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TWO-SYNODIC-PERIOD EARTH-MARS CYCLERS WITH INTERMEDIATE EARTH ENCOUNTER*

T. Troy McConaghy,† Chit Hong Yam,‡ Damon F. Landau,§ and James M. Longuski¶

Earth-Mars cycler trajectories (cyclers) may play an important role in a future human transportation system to Mars. Using a circular coplanar solar system model, cyclers which repeat every two synodic periods and have one intermediate Earth encounter are analyzed. One such cycler, the "ballistic S1L1 cycler," is very promising. It requires no propulsive maneuvers, has short transfer times between Earth and Mars, and has low V_∞ at Earth and Mars. A method is described to convert a ballistic cycler in the cicular coplanar model to a ballistic cycler in an ephemeris model (if possible). The method is applied to find and characterize examples of ballistic S1L1 cyclers.

INTRODUCTION

A future Earth-Mars transportation system will probably use many different kinds of spacecraft trajectories. For example, some trajectories are well-suited for human transportation whereas others are better suited for ferrying supplies. To develop an efficient system, designers should know what trajectories are available.

One potentially useful type of trajectory is the Earth-Mars cycler trajectory, or cycler. A spacecraft on a cycler regularly passes close to both Earth and Mars (but never stops at either). Cyclers which require propulsive maneuvers are referred to as powered cyclers, whereas cyclers that rely only on gravitational forces are referred to as ballistic cyclers.

It appears that Rall was the first to systematically investigate Earth-Mars cyclers, for his doctoral thesis in 1969.[1] This research was later summarized by Rall and Hollister.[2] The cycler idea languished until 1985, when Aldrin first proposed the "Aldrin cycler"[3] and Niehoff first proposed the VISIT 1 and VISIT 2 cyclers.[4–6] The existence of the Aldrin cycler was verified by Byrnes et al.[7] and the Aldrin cycler was compared to the VISIT cyclers by Friedlander et al.[8]

More recently, McConaghy et al.[9] showed that the Aldrin cycler and the VISIT cyclers are special cases in a larger family of cyclers. Several new cyclers were also identified by Byrnes et al.,[10] Chen et al.,[11,12] McConaghy et al.,[9] Rauwolf et al.,[13] and Russell and Ocampo.[14] We also note that there are some variations on cyclers, in which the spacecraft enters a temporary parking orbit at Mars (semi-cyclers),[15,16] at Earth (reverse semi-cyclers),[17] or at both Earth and Mars (stop-over cyclers).[18,19]

In this paper, we analyze all cyclers that repeat every two synodic periods and have one intermediate Earth encounter. This analysis is a logical extension of the analysis described in McConaghy et al.[9] Several new and potentially useful cyclers are presented.

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METHODOLOGY

We begin by making the following simplifying assumptions (some of which will be relaxed later):

1. The Earth-Mars synodic period, S, is $2\frac{1}{7}$ years.
2. Earth's orbit, Mars' orbit, and the cycler trajectory lie in the ecliptic plane.
3. Earth and Mars have circular orbits.
4. The cycler trajectory is conic and prograde (direct).
5. Only the Earth has sufficient mass to provide gravity-assist maneuvers.
6. Gravity-assist maneuvers occur instantaneously.

We note that assumption 1 is equivalent to assuming that the orbital period of Mars is 1.875 years (whereas a more accurate value is 1.881 years).

In order for a trajectory to be an Earth-Mars cycler, it must return to Earth after an integer number of synodic periods. In this paper, we only consider trajectories that return to Earth after two synodic periods, or $4\frac{2}{7}$ years. We also assume that the trajectory encounters Earth at an intermediate time τ, as illustrated in Fig. 1. We refer to such cycler trajectories as (2, 1) cyclers (for two synodic periods, one intermediate Earth).

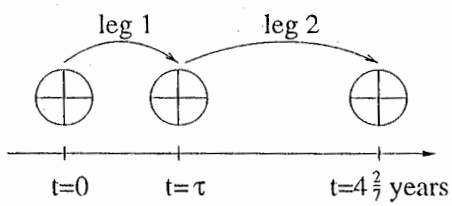

Figure 1 Assumed timing of the Earth encounters.

Since we're assuming that the orbits of Earth and Mars are circular, we can choose our Cartesian coordinate system so that the position vector of the Earth at time $t = 0$ is $[1, 0]$. Since the angular velocity of the Earth is 2π radians per year, the position of the Earth at any later time t will be $\mathbf{R}_{\text{Earth}}(t) = [\cos(2\pi t), \sin(2\pi t)]$. Since the spacecraft encounters the Earth at times $t = 0$, $t = \tau$, and $t = 4\frac{2}{7}$, the position vectors of the spacecraft at the first, second, and third Earth encounter are, respectively:

$$
\begin{aligned}
\mathbf{R}_1 &= [1, 0] \\
\mathbf{R}_2 &= [\cos(2\pi\tau), \sin(2\pi\tau)] \\
\mathbf{R}_3 &= \left[\cos\left(2\pi \cdot 4\frac{2}{7}\right), \sin\left(2\pi \cdot 4\frac{2}{7}\right)\right] \approx [-0.223, 0.975]
\end{aligned}
$$

To ensure that there is a unique trajectory for leg 1 (connecting $\mathbf{R}_1$ to $\mathbf{R}_2$ in a time of flight τ), we need to make some additional assumptions. First, we need to specify the number of complete revolutions r_1 that the spacecraft makes between $\mathbf{R}_1$ and $\mathbf{R}_2$. The value of r_1 could be 0, 1, 2, etc. Second, if r_1 is greater than zero, then we also need to specify whether we want the short-period solution or the long-period solution to the Lambert problem. This parameter, which we call P_1

can take values of 'S', 'L', or 'U', which stand for for short-period, long-period, or unique-period solutions, respectively. To ensure that there is exactly one solution for leg 2, we also need to specify r_2 and P_2, which have the same meanings as r_1 and P_1 (except they are for leg 2).

In summary, five parameters (P_1, r_1, P_2, r_2, and τ) specify a unique cycler trajectory that repeats every two synodic periods and has one intermediate Earth encounter. We use the compact notation $P_1 r_1 P_2 r_2(\tau)$ to refer to the unique cycler with the parameters P_1, r_1, P_2, r_2, and τ. Examples are S1L2(1.9), L1L1(2.7) and S2U0(3.2).

There are some exceptional cases where the five parameters P_1, r_1, P_2, r_2, and τ are not enough to uniquely determine a solution. These cases occur when the total transfer angle on one of the legs is a multiple of π radians. We refer to such transfers as $n\pi$ transfers. Since we are considering transfers from Earth to Earth (which has a period of one year), an $n\pi$ transfer occurs whenever one of the two legs has a time of flight (TOF) that is n half years. Specifically, the first leg will be an $n\pi$ transfer whenever $\tau = \frac{1}{2},\ 1,\ 1\frac{1}{2},\ 2,\ 2\frac{1}{2},\ 3,\ 3\frac{1}{2}$, or 4 years and the second leg will be an $n\pi$ transfer whenever $(4\frac{2}{7} - \tau) = \frac{1}{2},\ 1,\ 1\frac{1}{2},\ 2,\ 2\frac{1}{2},\ 3,\ 3\frac{1}{2}$, or 4 years. These special values of τ must be analyzed separately.

EVALUATING CYCLERS

Not all cycler trajectories are practical for applications. There are a number of criteria to be considered when evaluating a given cycler trajectory.

One of the most important considerations is the number of cycler vehicles required to ensure a short Earth-Mars trip and a short Mars-Earth trip every synodic period. For cyclers that repeat every two synodic periods, the number of cycler vehicles required is typically four. During each synodic period, there is one vehicle, the "outbound vehicle," that provides a short Earth-Mars leg and another vehicle, the "inbound vehicle," that provides a short Mars-Earth leg. Since those short legs generally don't occur again for another two synodic periods, two other vehicles are required to provide short outbound and inbound trips in the next synodic interval.

