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A POWERED EARTH-MARS CYCLER WITH THREE SYNODIC-PERIOD REPEAT TIME*

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We discuss preliminary results on constructing a powered Cycler by patching a series of Three Synodic-Period Semi-Cycler trajectories together. We present a powered Cycler with acceptable transfer times and moderate encounter velocities. Another attractive feature of this Cycler is that both short inbound and outbound legs occur within each Semi-Cycler segment, thus reducing the number of required vehicles to three. Even though the propellant usage is not insignificant, we believe that this Cycler still compares favorably with ballistic Cyclers which require four (or more) vehicles, especially when considering the long-term cost to supply and maintain each vehicle.

INTRODUCTION

Let us suppose that the first wave of human exploration is a success, and groups of settlers from Earth are now calling Mars their home. Most likely Mars will not provide everything they need to survive, so some necessities must come from Earth. In addition, personnel and scientific samples on Mars will be transported back to Earth. Thus we can assume that there will be frequent, two-way traffic between the two planets.

The simplest way to accomplish this flow of traffic is to launch goods and crews directly from one planet to another. Using this method, the spacecraft will leave the home planet (Earth), travel to and eventually land on the destination planet (Mars). When a return trip is desired, another spacecraft leaves Mars and comes back to Earth in the same manner. Over time, the cost to maintain, repair, and re-launch these interplanetary vehicles will quickly add up.

Alternative methods exist. Instead of landing on the destination planet each time, we can place the spacecraft on a trajectory that cycles back and forth between Earth and

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Mars. A trajectory that circulates between two or more planets indefinitely is called a Cycler. Once placed on a Cycler, the spacecraft may need to perform periodic trajectory-correction maneuvers to maintain the orbit, but since the vehicle never lands on the planets, we only need to launch the spacecraft once. Numerous Cycler trajectories have already been discovered; many of them employ gravity assists to reshape and reorient the orbits at each planetary encounter.[1-9]

The most well-known Cycler trajectory is the Aldrin Cycler, proposed by Aldrin in 1985.[4] The Aldrin Cycler can provide fast transfers to Mars (in which case it is called the Outbound Cycler) or fast transfers back to Earth (in which case it is called the Inbound Cycler). Two Cycler vehicles, one on each Cycler trajectory, would allow a visit to each planet every Earth-Mars synodic period (about 2.14 years). The transfer time is typically less than 6 months. At each flyby, smaller vehicles, called "Taxis," rendezvous with the Cycler vehicles to transport astronauts and goods to and from each planet. The main disadvantage of the Aldrin Cycler is the moderate to high flyby V_∞ at Mars (ranging from about 7 km/s to nearly 12 km/s). High V_∞ can make Taxi rendezvous with the Cycler vehicle very costly.[8]

Another type of circulating trajectory is the Semi-Cycler. These trajectories are similar to the Cyclers, with one main difference – the Semi-Cyclers remain in an orbit about Mars for a period of time before returning to the Earth, while the Cyclers perform only flybys at each planet.[7, 10] Since the Semi-Cycler is placed in an orbit about Mars, the Taxi rendezvous is less costly, since the V_∞ are effectively zero. The main disadvantage of the Semi-Cyclers is the propellant cost for planetary captures and escapes.

Designing Cyclers is generally more difficult than designing Semi-Cyclers (which are free-return trajectories connected by parking orbits about Mars) because of the more stringent timing constraints. If we replace the orbit insertion about Mars by a phasing orbit about the Sun, we can "patch" together series of Semi-Cyclers to form a continuous orbit – a Cycler. The resulting trajectory combines the advantages of the Cyclers and the Semi-Cyclers, while lessening the undesired features of both.

