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## Design of Low-Thrust Gravity-Assist Trajectories to the Outer Planets

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**A Nuclear electric propulsion engine is being designed for the Jupiter Icy Moons Orbiter (JIMO) mission. In this paper, we consider other missions—to the outer planets—that could be achieved with this propulsion system. We incorporate gravity assists at Venus and Earth to deliver greater mass (to Jupiter, Saturn, Uranus, Neptune and Pluto) than achievable with the JIMO engine alone. For example, we have found a trajectory to Pluto via gravity assist with Earth and Jupiter that launches in the year 2015 and rendezvouses with Pluto in 9.7 years. We also investigate the trade off in which final mass increases as the time-of-flight increases for three families of trajectories to Pluto (Earth-Earth-Jupiter-Pluto, Earth-Venus-Earth-Jupiter-Pluto and Earth-Venus-Venus-Jupiter-Pluto).**

### Nomenclature

$a_0$	=	initial acceleration of the spacecraft, mm/s <sup>2</sup>
$g$	=	standard acceleration due to gravity, m/s <sup>2</sup>
$I_{sp}$	=	specific impulse, s
$\dot{m}$	=	propellant mass flow rate, mg/s
$m_f$	=	final spacecraft mass, kg
$m_0$	=	initial spacecraft mass, kg
$P$	=	power, kW
$R_J$	=	radius of Jupiter
$T$	=	engine thrust, N
$V_\infty$	=	hyperbolic excess speed, km/s
$\Delta V$	=	magnitude of a change in velocity, km/s
$\eta$	=	overall engine efficiency

### I. Introduction

**A**N ambitious mission to Jupiter is being planned—the Jupiter Icy Moons Orbiter<sup>1,2</sup> (JIMO) mission. Although the details are uncertain, the JIMO spacecraft will be massive (as much as 20,000 kg after escape from Earth) and will use nuclear electric propulsion<sup>3</sup> (having a total engine power of about 100 kW) with a high specific impulse (about 6,000 s). The launch is expected in 2011.<sup>1</sup> Upon arrival at Jupiter, the spacecraft will spiral into orbit around Callisto, then Ganymede, and finally Europa—the Icy Moons of Jupiter.

If such a potent propulsive system is built, then a natural question to ask is what other missions might be flown. In this paper we will focus not on the Jupiter Icy Moons Orbiter Mission, but instead on innovative missions to the outer planets that are made possible by this new propulsion technology.

Our principal goal is to consider how a massive nuclear electric propulsion (NEP) engine, used in conjunction with gravity assists from the inner planets, can deliver a spacecraft to the outer planets (Jupiter, Saturn, Uranus,

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Neptune and Pluto). The technique of reaching the outer planets via gravity assists (but without electric propulsion) has been discussed for many decades beginning in the 1960’s with the pioneering work of Minovitch,<sup>4,5</sup> Flandro,<sup>6</sup> Deerwester,<sup>7</sup> and Niehoff.<sup>8</sup> In the 1970’s, additional contributions and applications of the gravity-assist technique were developed by Hollenbeck,<sup>9</sup> Stancati et al.<sup>10</sup> and Wallace.<sup>11</sup> In the 1980’s, Wallace et al.<sup>12</sup> and Diehl and Myers<sup>13</sup> presented detailed trajectory computations that used gravity assist to reach the outer solar system. An analysis of the efficacy of delta-V gravity-assist trajectories and a collection of practical results are discussed in the doctoral thesis of Sims.<sup>14</sup>

As early as the 1970’s, the use of electric propulsion in conjunction with a gravity-assist from the Earth was investigated by Meissinger,<sup>15</sup> and Atkins et al.<sup>16</sup> In 1979, Sauer<sup>17</sup> described how solar electric propulsion (SEP) combined with an Earth gravity assist could deliver spacecraft to the outer planets. More recent work was published by Kawaguchi,<sup>18</sup> Sauer,<sup>19</sup> Williams and Coverstone-Carroll,<sup>20</sup> and Maddock and Sims.<sup>21</sup> In solar electric propulsion missions, the low thrust trajectory arcs are restricted to a range of only a few astronomical units (AU) from the Sun. Beyond a few AU, the available power drops rapidly and becomes insufficient to operate the engines.

The advantage of NEP is that power is always available, although at a cost of additional hardware mass. NEP missions are discussed by Jones and Sauer,<sup>22</sup> Kluever,<sup>23</sup> and Fedotov et al.<sup>24</sup>

In 1999, Sims and Flanagan<sup>25</sup> developed a parameter optimization technique in which a low-thrust gravity-assist (LTGA) trajectory could be modeled as a series of impulsive  $\Delta V$ s patched together by conic arcs. Their method can be used to find optimal SEP and NEP gravity-assist trajectories with maximum final mass. Based on their technique, an LTGA optimization program was written called the Gravity-Assist, Low-Thrust, Local Optimization Program<sup>26</sup> (GALLOP). Also in 1999, Petropoulos et al.<sup>27</sup> developed a shape-based method to represent low-thrust gravity-assist trajectories in a broad search, propagation program (STOUR-LTGA). Petropoulos and Longuski<sup>28,29</sup> demonstrated how LTGA trajectories could be automatically designed to provide preliminary guesses to GALLOP. This method is described in Petropoulos’ doctoral thesis.<sup>30</sup> An extension of the LTGA trajectory design work was presented by McConaghy et al.<sup>26</sup> in 2003. In that same year, low-thrust trajectories to the outer solar system were presented by Cupples et al.,<sup>31</sup> Woo et al.,<sup>32</sup> and Vasile et al.<sup>33,34</sup>

In the current work, we employ the concepts of the aforementioned researchers to design LTGA rendezvous missions to the outer planets. We assume the spacecraft uses an engine similar in capability to the JIMO engine.

## II. Assumptions

We consider a spacecraft launched with a fictitious launch vehicle, which we will refer to as “the heavy lifter.” The upper stage of the heavy lifter injects the spacecraft into an interplanetary escape orbit with chemical propellant having an  $I_{sp}$  of 404 s. We assume the upper stage of the vehicle is able to inject a spacecraft with a mass of 20,000 kg to parabolic escape (i.e.  $V_\infty = 0$ ) or a lesser mass to hyperbolic speed ( $V_\infty > 0$ ). Figure 1 shows the injection capability of this hypothetical launch vehicle. As we can see in Fig. 1, the heavy lifter capability is 115% greater than that of the Boeing Delta 4050H-19\* launch vehicle.

We study cases where a spacecraft rendezvous with the outer planets. By rendezvous with the outer planets, we mean the spacecraft’s position and velocity match that of the target planet (i.e. the arrival  $V_\infty$  is 0). As a potential follow-on mission to JIMO, we consider the launch years 2014–2025. For the purpose of demonstrating mission feasibility, we select parameters based on the currently developing Nuclear Electric Xenon Ion System<sup>3</sup> (NEXIS). See Table 1. We assume (somewhat arbitrarily) that an acceptable final-mass-to-initial-mass ratio could be as low as 45%.