If a cycler trajectory is to be used for transportation between Earth and Mars, then at least one of the two legs must cross the orbit of Mars. This can be determined by calculating the aphelion radii of the two legs and making sure that at least one is greater than the orbital radius of Mars (1.52 AU).

If a taxi vehicle must rendezvous with the cycler vehicle at either Earth or Mars, then the flyby V_∞ should be reasonably low; values greater than about 10 km/s are probably too high.

Ideally, a cycler trajectory should be ballistic. A cycler is ballistic if the incoming and outgoing V_∞ are equal and the required Earth flyby altitude is greater than 300 km. Most cycler trajectories don't meet these requirements, so we compare the required ΔV per flyby instead.

We approximate the required ΔV per flyby as follows. For any given cycler $P_1 r_1 P_2 r_2(\tau)$, the incoming $\mathbf{V}_\infty$ and the outgoing $\mathbf{V}_\infty$ at the Earth encounters can be calculated. These two vectors must be connected using as little ΔV as possible. We can use a gravity-assist maneuver and we assume that all ΔV maneuvers occur far from Earth (because making accurate ΔV maneuvers during a gravity-assist maneuver is impractical).

Let us call the largest of the incoming and outgoing $\mathbf{V}_\infty$ vectors "$\mathbf{V}_L$." Call the shortest "$\mathbf{V}_S$." In order to get the greatest ΔV from the gravity-assist maneuver, we use it to rotate $\mathbf{V}_S$ so that it becomes as close to collinear with $\mathbf{V}_L$ as is possible (assuming a minimum allowable flyby altitude of 300 km). The approximated ΔV is the length of the vector from tip of the rotated $\mathbf{V}_S$ to the tip of $\mathbf{V}_L$, as illustrated in Fig. 2.

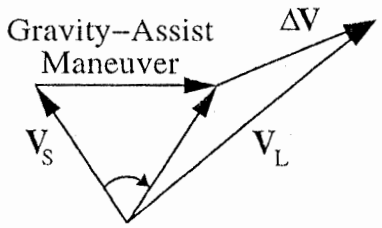

Figure 2 Required ΔV at a flyby.

Sometimes one of the legs of a cycler is identical to the orbit of the Earth (so the V_∞ at Earth is zero). If we don't ignore these cyclers (which are actually reverse semi-cyclers), then we need to decide what the cycler vehicle does while it follows Earth. In this case, we assume that the cycler vehicle goes into a 7-day elliptical Earth orbit with a perigee of 300 km. We approximate the ΔV required to transfer from the flyby hyperbola to the ellipse (and vice versa) by assuming the maneuver occurs at perigee.

To approximate the total ΔV required every two synodic periods, we just multiply the approximated ΔV per Earth flyby by two (since there are two Earth flybys every two synodic periods). To approximate the average ΔV per synodic period, we divide the ΔV per two synodic periods by two. The average ΔV per synodic period is therefore equal to the ΔV per Earth flyby.

ANALYSIS

We begin our analysis of the (2, 1) cyclers with those which can be completely specified by P_1, r_1, P_2, r_2, and τ. (The exceptional cases occur when one of the legs is an $n\pi$ transfer.) Since the cycler $P_1r_1P_2r_2(\tau)$ has the same characteristics as the cycler $P_2r_2P_1r_1(4\frac{2}{7}-\tau)$, we only need to consider τ between $2\frac{1}{7}$ and $4\frac{2}{7}$ years. (In this interval, the exceptional cases occur when $\tau = 2\frac{2}{7}$, $2\frac{1}{2}$, $2\frac{11}{14}$, 3, $3\frac{2}{7}$, $3\frac{1}{2}$, $3\frac{11}{14}$, or 4 years.)

We refer to all the (2, 1) cyclers with a given P_1, r_1, P_2, and r_2 as the $P_1r_1P_2r_2$ family. Each member of a family has a different τ. We investigate all families with $r_1, r_2 \leq 4$. We are only interested in cycler trajectories that have a Mars-crossing leg and which require less than 2.5 km/s of ΔV per Earth flyby. The seven families that contain such cyclers are indicated by a check mark (✓) in Table 1 [assuming $\tau \in [2\frac{1}{7}, 4\frac{2}{7})$]; the τ intervals where their $\Delta V < 2.5$ km/s are given in Table 2.

For each of the seven families of interest (S1S2, S1S1, S1L1, U0L1, L1L1, L2U0, and L3U0), one can make a plot showing how the approximated ΔV per Earth flyby depends on τ (for τ between $2\frac{1}{7}$ and $4\frac{2}{7}$ years). Figure 3 shows this plot for the S1S2 family. The absence of a data point for a particular τ means that there is no corresponding solution. The four dashed vertical lines indicate τ values where one of the legs has a duration equal to an integer multiple of a year (when $\tau = 2\frac{2}{7}$, 3, $3\frac{2}{7}$, or 4 years). The properties of the cycler trajectory often change dramatically as τ crosses one of these values.

There are three distinct groups of solutions in Fig. 3, but only one group has cyclers with a ΔV per flyby that is less than 2.5 km/s (the group with τ between about 2.479 and 2.492 years). Among the cyclers in that group, only the first leg of the cycler trajectory crosses the orbit of Mars. The τ that leads to the lowest ΔV (0.41 km/s) is 2.4885 km/s. The characteristics of cycler S1S2(2.4885) are summarized in Table 3.

Table 1 Cycler families with a Mars-crossing cycler that requires less than 2.5 km/s ΔV per Earth flyby

	P_1r_1								
P_2r_2	S4	S3	S2	S1	U0	L1	L2	L3	L4
S4	×[a]	×	×	×	×	×	×	×	×
S3	×	×	×	×	×	×	×	×	×
S2	×	×	×	✓[b]	×	×	×	×	×
S1	×	×	×	✓	×	×	×	×	×
U0	×	×	×	×	×	×	✓	✓	×
L1	×	×	×	✓	✓	✓	×	×	×
L2	×	×	×	×	×	×	×	×	×
L3	×	×	×	×	×	×	×	×	×
L4	×	×	×	×	×	×	×	×	×

[a] An × indicates that no cyclers in the family meet the criteria (of the table caption).

[b] A check mark (✓) indicates that at least one cycler in the family meets the criteria.

One can analyze the other six families of interest (S1S1, S1L1, U0L1, L1L1, L2U0, and L3U0) in a similar way. Each of those families has a short range of τ in which the ΔV per Earth flyby is less than 2.5 km/s. Those ranges for τ are summarized in Table 2. Figures 4–9 show how the approximated ΔV per Earth flyby depends on τ in those ranges.

Table 2 Families which have cyclers with a ΔV less than 2.5 km/s

Family	Range of τ where $\Delta V < 2.5$ km/s, yr	Legs which cross Mars' orbit
S1S2	2.479 to 2.492	first leg only
S1S1	2.894 to 2.941	first leg only
S1L1	2.794 to 2.860	first leg only
U0L1	2.708 to 2.796	see Fig. 6[a]
L1L1	$2\frac{1}{7}$ to 2.210	both legs
L2U0	2.504 to 2.580	second leg only
L3U0	2.751 to 2.764	second leg only

[a] For $\tau < 2.758$ years, both legs cross Mars' orbit and for $\tau > 2.758$ years, only the first leg crosses Mars' orbit.

Each of the families of interest has one member (i.e. one value of τ) with the minimum ΔV per Earth flyby. The characteristics of the ΔV-minimizing member from each family are summarized in Table 3.

The L1L1 family is of particular interest because cycler L1L1($2\frac{1}{7}$) is the Aldrin cycler.[7] The L1L1 family members with $\tau \approx 2\frac{1}{7}$ years are similar to the Aldrin cycler. As can be seen in Fig.7, the Aldrin cycler is not the L1L1 family member with the lowest ΔV per Earth flyby. That honor goes to L1L1(2.1604). Table 3 gives the details of the cyclers L1L1($2\frac{1}{7}$) and L1L1(2.1604).