METHODOLOGY

Before we patch together any Semi-Cyclers, suitable candidate trajectories must first be obtained. Let us consider those designed by Bishop et al.[7] and Aldrin et al.[10]

The Aldrin et al.[10] proposal includes two versions of Semi-Cyclers. In the first version (Version I), the Cycler vehicle leaves from an orbit about Mars, encounters the Earth twice, then returns to an orbit about Mars. The transit times between the two planets range from about 6 months up to about 9 months, and the entire sequence takes two synodic periods. A drawback of the Version I Semi-Cycler is that the trajectory does not exist for every synodic opportunity as shown in Ref. 10. The second version (Version II) is similar to the first one, except there are three Earth flybys separating the Mars departure and arrival. The time of flight between the first two Earth encounters is 1 year, while the time between the second and third Earth encounters is 6 months. A key feature of the Version II Semi-Cycler is that it provides short time-of-flight (TOF) legs from Earth to Mars and from Mars to Earth. Version II Semi-Cyclers have Mars flyby V_∞ that range from about 2 km/s to 7 km/s, and Earth encounter V_∞ that range from about 3 km/s to about 5 km/s. The entire sequence takes three synodic periods, thus requiring at least three vehicles to provide Earth-Mars and Mars-Earth transfers every synodic period.

In the Bishop et al.[7] proposal, the Cycler vehicle leaves from an orbit around Mars (which requires a propulsive maneuver), encounters the Earth five times (separated by a year each), and then returns to Mars. Upon Mars arrival, the Cycler Vehicle performs a maneuver and is captured into orbit around Mars. The transit times between the two planets are about 6 months each, and the entire sequence completes in about 5 years. The Mars flyby V_∞ can range anywhere from about 3 km/s to about 5 km/s, but the Earth encounter V_∞ can be as high as 9 km/s. A total of three Cycler vehicles are needed to provide a transfer opportunity between Earth and Mars every synodic period (every 2.14 years). A disadvantage of the Bishop Semi-Cycler is the complication of the numerous Earth-flybys. These extra Earth flybys impose timing and phasing constraints which make Cycler design more difficult. The other concern is that the Earth flyby V_∞ are sometimes very high.

Out of these candidate Semi-Cyclers, we choose to patch together Version II Semi-Cyclers into a Cycler. Other Semi-Cyclers (such as that given by Bishop[7]) may have merit in constructing Cyclers, but these considerations will not be addressed in this paper.

Patching Semi-Cyclers

To patch together series of Version II Semi-Cyclers, we replace the parking orbit at Mars by a heliocentric orbit that takes the same amount of time (about two Mars years, or roughly 1,374 days). The heliocentric orbit that we choose to use is a 3:2 resonance orbit with Mars (i.e. three spacecraft revolutions and two Mars revolutions about the Sun). We note that this 3:2 resonance may not be optimal, but it is convenient for preliminary Cycler construction. Our optimization program is capable of adjusting the TOF of this orbit (e.g. are capable of departing from our resonance initial guess) if needed. Although the Semi-Cyclers are launched from Mars, we assume that the Cycler is launched from the Earth in the optimization of the trajectory.

Our patched Cycler now has a flight sequence of EMMEE-EMMEE-EMMEE— where E denotes Earth and M denotes Mars. The time from the Earth launch of the first cycle to the next cycle is three Earth-Mars synodic periods (about 76 months, or about 2,340 days). This Cycler thus has a repeat time of three Earth-Mars synodic periods (consistent with its Semi-Cycler source), and so a minimum of three vehicles are needed to provide transfer opportunities to and from each planet every synodic period.

To show that the Cycler continues forever, we would have to computationally propagate the trajectory for an infinite amount of time. This is of course impossible to do, so we instead calculate the trajectory until the inertial positions of Earth, Mars and the Cycler vehicle repeat. The inertial geometry of Earth-Mars configuration repeat every 7 synodic periods, or about 15 years. Since our patched Cycler has a repeat time of three synodic periods, at least 7 repeat intervals (i.e. EMMEE sequences) are needed, because the entire flight time must be an integer multiple of 7 synodic periods. This means that we must design and simulate at least 21 synodic-periods (about 45 years) of patched Cyclers to demonstrate the likelihood of perpetual repeatability. We recall that we will need a total of three such vehicles, each starting at a different mission time.