**Table 1: Nuclear Electric Spacecraft Parameters**

Parameter	Values
Power Available to the Thrusters , $P$	95 kW <sup>a</sup>
Specific Impulse, $I_{sp}$	6,000 s
Overall Efficiency, $\eta$	70 %
Thrust, $T$	2.26 N
Mass Flow Rate, $\dot{m}$	38.4 mg/s
Injected Mass at Zero Launch $V_\infty$ , $m_0$	20,000 kg

<sup>a</sup> The spacecraft is propelled by five NEXIS thrusters.

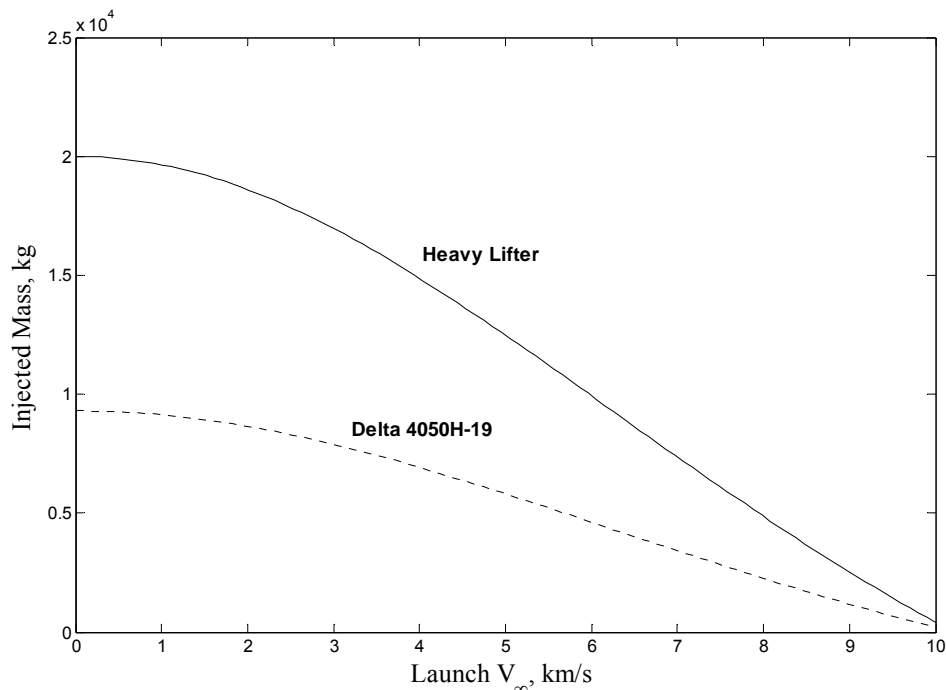
\* *The Delta IV Payload Planners Guide*, The Boeing Company, Huntington Beach, CA, 2000.

Although these parameters are rather specific, we note that they can be scaled to smaller (and even larger) values. Two scaling parameters govern the problem: the initial acceleration of the spacecraft and the specific impulse of the engine. The initial acceleration  $a_0$  and the specific impulse  $I_{sp}$  can be written as:

$$a_0 = T / m_0 = (2\eta P) / (m_0 g I_{sp}) \quad (1)$$

$$I_{sp} = T / (\dot{m} g) \quad (2)$$

For missions with the same values of  $a_0$  and  $I_{sp}$ , the resulting trajectories will be the same as the ones we present in this paper. (In our problem,  $I_{sp} = 6,000$  s and  $a_0 = 0.11$  mm/s<sup>2</sup>.)



**Figure 1: Launch capability of the heavy lifter, a fictitious launch vehicle, compared to the Delta 4050H-19.**

### III. Methodology for LTGA Trajectory Design

#### A. Broad Search

As a first step, we perform broad searches of low-thrust trajectories using a shape-based, automated approach as described in Refs. 27-30. The shape-based method provides an analytic representation of the low-thrust trajectory arc, which allows rapid searches over a wide range of the design space. We assume a two-body model, with coast and thrust arcs patched together between flybys of celestial bodies. We employ conic sections for coasting arcs and exponential sinusoids<sup>27</sup> for thrusting arcs. The polar equation of an exponential sinusoid is given by  $r = k_0 \exp[k_1 \sin(k_2 \theta + \varphi)]$ ; where  $k_0$ ,  $k_1$ ,  $k_2$ , and  $\varphi$  are constants. The required thrust acceleration at each point of the trajectory can be determined from the trajectory shape.

The shape-based approach is implemented as software named STOUR-LTGA. To perform a broad search in STOUR-LTGA, the user specifies the encounter sequence together with the launch date and launch  $V_\infty$  ranges and step sizes. Numerous trajectories can be analyzed and selected by a MATLAB toolbox called STOUR Interactive Toolbox (SIT). In general, we select trajectories with low propellant mass fraction, low TOF, low launch  $V_\infty$  and acceptable thrust acceleration.

## B. Optimization

Candidate trajectories selected from the broad search are used as the initial guesses for a low-thrust gravity-assist trajectory optimization program called GALLOP.<sup>26</sup> The trajectory model in GALLOP divides each planet-planet leg of the trajectory into segments of equal duration. (For a direct mission without gravity assist, there is only one leg.) The thrusting on each segment is modeled by an impulse at the midpoint of the segment, with conic arcs between the impulses. Each leg is propagated half-way forward from the initial body and half-way backward from the final body. In order to have a feasible trajectory, the forward- and backward-propagated half-legs must meet at a match-point time in the middle of the leg.

The optimization variables in GALLOP include the following: 1) the impulsive  $\Delta V$  on each segment, 2) the Julian dates at the launch, flyby, and destination bodies, 3) the launch  $V_\infty$ , 4) the incoming inertial velocity vectors at all of the postlaunch bodies, 5) the spacecraft mass at each body, 6) the flyby periapsis altitude at the gravity-assist bodies, and 7) the B-plane angle at the gravity-assist bodies.

The optimization program can alter these variables to find a feasible and optimal solution of the given problem. A feasible solution means the variables satisfy the constraints. These constraints include upper bounds on the impulsive  $\Delta V$  on each of the segments, the launch- $V_\infty$  magnitude, and the encounter dates at the bodies. Within the feasible set of solutions, the optimizer can find a solution which maximizes the final mass of the spacecraft.

## IV. Numerical Results

### A. LTGA Trajectory to Jupiter via Mars Gravity Assist

To illustrate the shape-based approach, we study an Earth-Mars-Jupiter mission with a zero-launch- $V_\infty$  constraint. We begin with a broad search in STOUR, with launch dates from Jan 2011 to Dec 2023 and a 20-day step size. Since we are interested in finding a candidate trajectory with thrust acceleration close to the parameters given in Table 1, we focus on trajectories with a small acceleration. For missions with the zero-launch- $V_\infty$  constraint, we also select trajectories based on their launch  $V_\infty$ . Figure 2 shows 5,424 trajectories with maximum acceleration less than  $0.15 \text{ mm/s}^2$ . (For the parameters in Table 1, the initial acceleration is  $0.11 \text{ mm/s}^2$ .) Trajectories with the smallest launch  $V_\infty$  ( $0.2 \text{ km/s}$ ) are also indicated by circles in Fig. 2. (We note that STOUR cannot constrain the launch  $V_\infty$  to be exactly zero.) We observe promising groups of trajectories that launch around 2019 to 2022.