Table 3 reveals three ballistic cyclers (i.e. $\Delta V = 0$). Among them [S1L1(2.8277), U0L1(2.7540) and L2U0(2.5408)], only S1L1(2.8277) has an acceptably low V_∞ at Earth and Mars (4.7 and 5.0 km/s, respectively). We further analyze the ballistic S1L1 cycler in the section titled "THE BALLISTIC S1L1 CYCLER."

Table 3 Notable two-synodic-period cyclers with intermediate Earth encounter

$P_1 r_1 P_2 r_2(\tau)$	ΔV per flyby,[a] km/s	Aphelion,[b] AU leg 1, leg 2	Period, yr leg 1, leg 2	Earth V_∞, km/s leg 1, leg 2	Mars V_∞, km/s leg 1, leg 2	Mars orbit crossing times, yr
S1S2(2.4885)	0.41	1.83, 1.21	1.40, 0.71	13.9, 13.7	10.2, NA[c]	0.21, 0.88, 1.61, 2.28
S1S1(2.9124)	0.90	1.62, 1.07	1.50, 0.95	3.7, 3.1	4.7, NA[c]	0.47, 0.95, 1.96, 2.44
S1L1(2.8277)	0.00	1.64, 1.22	1.49, 1.07	4.7, 4.7	5.0, NA[c]	0.42, 0.92, 1.91, 2.41
U0L1(2.7540)	0.00	3.20, 1.54	2.93, 1.18	11.3, 11.3	14.0, 5.4	0.19, 2.57, 3.45, 3.59
L1L1($2\frac{1}{7}$)[d]	1.41	2.23, 2.23	2.02, 2.02	6.5, 6.5	9.8, 9.8	0.40, 1.74, 2.54, 3.89
L1L1(2.1604)	1.19	2.24, 2.22	2.03, 2.02	6.9, 6.2	9.9, 9.6	0.40, 1.76, 2.55, 3.89
L2U0(2.5408)	0.00	1.36, 2.20	1.08, 1.94	8.8, 8.8	NA[c], 10.3	2.78, 4.05
L3U0(2.7531)	1.00	1.31, 2.29	0.80, 1.82	15.0, 15.6	NA[c], 13.5	2.92, 4.12

[a] Minimum ΔV for the given family (except for the Aldrin cycler).
[b] Orbital radius of Mars is 1.52 AU.
[c] Leg does not cross Mars' orbit.
[d] The Aldrin cycler.[7]

(2, 1) CYCLERS WITH AN $n\pi$ TRANSFER LEG

When one of the two transfer legs is an $n\pi$ transfer (with a TOF of n half years), it has additional degrees of freedom. The parameters P_1 and r_1 (or P_2 and r_2) are no longer sufficient.

When n is 1, 3, 5, or 7, the Earth encounters occur 180 degrees apart. The Lambert problem can be solved as usual to determine the set of possible transfer types (U0, L1, S1, L2, etc.). However, for each type of solution, there is an additional degree of freedom—the transfer leg can have any inclination.

When n is 3, 5, or 7 (i.e. when the leg TOF is $1\frac{1}{2}$, $2\frac{1}{2}$, or $3\frac{1}{2}$ years), one of the solutions to the Lambert problem is a transfer leg with a period of 1 year. Such transfer legs will encounter the Earth every half year. For example, if the TOF is $1\frac{1}{2}$ years and the transfer leg with a period of 1 year is used, the cycler vehicle will encounter the Earth after half a year, after 1 year, and after $1\frac{1}{2}$ years. Since (in this paper) we are only looking for transfer legs with no intermediate Earth encounters, the 3π, 5π, and 7π transfers with a period of 1 year are ignored.

When a leg is an $n\pi$ transfer and n is 2, 4, 6, or 8, the Earth encounters at the ends of the leg occur at the same point in inertial space. In these cases, the transfer leg has three degrees of freedom, one discrete and two continuous. The discrete degree of freedom is in the period, $\mathcal{P}_T$, of

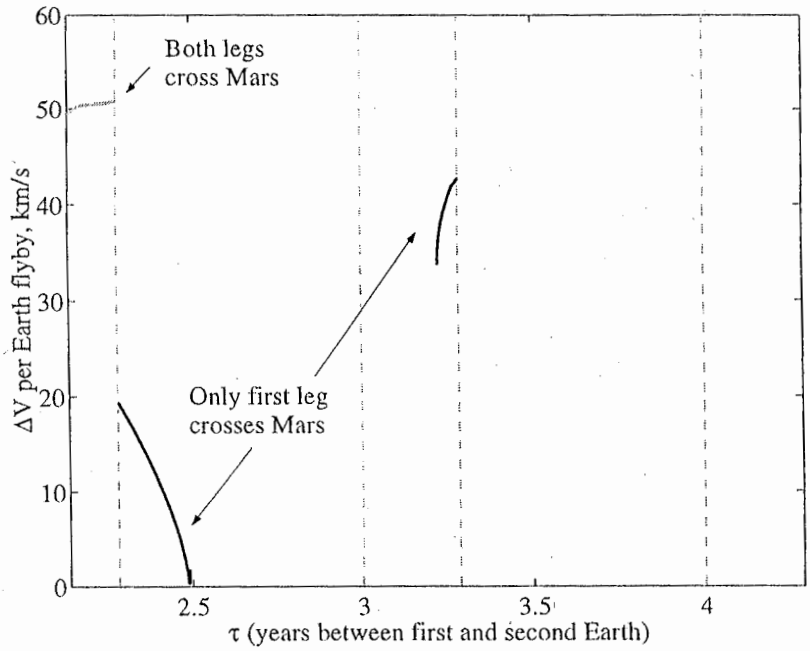

Figure 3 ΔV **per flyby for cyclers in the S1S2 family, where the minimum** ΔV **is 0.41 km/s when** $\tau = 2.4885$ **years.**

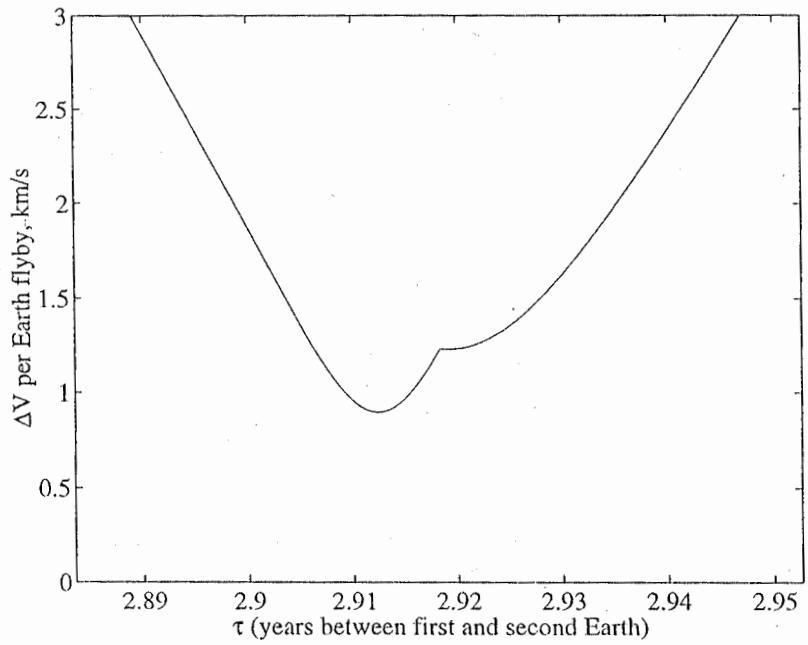

Figure 4 Notable cyclers in the S1S1 family, where the minimum ΔV **is 0.90 km/s when** $\tau = 2.9124$ **years.**

