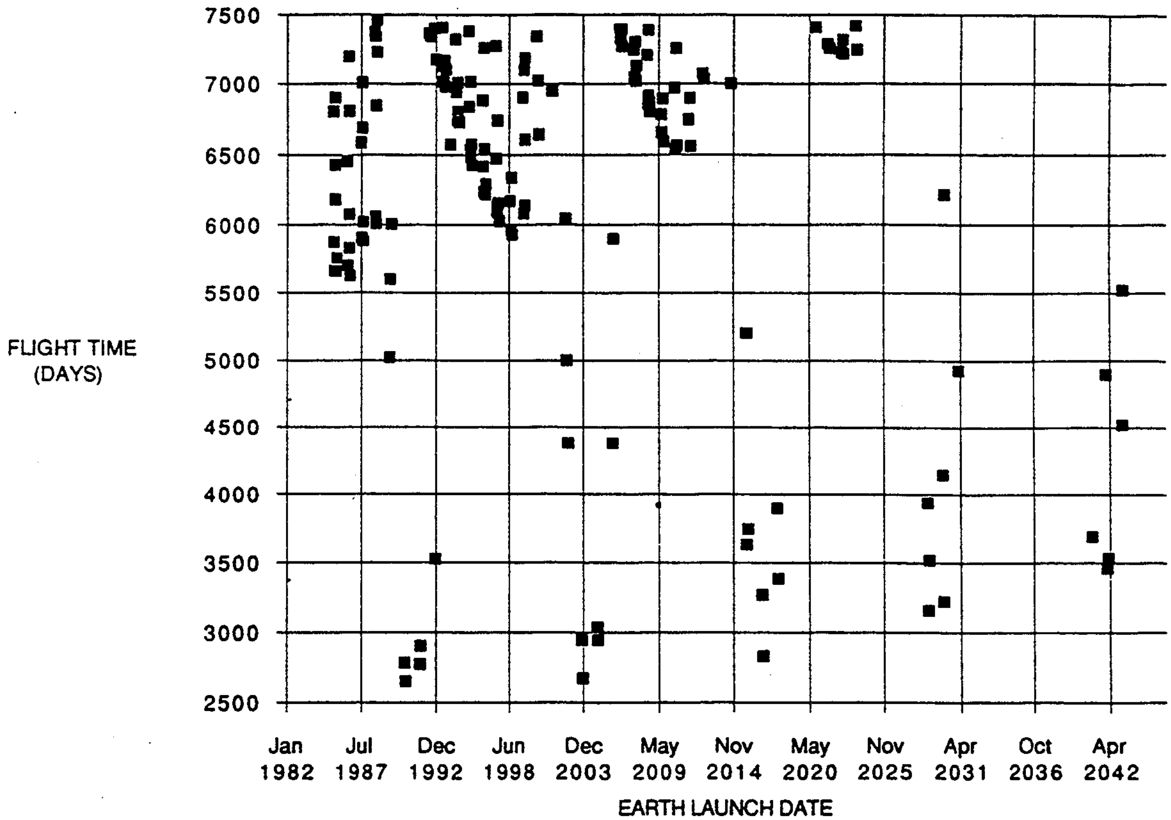

Fig. 8. Jupiter-Pluto trajectories (1985–2050).

tic trajectory. The earliest year in which a VEGA trajectory has a lower launch energy requirement than adjacent ballistics is in 2002. There is a four year gap between 2004 and 2008 in which neither mission mode is available (for vis viva < 12.25 km^2/s^2). VEGA trajectory launch energies in several years (2002, 2008, 2010, 2020, 2021, and 2023) are clearly less than nearby ballistic opportunities. Also, in several years (2004, 2012, and 2015), VEGA trajectories are comparable to nearby ballistics. In all cases, VEGA trajectories offer an additional set of launch windows which may be desirable for logistics reasons, or for the purpose of achieving different Mars arrival dates. Over the 30-year span the lowest ballistic launch energy occurred in 2018 (vis viva = 7.75 km^2/s^2) and the lowest VEGA in 2010 (vis viva = 7.59 km^2/s^2).

In addition to launch energy, several other parameters were examined in the study: arrival energy, flight time, and arrival date. These results are provided by Williams (1990).

6. Conclusions

We have discussed a tool with extraordinary potential for exploring new and exciting solar system missions. This automated design algorithm has been applied to several problems of current interest with some intriguing results.