Next, we perform a refined search of E-M-J trajectories from Jan 2019 to Dec 2022, with a step size of 10 days. Figure 3 shows the resulting trajectories with maximum acceleration less than  $0.15 \text{ mm/s}^2$ . A candidate trajectory with low propellant mass fraction and reasonable TOF (indicated by the arrow in Fig. 3) is selected as an initial guess for optimization in GALLOP. From Fig. 4, we can see that the trajectory of the initial guess and the optimal solution are similar in their shape. Table 2 gives a comparison of the trajectory characteristics of the STOUR initial guess and the optimized solution in GALLOP. We notice that the optimizer moved the Mars encounter date almost four months later.

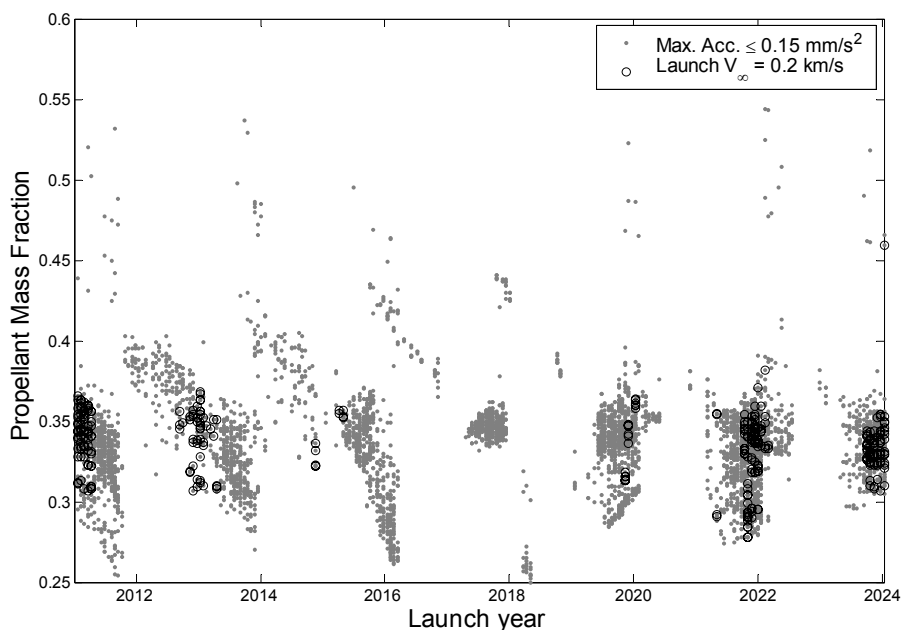
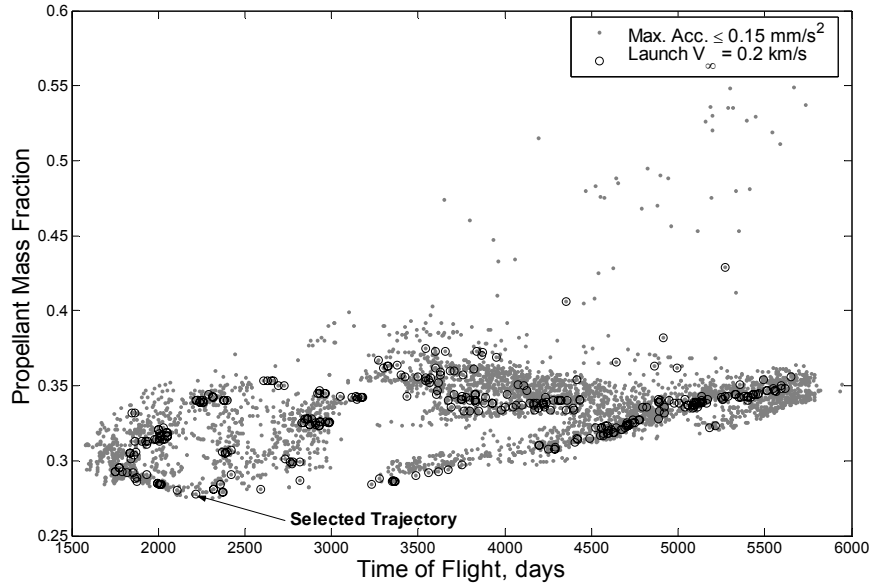
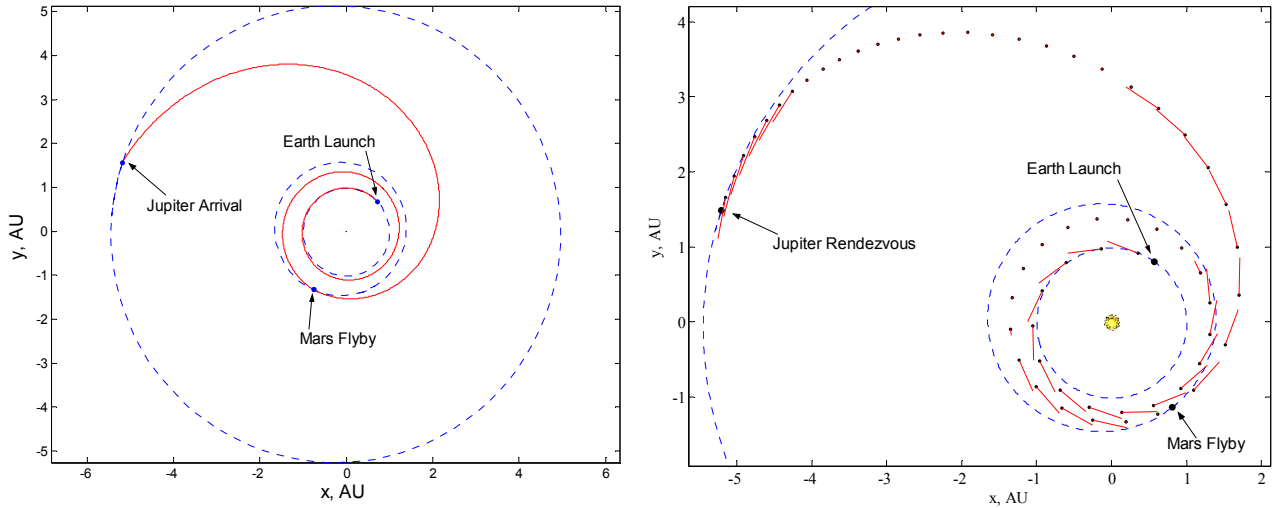


Figure 2: STOUR-LTGA broad search for Earth-Mars-Jupiter trajectories; launch date ranges from the year 2011 to 2023.



**Figure 3: STOUR-LTGA refined search for Earth-Mars-Jupiter trajectories; launch date ranges from the year 2019 to 2022.**



**Figure 4: Earth-Mars-Jupiter trajectory; left: STOUR initial guess, right: optimal solution in GALPOP.**

**Table 2: E-M-J rendezvous trajectory**

Characteristic	STOUR-LTGA	GALPOP
E-1 Launch date	Nov 6, 2021	Nov 16, 2021
E-1 Launch $V_{\infty}$ , km/s	0.2	0
M-2 Flyby date	Nov 28, 2023	Mar 20, 2024
M-2 Flyby $V_{\infty}$ , km/s	2.95	3.61
M-2 Flyby altitude, km	410	500 <sup>b</sup>
M-2 Flyby B-plane angle <sup>a</sup>	-21.6°	0°
J-3 Arrival date	Nov 15, 2027	Nov 26, 2027
J-3 Arrival $V_{\infty}$ , km/s	2.89	0
Total TOF, years	6.02	6.02
Initial mass, kg	20,000	20,000
Final mass, kg	14,420	16,026
Propellant mass fraction	27.9%	19.9%

<sup>a</sup> Fundamental plane taken as ecliptic of J2000.