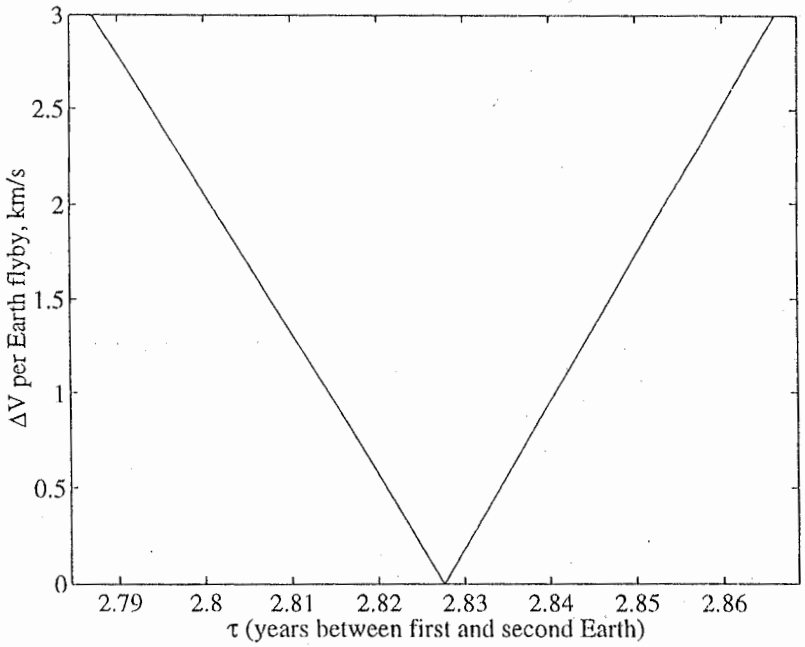

Figure 5 Notable cyclers in the S1L1 family, where a ballistic cycler occurs when $\tau = 2.8277$ years.

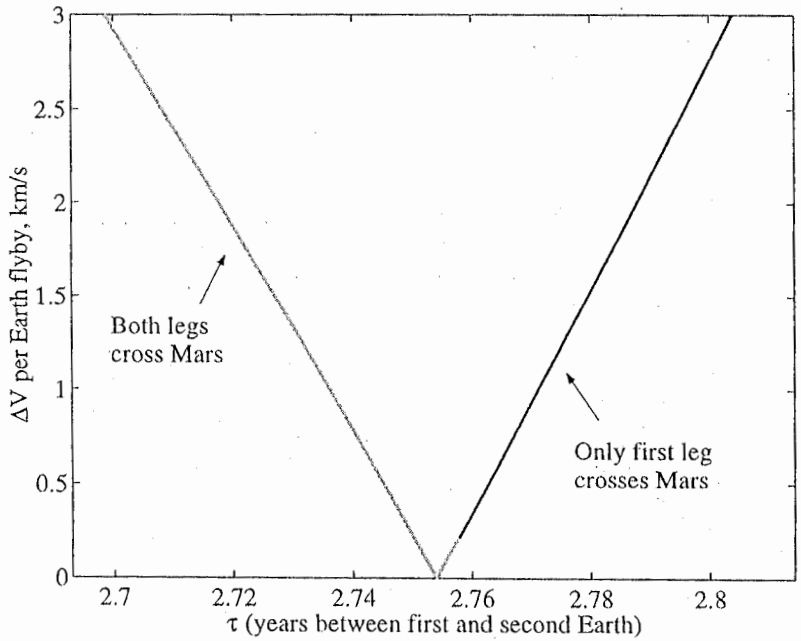

Figure 6 Notable cyclers in the U0L1 family, where a ballistic cycler occurs when $\tau = 2.7540$ years.

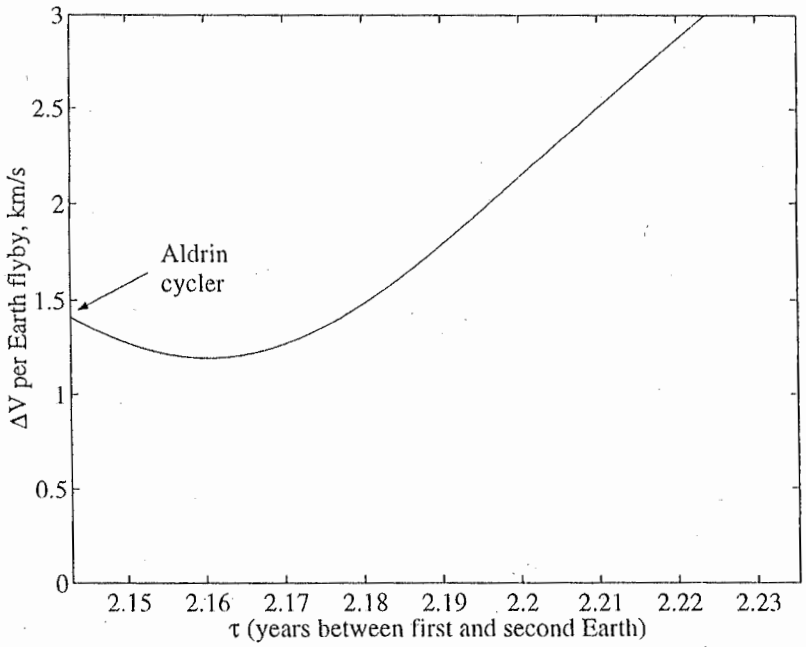

Figure 7 Notable cyclers in the L1L1 family, where the minimum ΔV is 1.19 km/s when $\tau = 2.1604$ years. The Aldrin cycler corresponds to $\tau = 2.1429$ years.

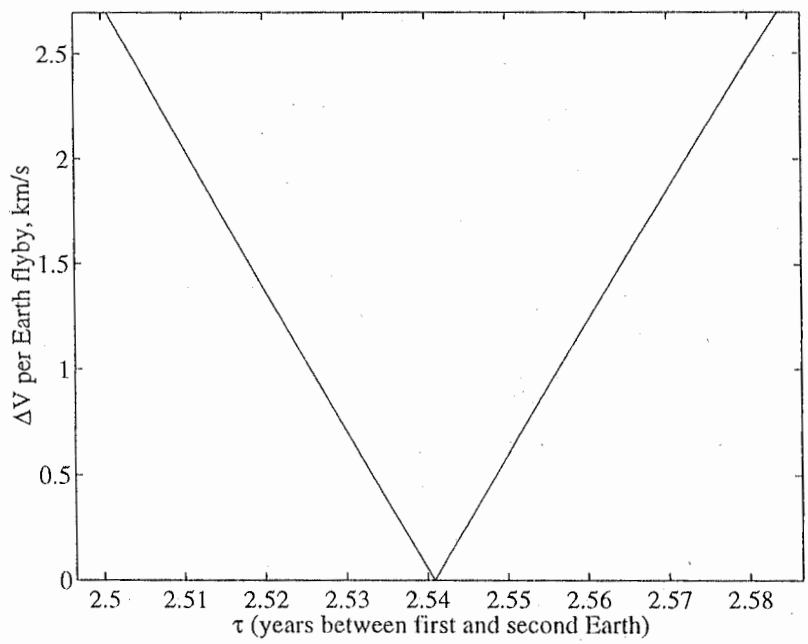

Figure 8 Notable cyclers in the L2U0 family, where a ballistic cycler occurs when $\tau = 2.5408$ years.