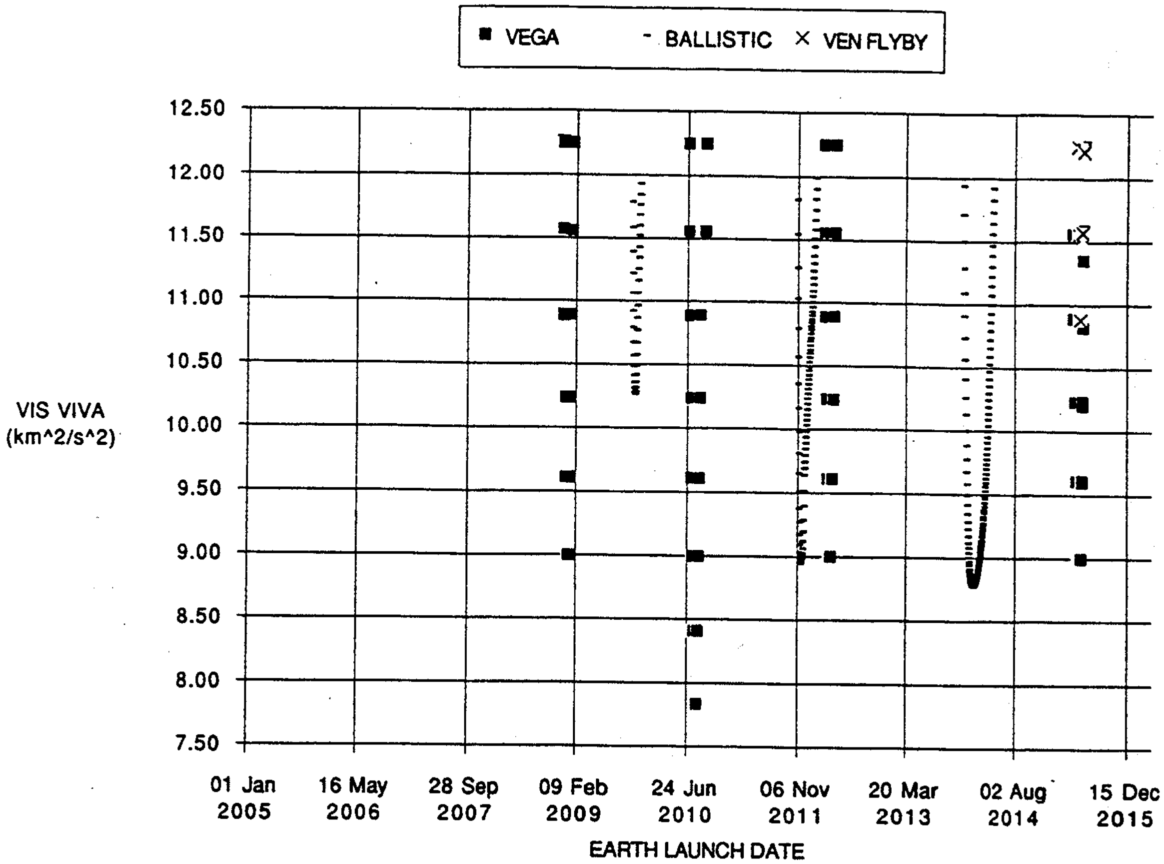

Fig. 9. Earth departure vis viva for Mars trajectories (2005–2015).

One important result is the identification of a large set of potential trajectories to the outer solar system spanning the next sixty years. Among these are: several three-planet tour opportunities; fast trajectories to Uranus, Neptune, and Pluto; as well as *the last opportunity until 2153 for a four-planet grand tour mission. This occurs in 1996 and includes Pluto.*

In the last few years there has been increasing interest in an expanded planetary exploration program possibly including human missions to Mars. With this in mind, VEGA (Venus-Earth Gravity-Assist) and Venus flyby trajectories to Mars were computed for the period 1995–2024 and compared to ballistic trajectories. In several years (2002, 2008, 2010, 2020, 2021, and 2023) VEGA *trajectories to Mars require less energy than nearby ballistic opportunities.* Of course the flight time and arrival energies are larger, but the lower energy launch windows may make these trajectories an attractive alternative for many future Mars missions.

Acknowledgements

The authors thank the following individuals for their assistance and continuing support: D.V. Byrnes, R.E. Diehl, E.A. Rinderle, R.S. Schlaifer, A.B. Sergeyevsky, D.L. Skinner, and D.S. Stetson. This work has been supported by National Aeronautics and Space Administration Grant NGT-50499 (NASA Grants Officer John E. Moore, NASA University Programs Manager John Lynch, JPL Program Administrators Harry I. Ashkenas and Carol L. Snyder, and JPL Technical Monitors Roger E. Diehl and Douglas S. Stetson). The results reported here could not have been obtained without the software and assistance provided by the Jet Propulsion Laboratory.

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