<sup>b</sup> On lower bound.

## B. Other LTGA Trajectories to Jupiter

We next study cases with an Earth gravity assist, a Venus-Earth gravity assist, and a Venus-Venus gravity assist. (The Venus-Venus gravity assist trajectory is provided as an alternative to using the Earth as a gravity-assist body.) Table 3 summarizes the trajectory characteristics of an Earth-Earth-Jupiter rendezvous mission. From Fig. 5, we observe that between launch and the Earth flyby, the spacecraft thrusts (on average) towards the inertial positive  $y$  direction. The Earth  $V_\infty$  increases from 0.71 km/s to 7.5 km/s in 1.2 years during this Earth-Earth transfer. The total mission takes 4.5 years with a final mass of 16.2 mt (metric tons). We notice that the Earth-to-Earth transfer in our trajectory is very similar to that suggested by Kawaguchi in Ref. 18.

An Earth-Venus-Earth-Jupiter rendezvous trajectory is shown in Fig. 6. The trajectory has three phases of thrusting. Launching from the Earth with a  $V_\infty$  of 2.2 km/s, the energy of the orbit is decreased via the spacecraft thrusting against its velocity. After the Venus flyby, the spacecraft coasts back to the Earth with a flyby  $V_\infty$  of 10 km/s. The second thrusting phase occurs after the Earth flyby to further increase the energy of the orbit. Finally, the spacecraft rendezvous with Jupiter during its third thrusting phase. Table 4 summarizes the trajectory characteristics of an E-V-E-J rendezvous mission. To compare the mission performance, the TOF of the E-V-E-J trajectory is kept the same as the E-E-J (4.5 years). We notice that with the same TOF, the E-E-J mission has a final mass of about 600 kg greater than that of the E-V-E-J mission.

Figure 7 shows the trajectory plot of the Earth-Venus-Venus-Jupiter rendezvous mission. We note that the spacecraft takes a 2:1 resonance orbit on the Venus-Venus leg. The Venus flyby  $V_\infty$  increases from 6 km/s to 10 km/s between V-2 and V-3. Table 5 summarizes the trajectory characteristic of the E-V-V-J mission. The trajectory rendezvous with Jupiter in 4.9 years and the remaining final mass is 13.8 mt, which is about 2,000 kg less than the E-V-E-J mission.

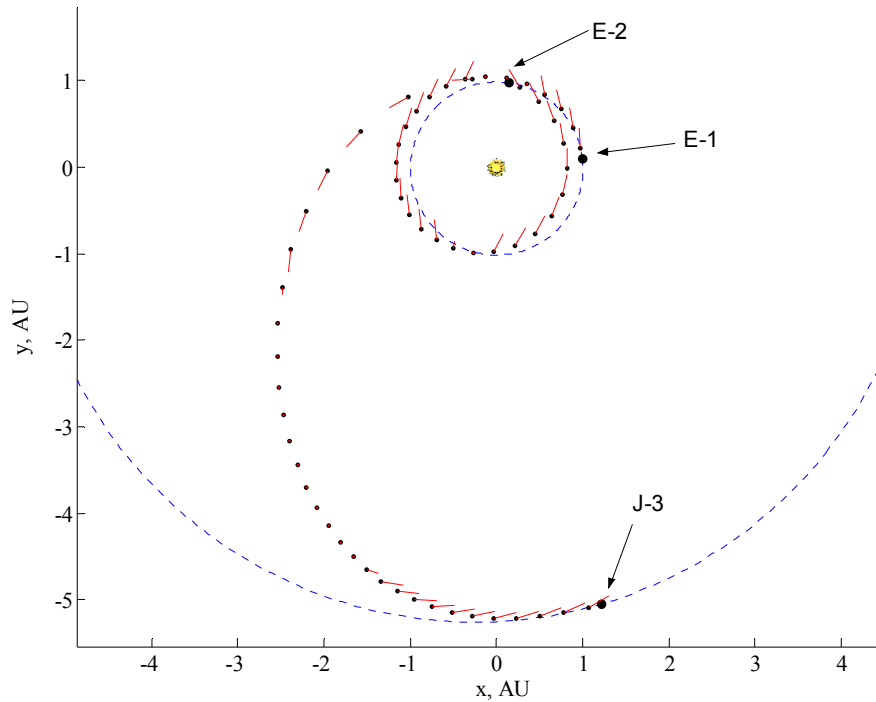


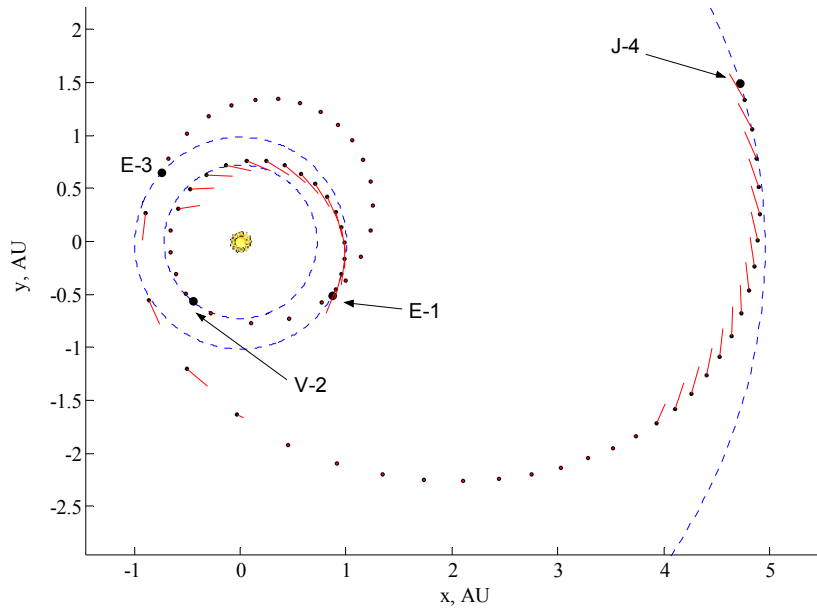
Figure 5: Trajectory plot of an Earth-Earth-Jupiter rendezvous mission.

**Table 3: E-E-J rendezvous trajectory**

Characteristic	Value
E-1 Launch date	Sep 29, 2015
E-1 Launch $V_\infty$ , km/s	0.71
E-2 Flyby date	Dec 13, 2016
E-2 Flyby $V_\infty$ , km/s	7.49
E-2 Flyby altitude, km	500 <sup>b</sup>
E-2 Flyby B-plane angle <sup>a</sup>	0°
J-3 Arrival date	Apr 5, 2020
Total TOF, years	4.52
Initial mass, kg	19,820
Final mass, kg	16,181
Propellant mass fraction	18%

<sup>a</sup> Fundamental plane taken as ecliptic of J2000.