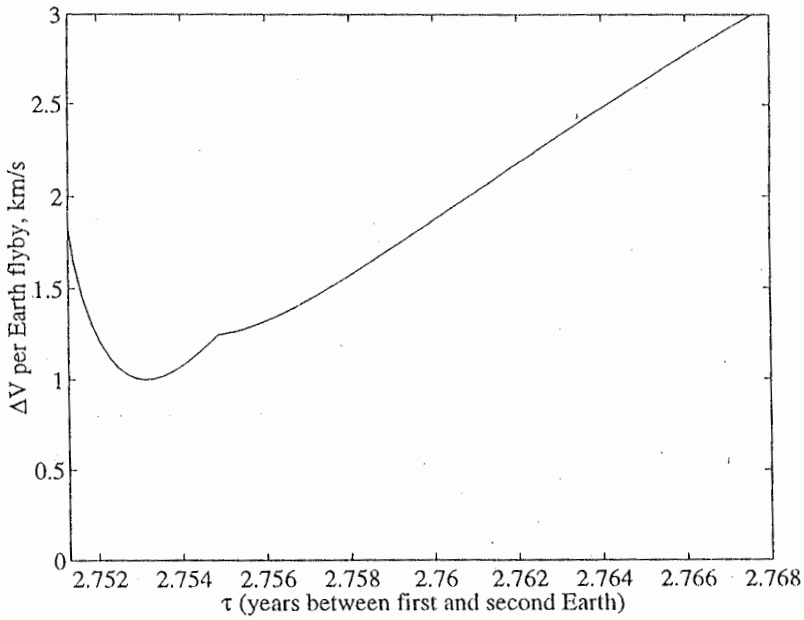

Figure 9 Notable cyclers in the L3U0 family, where the minimum ΔV is 1.00 km/s when $\tau = 2.7531$ km/s.

the transfer orbit (or equivalently, the number of spacecraft revolutions):

$$\mathcal{P}_T = \text{TOF}, \left(\frac{\text{TOF}}{2}\right), \left(\frac{\text{TOF}}{3}\right), \ldots, \left(\frac{\text{TOF}}{r_{\max}}\right) \tag{1}$$

[The sequence terminates when the semimajor axis of the transfer leg, $a_T = \mathcal{P}_T^{2/3}$ is less than 0.5 AU. That is, $r_{\max} = \lfloor \text{TOF}/(1/2)^{3/2} \rfloor$ (the largest integer less than or equal to $\text{TOF}/(1/2)^{3/2}$).]

For each choice of transfer-orbit period, the departing inertial velocity vector, $\mathbf{v}_T$, can lie anywhere on the surface of a sphere with radius v_T determined by the vis-viva equation:

$$v_T = \sqrt{\mu_{\text{Sun}} \left(\frac{2}{r_{\text{Earth}}} - \frac{1}{a_T}\right)} \tag{2}$$

(The two degrees of freedom on the surface of the sphere are the two continuous degrees of freedom.)

For the sake of brevity, a complete analysis of the (2, 1) cyclers with an $n\pi$ transfer leg is not included in this paper. Two-synodic-period cyclers with one $n\pi$ transfer leg and one generic (non-$n\pi$) transfer leg are included in the analysis of Russell and Ocampo[20] (but they do not consider cyclers with two different generic transfers, as we do in this paper).

FINDING BALLISTIC CYCLERS IN AN EPHEMERIS MODEL OF THE SOLAR SYSTEM

After finding ballistic cyclers in our simplified (circular coplanar) model of the solar system, we wanted to know if they exist and are still ballistic in an ephemeris model. By an ephemeris model,

we mean that the positions and velocities of the planets are obtained from an integrated ephemeris, such as JPL's DE405. We continue to ignore third-body gravitational effects, except during the planetary flybys. Gravity-assist maneuvers can occur at both Earth and Mars but are still modeled as an instantaneous rotation of the V_∞ vector.

In order to show that a cycler is ballistic in the ephemeris model, we would have to propagate it for an infinite time. Since that is impossible, we instead propagate it until the Earth, Mars and the cycler vehicle return to their initial state in inertial space (approximately). Two-synodic-period cyclers repeat inertially after 7 repeat intervals of $4\frac{2}{7}$ years, or 30 years. The orbits of Earth and Mars also repeat inertially every 30 years (after 30 Earth revolutions and 16 Mars revolutions). Therefore, if we can propagate a two-synodic-period cycler ballistically for 30 years or more, then we can be fairly confident that it will propagate ballistically into the distant future (in the ephemeris model).

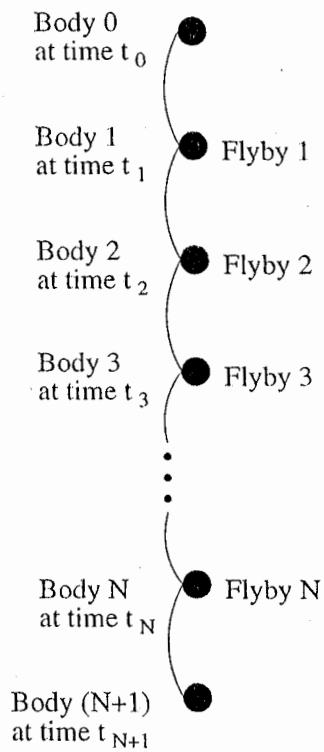

Figure 10 The $(N+2)$ bodies and the N flybys.

The problem of finding a 30+ year itinerary for a ballistic cycler can be formulated as finding a solution to a system of N equations and N inequalities in $(N+2)$ unknowns. The $(N+2)$ unknowns are the encounter times $t_0, t_1, \ldots, t_{N+1}$ at the bodies, as shown in Fig. 10. The type of transfer on each leg is known from the circular coplanar analysis so the body encounter times uniquely determine the incoming and outgoing V_∞ at the N intermediate flybys. In order for the ith flyby to be ballistic, the (magnitude of the) incoming V_∞ must equal the outgoing V_∞

$$V_{\infty\text{in},i}(t_{i-1}, t_i) - V_{\infty\text{out},i}(t_i, t_{i+1}) = 0 \qquad \text{for } i = 1, 2, \ldots, N \tag{3}$$

and the required flyby altitude h_i must be greater than or equal to 300 km.

$$h_i(t_{i-1}, t_i, t_{i+1}) - 300 \geq 0 \qquad \text{for } i = 1, 2, \ldots, N \tag{4}$$

Equations (3) and (4) can be neatly summarized using the vector notation

$$\begin{aligned} \mathrm{g(t)} &= 0 && (5) \\ \mathrm{h(t)} &\geq 0 && (6) \end{aligned}$$

where $\mathrm{t} = (t_0, t_1, \ldots, t_{N+1})$, Eq. (5) is the V_∞ constraint, and Eq. (6) is the altitude constraint. If none of the altitude constraints is active (i.e. if none of the flybys occurs at 300 km), then the solution to Eqs. (5) and (6) is not unique. In fact, since there are N equations in $(N+2)$ unknowns, we expect the set of solutions to have two degrees of freedom.

We used the software package SNOPT[21] (which stands for Sparse Nonlinear OPTimizer) to find a solution to Eqs. (5) and (6). Since SNOPT uses a quasi-Newton method to solve systems of equations and inequalities, it requires a good initial guess of the solution (i.e. the body-encounter times, t). At first, we tried to use the solution in the simplified (circular coplanar) model of the solar system as our initial guess, but SNOPT could not converge from that guess to a solution in the ephemeris model.

The problem is that the ephemeris model is too different from the simplified model, so the solution to the simplified model was a bad initial guess for the solution to the ephemeris model. We decided to use a continuation approach to surmount this problem. The orbital elements of Earth and Mars were collected together into a constant parameter vector which we call "a." The system of equations and inequalities then has the form

$$\begin{aligned} \mathrm{g(t, a)} &= 0 && (7) \\ \mathrm{h(t, a)} &\geq 0 && (8) \end{aligned}$$

For the simplified solar system model, $\mathrm{a} = \mathrm{a}_{\mathrm{simplified}}$, and for a more accurate solar system model, $\mathrm{a} = \mathrm{a}_{\mathrm{accurate}}$. (We note that the orbital elements in $\mathrm{a}_{\mathrm{simplified}}$ and $\mathrm{a}_{\mathrm{accurate}}$ are constants.) We already have a solution when $\mathrm{a} = \mathrm{a}_{\mathrm{simplified}}$ and we would like to find a solution when $\mathrm{a} = \mathrm{a}_{\mathrm{accurate}}$. (The solution when $\mathrm{a} = \mathrm{a}_{\mathrm{accurate}}$ could then be used as an initial guess for the ephemeris model.)