<sup>b</sup> On lower bound.



**Figure 6: Trajectory plot of an Earth-Venus-Earth-Jupiter rendezvous mission.**

**Table 4: E-V-E-J rendezvous trajectory**

Characteristic	Value
E-1 Launch date	Aug 22, 2018
E-1 Launch $V_\infty$ , km/s	2.19
V-2 Flyby date	Feb 26, 2019
V-2 Flyby $V_\infty$ , km/s	5.75
V-2 Flyby altitude, km	500 <sup>b</sup>
V-2 Flyby B-plane angle <sup>a</sup>	14°
E-3 Flyby date	Feb 8, 2020
E-3 Flyby $V_\infty$ , km/s	10.3
E-3 Flyby altitude, km	500 <sup>b</sup>
E-3 Flyby B-plane angle <sup>a</sup>	177°
J-4 Arrival date	Feb 27, 2023
Total TOF, years	4.52
Initial mass, kg	18,333
Final mass, kg	15,606
Propellant mass fraction	15%

<sup>a</sup> Fundamental plane taken as ecliptic of J2000.

<sup>b</sup> On lower bound.

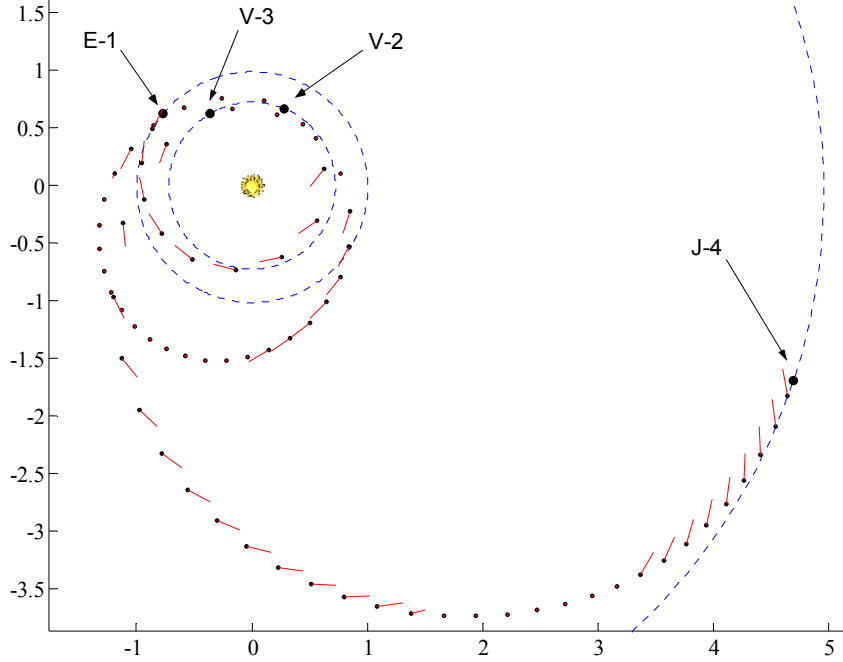


Figure 7: Trajectory plot of an Earth-Venus-Venus-Jupiter rendezvous mission.

Table 5: E-V-V-J rendezvous trajectory

Characteristic	Value
E-1 Launch date	Feb 10, 2017
E-1 Launch $V_\infty$ , km/s	2.64
V-2 Flyby date	Aug 24, 2017
V-2 Flyby $V_\infty$ , km/s	6.25
V-2 Flyby altitude, km	500 <sup>b</sup>
V-2 Flyby B-plane angle <sup>a</sup>	9°
V-3 Flyby date	Dec 19, 2018
V-3 Flyby $V_\infty$ , km/s	10.2
V-3 Flyby altitude, km	500 <sup>b</sup>
V-3 Flyby B-plane angle <sup>a</sup>	1°
J-4 Arrival date	Jan 15, 2022
Total TOF, years	4.93
Initial mass, kg	17,614
Final mass, kg	13,809
Propellant mass fraction	22%

<sup>a</sup> Fundamental plane taken as ecliptic of J2000.

<sup>b</sup> On lower bound.

### C. Analysis of Launching with a Non-Zero $V_\infty$

In Ref. 35, we noted that the optimal launch  $V_\infty$  is always nonzero. However, a nonzero launch  $V_\infty$  seems suboptimal because the heavy lifter has a much lower  $I_{sp}$  than the nuclear electric engine (404 s versus 6,000 s). One might think it would be propellant-optimal to thrust as little as possible with the lower- $I_{sp}$  engine. That is, one might think that the heavy lifter should boost the spacecraft to escape velocity (with a zero launch  $V_\infty$ ), leaving the higher- $I_{sp}$  engine to complete the remaining  $\Delta V$ .

Nevertheless, we observed that the optimal launch  $V_\infty$  is never zero.<sup>35</sup> To understand why, we note that a given  $\Delta V$ , exerted tangential to the spacecraft velocity vector, will cause an energy increase of  $\Delta E = V(\Delta V) + \Delta V^2/2$ . That is, when the velocity of the spacecraft is higher (near the Earth), a given  $\Delta V$  causes a greater increase in the spacecraft's Energy (and launch  $V_\infty$ ). Even though a given  $\Delta V$  may require more propellant near the Earth (because the heavy lifter has a lower  $I_{sp}$ ), it will be more effective at increasing the energy than a  $\Delta V$  executed far from the Earth. There is a choice between a highly-effective  $\Delta V$  at low  $I_{sp}$  or a less-effective  $\Delta V$  at high  $I_{sp}$ .

To decide where to do the  $\Delta V$ , we consider the following simplified model. Suppose the spacecraft begins in a circular parking orbit about the Earth. The heavy lifter performs a  $\Delta V$  to boost the spacecraft to an escape orbit (with a launch  $V_\infty \geq 0$ ). Once the spacecraft reaches deep space, it uses the nuclear electric engine to perform a second  $\Delta V$



to boost the  $V_\infty$  to a pre-specified target value,  $V_{\infty, \text{target}}$ . The problem is to choose the magnitudes of the first and second  $\Delta V$  so as to maximize the final mass and achieve the final target  $V_\infty$ . This constrained optimization problem can be solved; the optimal launch  $V_\infty$ ,  $V_\infty^*$  is

$$V_\infty^* = \begin{cases} V_{\infty, \text{target}} & \text{if } V_{\infty, \text{target}} \leq V_{\infty, \text{target}}^{\text{critical}} \\ V_{\infty, \text{target}}^{\text{critical}} & \text{if } V_{\infty, \text{target}} > V_{\infty, \text{target}}^{\text{critical}} \end{cases} \quad (3)$$

where

$$V_{\infty, \text{target}}^{\text{critical}} \equiv V_c \sqrt{\frac{2}{(I_{sp2} / I_{sp1})^2 - 1}} \quad (4)$$

and  $V_c$  is the circular velocity in the initial parking orbit,  $I_{sp1}$  is the specific impulse of the heavy lifter, and  $I_{sp2}$  is the specific impulse of the nuclear electric engine. (For the engine parameters we have been using in this paper, the critical target  $V_\infty$  is 0.744 km/s.)

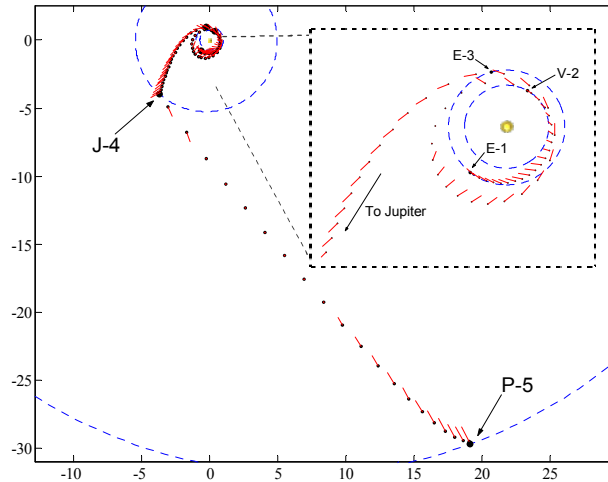
We note that the optimal launch  $V_\infty$  is almost never zero. There are only two special cases where it is zero: 1) when the target  $V_\infty$  is zero, or 2) when  $I_{sp2} \gg I_{sp1}$ .

#### D. LTGA Trajectories to Pluto

A mission to Pluto presents many challenges. Due to the great distance (Pluto is at 36 AU in 2025), it is extremely difficult to have a reasonable TOF for a practical propellant cost. While a direct Earth-Pluto mission can achieve a moderate TOF of just under 12 years, its propellant mass fraction of 72% is far from realistic.<sup>35</sup> It is thus clear that an acceptable mission to Pluto must employ gravity assists (for our system). To find a gravity-assist trajectory to Pluto, we convert the Jupiter rendezvous missions (E-E-J, E-V-E-J and E-V-V-J) to flyby missions and then add a Jupiter-Pluto leg.

For example, Fig. 8 shows an E-V-E-J-P trajectory. Starting from the Earth (thrusting in an inertially fixed direction), the spacecraft encounters Venus after about half a rev around the Sun. After a flyby at Venus and encountering the Earth again, a close swingby of Jupiter boosts the spacecraft to Pluto. Table 6 summarizes the trajectory characteristics. The total TOF of this trajectory is 10.5 years, and the PMF is 50% (significantly better than the direct mission).

We investigate the trade-off between TOF and final mass for the three gravity-assist sequences: E-V-E-J-P, E-E-J-P and E-V-V-J-P. Table 7 summarizes the key parameters of these three families of rendezvous trajectories. Figure 9 shows how the TOF varies with the final mass. We note that when the TOF is less than 11 years, the E-E-J-P sequence has a higher final mass than the E-V-E-J-P sequence. However, the difference in final mass for the E-E-J-P and E-V-E-J-P sequences is not significant. These two families launch in different time windows. Therefore, a mission designer can plan a mission using the E-V-E-J-P sequence (launching in May 2015) and if the launch does not happen for some reason, then the E-E-J-P sequence (launching in Dec 2015) can be used as a backup.



**Figure 8: Trajectory plot of an Earth-Venus-Earth-Jupiter-Pluto rendezvous mission.**

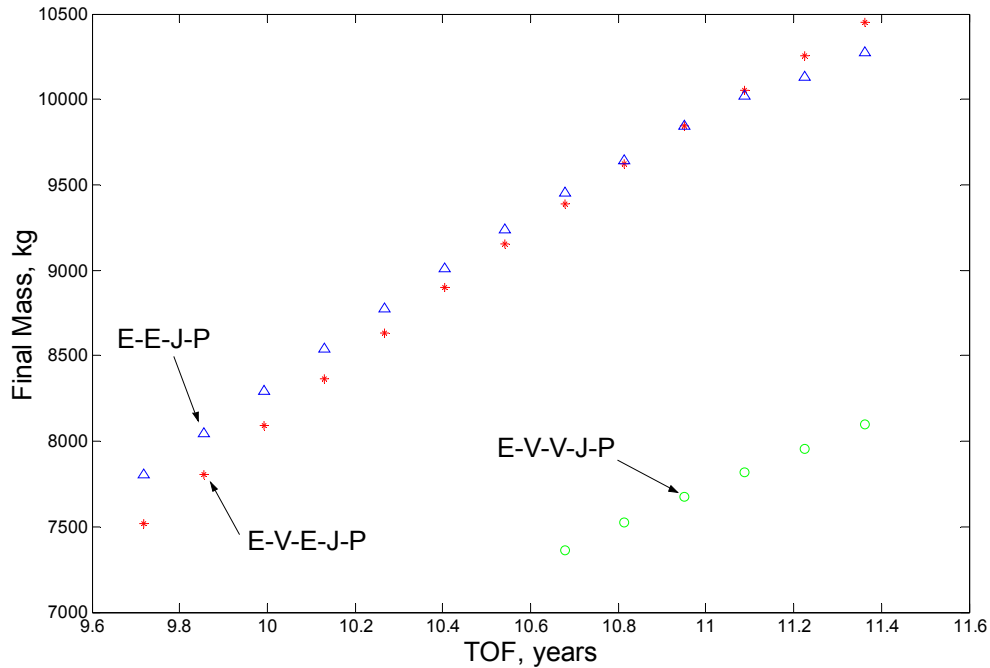
**Table 6: E-V-E-J-P rendezvous trajectory**

Characteristic	Value
E-1 Launch date	May 8, 2015
E-1 Launch $V_\infty$ , km/s	2.11
V-2 Flyby date	Oct 14, 2015
V-2 Flyby $V_\infty$ , km/s	5.37
V-2 Flyby altitude, km	6,398
V-2 Flyby B-plane angle <sup>a</sup>	110°
E-3 Flyby date	Jan 6, 2017
E-3 Flyby $V_\infty$ , km/s	11.3
E-3 Flyby altitude, km	500 <sup>b</sup>
E-3 Flyby B-plane angle <sup>a</sup>	-1°
J-4 Flyby date	Apr 29, 2018
J-4 Flyby $V_\infty$ , km/s	16.7
J-4 Flyby altitude, $R_J$ <sup>c</sup>	2.48
J-4 Flyby B-plane angle <sup>a</sup>	-4°
P-5 Arrival date	Nov 21, 2025
Total TOF, years	10.5
Initial mass, kg	18,442
Final mass, kg	9,155
Propellant mass fraction	50%

<sup>a</sup> Fundamental plane taken as ecliptic of J2000.

<sup>b</sup> On lower bound.

<sup>c</sup> Assuming a Jupiter radius of 71,492 km.



**Figure 9: Trade study of Pluto rendezvous missions.**

In Fig. 9, we notice that the final mass for the E-V-V-J-P mission is about 2,000 kg less than that of the E-E-J-P and the E-V-E-J-P missions. Therefore, if we cannot perform the Earth flyby, we can still reach Pluto in less than 11 years, with a final mass of 2,000 kg less than the Earth flyby cases.

Another non-Earth flyby trajectory is the Earth-Venus-Mars-Jupiter-Pluto sequence. Figure 10 and Table 8 provide the trajectory plot and characteristics of an E-V-M-J-P mission. The spacecraft takes 10.8 years to rendezvous with Pluto and the final mass is about 7,600 kg. With the same TOF (10.8 years), the E-V-M-J-P sequence is just slightly better than the E-V-V-J-P sequence.