We can define a sequence of problems with a varying from $\mathrm{a}_{\mathrm{simplified}}$ to $\mathrm{a}_{\mathrm{accurate}}$. By using the solution to the kth problem in the sequence as the initial guess for the $(k+1)^{\mathrm{st}}$ problem in the sequence, we can ultimately arrive at the solution to the problem where $\mathrm{a} = \mathrm{a}_{\mathrm{accurate}}$. The solution to the problem with $\mathrm{a} = \mathrm{a}_{\mathrm{accurate}}$ is then used as an initial guess to the problem in which the positions and velocities of the planets are determined using JPL's DE405 ephemerides. We found that this continuation method worked very well.

THE BALLISTIC S1L1 CYCLER

Table 4 gives a 33-year itinerary for one of the two outbound cycler vehicles on a ballistic S1L1 cycler trajectory. Ephemeris data is used to determine the positions and velocities of Earth and Mars (i.e. the itinerary is a solution in the ephemeris model). Tables 5–7 give example itineraries for the other three ballistic S1L1 trajectories (one outbound and two inbound). We note that the encounters at the beginning and the end of the tables do not match the pattern because they are not constrained to maintain the cyclic behavior (but the mismatch is easily remedied by increasing the number of flyby bodies and re-solving as described earlier).

Table 4 An itinerary for outbound cycler vehicle 1 on a ballistic S1L1 trajectory

Encounter	Date mm/dd/yyyy	V_∞, km/s	Closest approach distance, km	Leg TOF, days
Earth-1	08/13/2005	4.01	—	—
Mars-2	02/27/2006	3.02	4,816	198
Earth-3	06/09/2008	6.89	20,130	833
Earth-4	12/03/2009	6.90	31,110	541
Mars-5	06/06/2010	4.31	17,710	186
Earth-6	08/24/2012	6.42	26,490	809
Earth-7	02/14/2014	6.43	41,520	539
Mars-8	07/03/2014	7.14	12,190	138
Earth-9	12/09/2016	4.01	27,730	890
Earth-10	05/22/2018	4.03	19,920	530
Mars-11	09/15/2018	6.47	11,580	115
Earth-12	04/06/2021	4.61	22,990	934
Earth-13	09/20/2022	4.59	14,780	532
Mars-14	05/01/2023	2.77	7,601	223
Earth-15	07/02/2025	7.08	23,860	793
Earth-16	12/26/2026	7.09	35,120	542
Mars-17	06/14/2027	5.27	13,840	170
Earth-18	09/21/2029	5.80	26,850	830
Earth-19	03/12/2031	5.80	37,520	537
Mars-20	07/15/2031	7.85	8,802	125
Earth-21	01/15/2034	4.21	24,870	915
Earth-22	06/28/2035	4.20	2,756	529
Mars-23	11/12/2035	5.87	1,770	137
Earth-24	05/06/2038	7.23	—	906

Since none of the flybys occurs at 300 km (the lower bound), none of the inequality constraints [Eq. (6)] are active, so we expect the set of itineraries for each vehicle to be two-dimensional. Each itinerary has different characteristics. In order to understand how much the characteristics can vary from itinerary to itinerary, we found a large set of itineraries for outbound vehicle 1. (We looked for 27-body itineraries instead of 24-body itineraries like those given in Tables 4–7.)

The first itinerary [i.e. the first solution to Eq. (5)] was found using the continuation method described in the previous section. The second itinerary was found by fixing the launch date at a slightly different value and using the first itinerary as an initial guess. All other itineraries were found in a similar way (by fixing the launch date or the arrival date at a value close to a known solution and using the known solution as the initial guess). Figure 11 shows 2,549 (launch date, arrival date) pairs that we sampled and which correspond to feasible itineraries. The launch date range is 128 days and the arrival date range is 74 days. (We note that even though we expect the set of feasible itineraries to be two-dimensional, the launch date and arrival date may not the best choice of coordinates. Two different feasible itineraries could have the same launch and arrival date.)

Each feasible itinerary indicated in Fig. 11 has different characteristics. A mission designer is mainly concerned with the short Earth-Mars legs. We call the Earth flybys at the beginning of those legs the "taxi-Earth flybys," to distinguish them from the other Earth flybys. (All Mars flybys occur at the end of a short Earth-Mars leg, so we do not need to distinguish them as "taxi-Mars flybys.")

Table 5 An itinerary for outbound cycler vehicle 2 on a ballistic S1L1 trajectory

Encounter	Date mm/dd/yyyy	V_∞, km/s	Closest approach distance, km	Leg TOF, days
Earth-1	09/30/2007	4.52	—	—
Mars-2	05/19/2008	3.00	6,601	231
Earth-3	07/16/2010	7.05	25,020	788
Earth-4	01/09/2012	7.06	37,470	542
Mars-5	06/17/2012	5.89	9,791	160
Earth-6	10/11/2014	5.33	25,510	846
Earth-7	03/29/2016	5.31	35,250	535
Mars-8	07/25/2016	7.87	9,621	118
Earth-9	02/04/2019	3.99	22,760	924
Earth-10	07/17/2020	3.98	4,273	530
Mars-11	12/18/2020	4.36	5,149	154
Earth-12	05/25/2023	6.08	20,050	887
Earth-13	11/13/2024	6.10	27,320	538
Mars-14	06/01/2025	3.71	16,070	200
Earth-15	08/08/2027	6.72	25,930	799
Earth-16	01/30/2029	6.73	41,310	541
Mars-17	06/27/2029	6.62	16,700	148
Earth-18	11/14/2031	4.46	32,010	871
Earth-19	04/29/2033	4.47	24,950	532
Mars-20	08/18/2033	7.58	7,070	111
Earth-21	03/06/2036	4.72	22,300	930
Earth-22	08/20/2037	4.74	617	532
Mars-23	01/22/2038	5.66	1,454	155
Earth-24	05/11/2040	11.05	—	840

The three key characteristics on each Earth-Mars leg are the taxi-Earth flyby V_∞, the TOF, and the Mars flyby V_∞. Ideally, we would like all three values to be as small as possible for all Earth-Mars legs. That is, we want to minimize the maximum taxi-Earth V_∞, the maximum Earth-Mars TOF, and the maximum Mars V_∞. Figures 12–14 show how these three quantities are related (for the itineraries of Fig. 11).

Among all the itineraries, the smallest maximum taxi-Earth V_∞ is 7.0864 km/s (see Figs. 12 and 13), the smallest maximum Earth-Mars TOF is 222.6 days (see Figs. 13 and 14), and the smallest maximum Mars V_∞ is 7.6897 km/s (see Figs. 12 and 14). Surprisingly, there are many itineraries which come close to having all of these smallest maxima. An example of one such itinerary is given in Table 8, where the maxima are 7.086 km/s (at Earth), 222.6 days, and 7.695 km/s (at Mars), as indicated by the boxes.

Table 6 An itinerary for inbound cycler vehicle 1 on a ballistic S1L1 trajectory

Encounter	Date mm/dd/yyyy	V_∞, km/s	Closest approach distance, km	Leg TOF, days
Earth-1	04/01/2005	3.33	—	—
Mars-2	10/05/2007	7.25	12,140	918
Earth-3	02/15/2008	6.22	44,630	133
Earth-4	08/07/2009	6.23	19,040	539
Mars-5	11/05/2011	4.78	6,710	820
Earth-6	04/30/2012	7.05	29,830	177
Earth-7	10/24/2013	7.05	24,830	542
Mars-8	01/08/2016	2.75	9,870	805
Earth-9	08/13/2016	4.15	13,410	218
Earth-10	01/25/2018	4.17	22,550	530
Mars-11	08/20/2020	7.19	10,440	938
Earth-12	12/06/2020	4.69	31,100	108
Earth-13	05/22/2022	4.67	16,020	532
Mars-14	10/16/2024	6.64	3,854	877
Earth-15	03/11/2025	6.72	40,750	146
Earth-16	09/03/2026	6.71	22,390	541
Mars-17	11/11/2028	3.79	10,580	800
Earth-18	05/29/2029	6.19	25,610	199
Earth-19	11/18/2030	6.18	30,640	539
Mars-20	04/07/2033	3.77	15,350	870
Earth-21	09/16/2033	3.87	10,950	162
Earth-22	02/27/2035	3.84	24,270	529
Mars-23	09/08/2037	7.71	678	924
Earth-24	12/13/2037	9.12	—	96

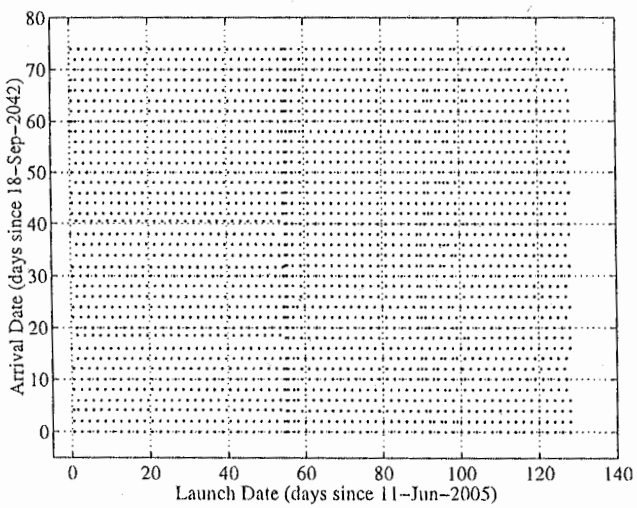

Figure 11 Launch and arrival dates of feasible itineraries for outbound vehicle 1 on a ballistic S1L1 trajectory.

Table 7 An itinerary for inbound cycler vehicle 2 on a ballistic S1L1 trajectory

Encounter	Date mm/dd/yyyy	V_∞, km/s	Closest approach distance, km	Leg TOF, days
Earth-1	05/29/2007	4.29	—	—
Mars-2	10/17/2009	5.83	4,553	872
Earth-3	03/25/2010	6.94	40,590	159
Earth-4	09/18/2011	6.95	23,390	542
Mars-5	11/17/2013	3.25	11,950	791
Earth-6	06/15/2014	5.81	23,670	210
Earth-7	12/04/2015	5.79	18,320	537
Mars-8	05/27/2018	4.60	6,957	905
Earth-9	10/15/2018	3.78	8,146	141
Earth-10	03/26/2020	3.79	26,090	529
Mars-11	09/28/2022	7.72	12,520	916
Earth-12	01/27/2023	6.05	43,350	121
Earth-13	07/18/2024	6.07	11,170	538
Mars-14	11/03/2026	5.57	2,099	838
Earth-15	04/17/2027	7.16	31,070	165
Earth-16	10/10/2028	7.15	24,350	542
Mars-17	12/08/2030	2.76	9,644	788
Earth-18	07/21/2031	4.66	17,280	225
Earth-19	01/03/2033	4.66	21,780	532
Mars-20	07/26/2035	6.34	9,423	934
Earth-21	11/19/2035	3.96	19,000	116
Earth-22	05/02/2037	3.95	25,160	530
Mars-23	10/14/2039	7.08	41,820	895
Earth-24	04/07/2040	3.40	—	176

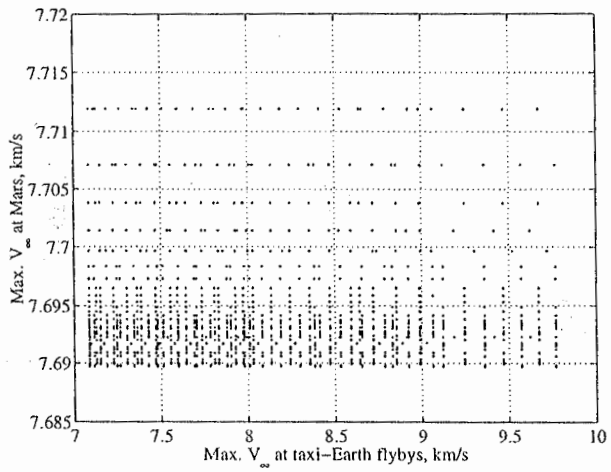

Figure 12 Maximum taxi-Earth V_∞ and maximum Mars V_∞ of feasible itineraries for outbound vehicle 1 on a ballistic S1L1 trajectory.

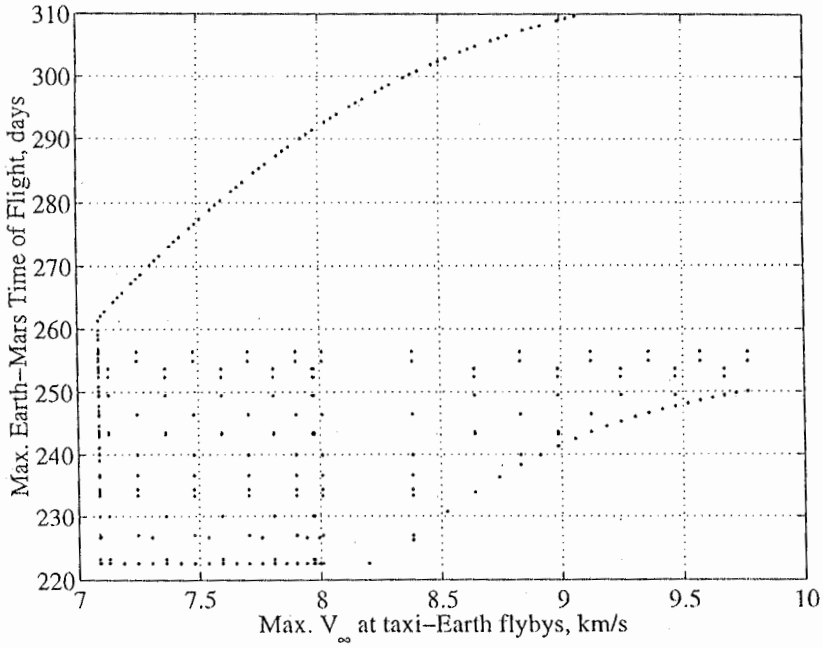

Figure 13 Maximum Earth-Mars TOF and maximum taxi-Earth V_∞ of feasible itineraries for outbound vehicle 1 on a ballistic S1L1 trajectory.

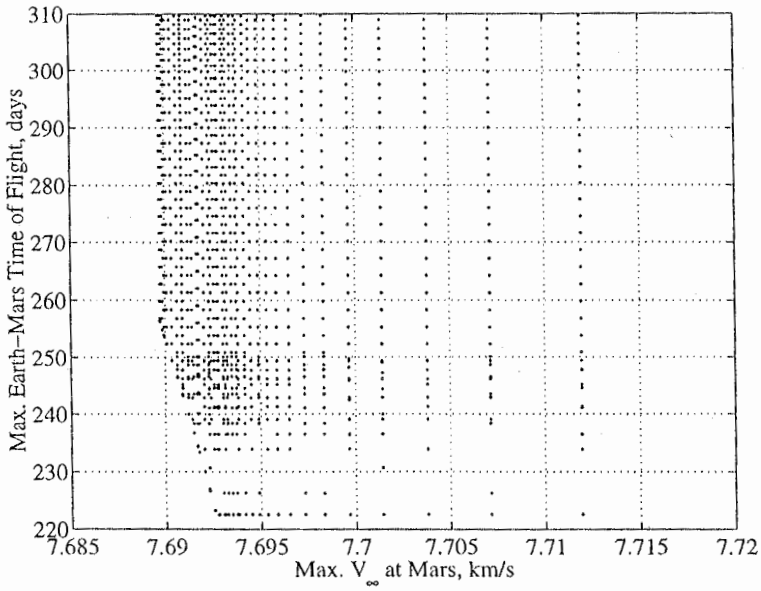

Figure 14 Maximum Earth-Mars TOF and maximum Mars V_∞ of feasible itineraries for outbound vehicle 1 on a ballistic S1L1 trajectory.