**Table 7: Trajectory parameters of three families of rendezvous missions to Pluto**

Launch Date	Launch $V_{\infty}$ , km/s	Injected Mass, kg	Flight Times for each leg, days	Total Time of Flight, years	Final Mass, kg	Propellant Mass Fraction
<i>E-V-E-J-P</i>						
June 13, 2015	2.98	17,007	135, 440, 456, 2519	9.72	7,515	55.8 %
June 7, 2015	2.78	17,367	139, 441, 462, 2558	9.86	7,805	55.1 %
May 29, 2015	2.51	17,832	146, 445, 465, 2595	9.99	8,090	54.6 %
May 23, 2015	2.38	18,047	149, 446, 470, 2635	10.1	8,365	53.6 %
May 29, 2015	2.52	17,823	145, 444, 468, 2693	10.3	8,632	51.6 %
May 24, 2015	2.38	18,046	148, 445, 473, 2733	10.4	8,898	50.7 %
May 8, 2015	2.11	18,442	160, 450, 478, 2763	10.5	9,155	50.4 %
May 2, 2015	2.06	18,525	163, 450, 482, 2804	10.7	9,389	49.3 %
May 3, 2015	2.06	18,514	163, 450, 482, 2855	10.8	9,625	48.0 %
Apr 28, 2015	2.03	18,563	167, 451, 482, 2900	11.0	9,846	47.0 %
Apr 13, 2015	2.01	18,593	170, 452, 486, 2942	11.1	10,053	45.9 %
Apr 24, 2015	2.01	18,595	169, 451, 487, 2992	11.2	10,253	44.9 %
Apr 25, 2015	2.01	18,591	169, 451, 487, 3043	11.4	10,449	43.8 %
<i>E-E-J-P</i>						
Dec 29, 2015	3.28	16,418	363, 533, 2654	9.72	7,808	52.4 %
Dec 18, 2015	2.93	17,098	374, 546, 2680	9.86	8,048	52.9 %
Dec 24, 2015	3.05	16,866	367, 544, 2740	9.99	8,295	50.8 %
Dec 19, 2015	2.88	17,196	371, 551, 2778	10.1	8,539	50.3 %
Dec 14, 2015	2.70	17,515	376, 557, 2817	10.3	8,779	49.9 %
Dec 9, 2015	2.52	17,827	381, 564, 2855	10.4	9,012	49.4 %
Dec 3, 2015	2.33	18,120	386, 572, 2893	10.5	9,236	49.0 %
Nov 28, 2015	2.15	18,399	390, 579, 2931	10.7	9,452	48.6 %
Nov 23, 2015	1.97	18,640	395, 585, 2970	10.8	9,645	48.2 %
Nov 12, 2015	1.70	18,980	406, 593, 3002	11.0	9,848	48.1 %
Oct 25, 2015	1.37	19,334	423, 600, 3027	11.1	10,021	48.2 %
Sep 11, 2015	1.42	19,289	469, 584, 3048	11.2	10,130	47.5 %
July 23, 2015	1.51	19,198	521, 560, 3069	11.4	10,273	46.5 %
<i>E-V-V-J-P</i>						
June 15, 2015	3.72	15,494	89, 465, 584, 2762	10.7	7,365	52.5 %
June 12, 2015	3.61	15,745	91, 465, 594, 2799	10.8	7,523	52.2 %
June 11, 2015	3.46	16,061	94, 464, 610, 2832	11.0	7,678	52.2 %
June 11, 2015	3.46	16,053	94, 464, 610, 2883	11.1	7,819	51.3 %
June 9, 2015	3.37	16,244	96, 465, 619, 2921	11.2	7,956	51.0 %
June 9, 2015	3.34	16,305	96, 464, 622, 2967	11.4	8,097	50.3 %

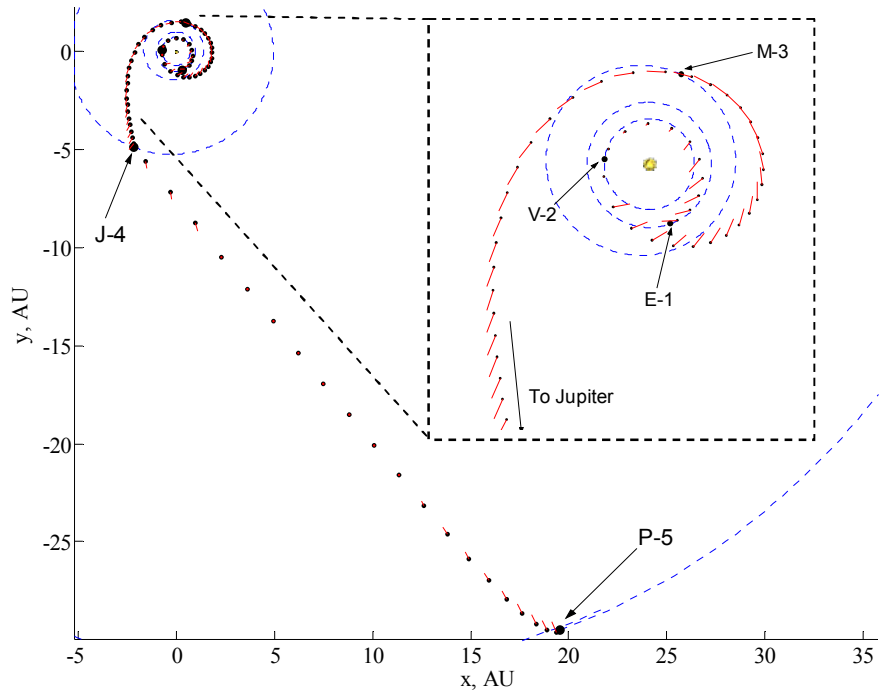


Figure 10: Trajectory plot of an Earth-Venus-Mars-Jupiter-Pluto rendezvous mission.

Table 8: E-V-M-J-P rendezvous trajectory

Characteristic	Value
E-1 Launch date	July 12, 2015
E-1 Launch $V_{\infty}$ , km/s	3.06
V-2 Flyby date	Dec 25, 2015
V-2 Flyby $V_{\infty}$ , km/s	7.39
V-2 Flyby altitude, km	1,599
V-2 Flyby B-plane angle <sup>a</sup>	-15°
M-3 Flyby date	Apr 6, 2017
M-3 Flyby $V_{\infty}$ , km/s	6.63
M-3 Flyby altitude, km	500 <sup>b</sup>
M-3 Flyby B-plane angle <sup>a</sup>	179°
J-4 Flyby date	Dec 29, 2018
J-4 Flyby $V_{\infty}$ , km/s	15.5
J-4 Flyby altitude, $R_J$ <sup>c</sup>	10.1
J-4 Flyby B-plane angle <sup>a</sup>	-6°
P-5 Arrival date	May 5, 2026
P-5 Arrival $V_{\infty}$ , km/s	0
Total TOF, years	10.8
Initial mass, kg	16,847
Final mass, kg	7,613
Propellant mass fraction	55%

<sup>a</sup> Fundamental plane taken as ecliptic of J2000.

<sup>b</sup> On lower bound.