Table 8 A remarkable itinerary for outbound cycler vehicle 1 on a ballistic S1L1 trajectory

Encounter	Date mm/dd/yyyy	V_∞,[a] km/s	Closest approach distance, km	Leg TOF,[b] days
Earth-1	09/09/2005	5.32	—	—
Mars-2	03/03/2006	3.00	9961	175
Earth-3	06/09/2008	6.92	20543	830
Earth-4	12/03/2009	6.93	31192	542
Mars-5	06/06/2010	4.31	17762	185
Earth-6	08/24/2012	6.42	26495	809
Earth-7	02/14/2014	6.43	41528	540
Mars-8	07/03/2014	7.14	12174	138
Earth-9	12/09/2016	4.01	27726	890
Earth-10	05/22/2018	4.03	19923	530
Mars-11	09/15/2018	6.47	11570	115
Earth-12	04/06/2021	4.61	22992	934
Earth-13	09/20/2022	4.59	14780	532
Mars-14	05/01/2023	2.77	7593	$\boxed{223}$
Earth-15	07/02/2025	7.08	23858	793
Earth-16	12/26/2026	$\boxed{7.09}$	35164	542
Mars-17	06/14/2027	5.26	13751	170
Earth-18	09/21/2029	5.78	26818	830
Earth-19	03/12/2031	5.78	39051	537
Mars-20	07/15/2031	$\boxed{7.70}$	10573	125
Earth-21	01/15/2034	3.78	22970	915
Earth-22	06/28/2035	3.76	9633	529
Mars-23	11/13/2035	4.68	15695	138
Earth-24	05/08/2038	5.54	27049	907
Earth-25	10/25/2039	5.56	20455	536
Mars-26	04/28/2040	4.20	23678	186
Earth-27	11/13/2042	7.87[c]	—	928

[a] The average taxi-Earth V_∞ is 5.50 km/s and the average Mars V_∞ is 5.06 km/s.

[b] The average Earth-Mars TOF is 162 days.

[c] The V_∞ at the end can be ignored, as explained in the text.

FUTURE WORK AND CONCLUSIONS

The analytical method used in this paper has yet to be applied to investigate many classes of cyclers [such as (3, 2) cyclers]. These analytical methods can be extended to include: 1) using gravity-assist maneuvers at Mars, Venus, or the Moon, or 2) allowing for propulsive ΔV maneuvers at any time. There are also some open questions relating to the ballistic cyclers. Is there a way to get a more thorough sampling of the two-dimensional set of solutions? Are the ballistic cyclers stable? If not, how hard is it to stabilize them?

The human exploration and development of Mars is an exciting prospect and cyclers may play a key role. Analytical methods have been used to construct and analyze many cyclers, including

some with attractive characteristics for a human transportation system. Future applications of our method may yield other interesting cyclers.

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REFERENCES

1. Rall, C. S., "Free-Fall Periodic Orbits Connecting Earth and Mars," Sc.D. Thesis, Department of Aeronautics and Astronautics, M.I.T., Cambridge, MA, Oct. 1969.
2. Rall, C. S., and Hollister, W. M., "Free-Fall Periodic Orbits Connecting Earth and Mars," AIAA Paper 71-92, Jan. 1971.
3. Aldrin, E. E., "Cyclic Trajectory Concepts," SAIC presentation to the Interplanetary Rapid Transit Study Meeting, Jet Propulsion Laboratory, Oct. 1985.
4. Niehoff, J., "Manned Mars Mission Design," *Steps to Mars*, Joint AIAA/Planetary Society Conference, National Academy of Sciences, July 1985.
5. Niehoff, J., "Integrated Mars Unmanned Surface Exploration (IMUSE), A New Strategy for the Intensive Science Exploration of Mars," Space Science Board, Woods Hole, MA, July 1985.
6. Niehoff, J., "Pathways to Mars: New Trajectory Opportunities," American Astronautical Society, AAS Paper 86-172, July 1986.
7. Byrnes, D. V, Longuski J. M, and Aldrin B., "Cycler Orbit Between Earth and Mars," *Journal of Spacecraft and Rockets*, Vol. 30. No. 3, May–June 1993, pp. 334–336.
8. Friedlander, A. L., Niehoff, J. C., Byrnes, D. V., and Longuski, J. M., "Circulating Transportation Orbits between Earth and Mars," AIAA Paper 86-2009, Aug. 1986.
9. McConaghy, T. T., Longuski, J. M., and Byrnes, D. V., "Analysis of a Broad Class of Earth-Mars Cycler Trajectories," AIAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2002-4420, Monterey, CA, Aug. 5–8, 2002.
10. Byrnes, D. V., McConaghy, T. T., and Longuski, J. M., "Analysis of Various Two Synodic Period Earth-Mars Cycler Trajectories," AIAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2002-4420, Monterey, CA, Aug. 5–8, 2002.
11. Chen, K. J., McConaghy, T. T., Okutsu, M., and Longuski, J. M., "A Low-Thrust Version of the Aldrin Cycler," AIAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2002-4421, Monterey, CA, Aug. 5–8, 2002.
12. Chen, K. J., Landau, D. F., McConaghy, T. T., Okutsu, M., Longuski, J. M., and Aldrin, B., "Preliminary Analysis and Design of Powered Earth-Mars Cycling Trajectories," AIAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2002-4422, Monterey, CA, Aug. 5–8, 2002.
13. Rauwolf, G. A., Friedlander, A. L., and Nock, K. T., "A Mars Cycler Architecture Utilizing Low-Thrust Propulsion," AIAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2002-5046, Monterey, CA, Aug. 5–8, 2002.
14. Russell, R. P. and Ocampo, C. A., "A Systematic Method for Constructing Earth-Mars Cyclers Using Direct Return Trajectories," AAS/AIAA Spaceflight Mechanics Meeting, AAS Paper 03-145, Ponce, Puerto Rico, Feb. 9–13, 2003.
15. Bishop, R. H., Byrnes, D. V., Newman, D. J., Carr, C. E., and Aldrin, B., "Earth-Mars Transportation Opportunities: Promising Options for Interplanetary Transportation," The Richard H. Battin Astrodynamics Conference, AAS Paper 00-255, College Station, TX, March 20–21, 2000.
16. Aldrin, B., Byrnes, D. V., Jones, R., and Davis, H., "Evolutionary Space Transportation Plan for Mars Cycling Concepts," AIAA Space 2001 Conference and Exhibition, AIAA Paper 2001-4677, Albuquerque, NM, Aug. 28–30, 2001.
17. Landau, D. F., and Longuski, J. M., "Comparative Assessment of Human Missions to Mars," AAS/AIAA Astrodynamics Specialist Conference, AAS Paper 03-513, Big Sky, MT, Aug. 4–7, 2003.
18. Niehoff, J., Friedlander, A., and McAdams, J., "Earth-Mars Transportation Cycler Concepts," 42nd

IAF International Astronautical Congress, Montreal, Canada, Oct. 5–11, 1991.
19. Penzo, P. A., and Nock, K. T., "Earth-Mars Transportation Using Stop-Over Cyclers," AIAA/AAS Astrodynamics Specialist Conference, AIAA Paper 2002-4424, Monterey, CA, Aug. 5–8, 2002.
20. Russell, R. P. and Ocampo, C. A., "A Geometric Analysis of Half and Full-Revolution Return Trajectories via Planetary Flybys," AAS/AIAA Astrodynamics Specialist Conference, AAS Paper 03-508, Big Sky, MT, Aug. 4–7, 2003.
21. Gill, P. E., Murray, W., and Saunders, M. A., "SNOPT: An SQP algorithm for large-scale constrained optimization," *SIAM Journal on Optimization*, Vol. 12, 2002, pp. 979–1006.