<sup>c</sup> Assuming a Jupiter radius of 71,492 km.

## V. Summary of Rendezvous Trajectories to the Outer Planets

The goal of these missions is to deliver spacecraft to the outer planets in a reasonable flight time and with an acceptable final mass. In the launch years from 2014 to 2025, we study rendezvous missions for cases with and without gravity assist. (The analysis of missions without gravity assist is provided in Ref. 35.) Table 9 summarizes the mission performance of the trajectories we have studied. All trajectories in Table 9 have TOF constraints with the encounter dates being free.

**Table 9: Rendezvous missions to the outer planets**

Encounter Sequence	Launch Date	Launch $V_{\infty}$ , km/s	Flight Times for each leg, days	Total Time of Flight, years	Final Mass, kg
<i>Jupiter</i>					
E-J <sup>a</sup>	Apr 13, 2024	0	2019	5.53	13,301
E-J	Apr 24, 2024	1.23	2019	5.53	13,709
E-M-J <sup>a</sup>	Jan 12, 2022	0	771, 1038	4.95	13,996
E-M-J	Feb 17, 2022	1.04	759, 1050	4.95	14,879
E-E-J	Sep 6, 2015	0.59	460, 1340	4.93	16,260
E-E-J	Sep 29, 2015	0.71	441, 1209	4.52	16,181
E-V-E-J	Aug 22, 2018	2.19	188, 347, 1115	4.52	15,606
E-V-V-J	Feb 10, 2017	2.64	195, 482, 1122	4.93	13,809
<i>Saturn</i>					
E-S <sup>a</sup>	Oct 9, 2022	0	2631	7.20	11,285
E-S	Jan 24, 2023	1.30	2631	7.20	12,653
E-M-S	Feb 16, 2022	1.20	587, 2043	7.20	13,943
E-M-S	Apr 14, 2022	2.17	541, 1759	6.30	12,444
E-V-E-S	Oct 21, 2021	1.95	172, 324, 1804	6.30	14,306
E-V-E-J-S	June 4, 2015	2.24	179, 340, 648, 1283	6.71	12,872
<i>Uranus</i>					
E-U <sup>a</sup>	Oct 20, 2020	0	3537	9.68	8,268
E-U	June 5, 2021	2.32	3537	9.68	10,402
E-E-J-U	Jan 28, 2019	1.02	423, 599, 2378	9.31	12,912
E-E-J-U	Feb 11, 2020	0.94	445, 582, 2374	9.31	13,029
E-M-E-J-U	Dec 28, 2017	1.16	1039, 195, 515, 2450	11.5	14,097
E-V-E-J-U	Aug 25, 2018	2.21	184, 345, 587, 2284	9.31	12,172
E-V-E-J-U	Sep 27, 2018	2.81	172, 327, 526, 1976	8.21	9,225
<i>Neptune</i>					
E-N <sup>a</sup>	Aug 7, 2019	0	4114	11.3	6,347
E-N	Apr 21, 2020	2.63	4114	11.3	8,543
E-V-E-J-N	Aug 26, 2018	2.22	187, 348, 518, 3247	11.8	11,783
E-V-E-J-N	Sep 13, 2018	2.48	179, 338, 501, 2583	9.86	9,264
<i>Pluto</i>					
E-P <sup>a</sup>	May 28, 2015	0	4320	11.8	5,661
E-P	Aug 20, 2015	2.75	4320	11.8	6,890
E-J-P	May 29, 2014	1.11	1675, 3125	13.1	9,109
E-M-E-J-P	Mar 19, 2014	2.06	737, 285, 470, 2799	11.7	9,162
E-V-V-J-P	June 12, 2015	3.60	91, 465, 594, 2799	10.8	7,523
E-V-M-J-P	July 12, 2015	3.06	166, 468, 632, 2684	10.8	7,613
E-V-E-J-P	May 13, 2015	2.06	163, 450, 482, 2855	10.8	9,625
E-E-J-P	Nov 23, 2015	1.97	395, 585, 2970	10.8	9,645

<sup>a</sup> The minimum TOF trajectory with launch  $V_{\infty}$  constrained to be zero.

For direct missions, we considered (in Ref. 35) both zero and non-zero launch  $V_\infty$  cases as baselines for comparison with the gravity-assist trajectories. The final mass is acceptable only for the cases to Jupiter, Saturn and Uranus. For Neptune and Pluto, not only is the final mass too small, the TOF is too long (over eleven years). We therefore conclude that missions to Neptune or Pluto without gravity assist are infeasible.

We are interested to know what gravity-assist sequence would give us the best results (in terms of mission duration and propellant cost) to each outer planet. For Jupiter and Saturn, the case with a single Mars gravity assist allows us to have a shorter TOF and a higher final mass than the direct mission.

We also explore the potential of Earth (E-E) and Venus-Earth (E-V-E) gravity-assist sequences. From Table 9, we notice that both of the E-E and E-V-E trajectories are more effective than a single Mars gravity assist. In particular, for the mission to Jupiter with a flight-time of 4.5 years, the E-E-J final mass is 600 kg greater than the final mass of the E-V-E-J case. Therefore, the E-E-J trajectory is a better option than the E-V-E-J (when the TOF is 4.5 years). For a target beyond Saturn, we employ Jupiter as a gravity-assist body in addition to the E-E and E-V-E sequences. In a mission to Uranus with a TOF of 9.3 years, the E-E-J-U trajectory is found to be more effective than the E-V-E-J-U. For missions to Pluto with a TOF of 10.8 years, the E-E-J-P and E-V-E-J-P sequences have similar final mass.

For missions to Saturn and Neptune, the best trajectories we have found are the E-V-E-S and the E-V-E-J-N respectively. However, this conclusion is made based upon the limited trajectories we have investigated.

## VI. Future Work

1. Clearly, we have not exhausted all the gravity-assist sequences (such as E-V-E-E-J and E-V-V-V-J) that could improve mission performance (TOF and final mass).
2. Because of the efficiency of the Earth-Earth gravity-assist trajectories in the case of Jupiter, Uranus and Pluto, we expect that missions to Saturn and possibly to Neptune could benefit from the sequence.
3. There may be launch windows where Jupiter is not well positioned for gravity assist to the far outer planets, which suggests we should find trajectories that do not use Jupiter gravity assist.
4. There are other scientific targets of interest in the solar system, which include missions to Mercury, asteroids, comets and other solar system bodies. There is also interest in missions to escape the solar system such as an interstellar probe<sup>36</sup> and the 1000 AU mission.<sup>37</sup>

## VII. Conclusions

The nuclear electric propulsion system being designed for the JIMO spacecraft combined with gravity assist will enable a variety of missions to the outer planets. The inclusion of gravity assist with Venus, Earth or Mars in conjunction with a Jupiter gravity-assist further improves the mission performance to Uranus, Neptune and Pluto. We study three families of rendezvous trajectories to Pluto. The Earth-Venus-Earth-Jupiter-Pluto family and the Earth-Earth-Jupiter-Pluto family are among the best sequences to Pluto in terms of final mass and time-of-flight. The Earth-Venus-Venus-Jupiter-Pluto family provides an alternative non-Earth flyby sequence, though with an extra cost of 2,000 kg.

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