Reference 10 lists the trajectory itineraries for several Version II Semi-Cyclers that provide us with good starting guesses. However, Ref. 10 does not have itineraries beyond the second repeat interval for any of the three vehicles, thus we had to first calculate the remaining flyby dates based on trends observed from the listed data. We note that these calculations are by no means precise; the purpose they serve is to give us some reasonable starting guesses to use in our low-thrust trajectory optimizer.

The low-thrust trajectory optimizer we used is called GALLOP[11-14] (which stands for Gravity-Assist Low-thrust Local Optimization Program). GALLOP transforms the trajectory optimization problem into a nonlinear programming problem (NLP) and maximizes the final spacecraft mass; it is driven by a sequential quadratic programming (SQP) algorithm, SNOPT.[15]

RESULTS

In our first attempt, we were very optimistic and decided to optimize the entire 21 synodic-period flyby sequence. The resulting numerical problem was enormous. Even with a rather coarse discretization size (30-day segments), our problem ended up with 1,968 optimization variables and 823 constraints (resulting in a matrix with 1,619,664 elements!). Since NLP run time increases with problem size, a problem of this size would take a long time to solve. Our first attempt took over 13 hours on our SunBlade 1000 Workstation, and the resulting trajectory was not even feasible. After several more attempts with no results to speak of, we decided that a new approach was needed.

The method we chose was to divide this gigantic problem (21 synodic periods, with flyby sequence of EMMEE-EMMEE-EMMEE—) into seven smaller problems (each one has a flyby sequence of EMMEEEM, lasting slightly over 3 synodic periods) and to optimize them separately. These smaller problems typically had about 300 optimization variables and about 120 constraints. The usual run time was around 10-20 minutes.

The seven smaller pieces ultimately must be connected back together to form a continuous Cycler. To enforce this continuity, we overlapped the seven pieces with one another — the last two encounters (i.e., EM) of each EMMEEEM piece are the first two bodies of the next EMMEEEM piece. Furthermore, we constrained the incoming V_∞ vectors to be the same at each overlapping Mars. We were thus able to construct all three Cycler trajectories.

Summaries of the resulting trajectories of the three required vehicles are listed in Tables 1-3. The numbering of the vehicles is completely arbitrary and has no significance besides clarifying references to a particular vehicle.

A portion of the trajectory plot is shown in Fig. 1 (the complete trajectory plot is too cluttered because of so many flybys). In Fig. 2, we use what we call a "Radial Distance Plot" which plots the distance of the spacecraft from the Sun versus time since launch. Figure 2 shows a portion of a typical Radial Distance Plot for vehicle 1 (this portion corresponds to E-1 through E-11 for vehicle 2). We note in Fig. 2 that the first Mars-Mars transfer (between the 280th and 1600th day) is a 3:2 resonance, while the second Mars-Mars transfer is not. This is a good example of the optimizer finding a non-resonance transfer that is better than a resonance orbit. In fact, glancing at Tables 1-3 indicates that most of the Mars-Mars portions of the Cycler are non-resonant (whenever the Mars-Mars time of flight is not 1,374 days). In Fig. 3 we show a portion of the thrust profile for vehicle 3 (E-21 through M-27).

The results in Tables 1-3 each cover the entire 21 synodic-period timeframe. After any integer multiple of seven Earth-Mars synodic periods, the inertial alignments of the two planets (essentially) repeat, so we can predict that the next set of 21 synodic periods of the Cycler will be similar to the results in Tables 1-3. Thus we have also demonstrated that the Cycler has a chance to repeat perpetually provided that propellant is available. [Even if the Cycler does not repeat exactly after 21 synodic periods, experience has shown that extending the Cycler trajectory is just a matter of constructing the next 7 (or more) repeat intervals using the same methodology.] We can thus construct the Cycler for an arbitrarily long period of time.

The cumulative ΔV for the three vehicles range from about 0.58 km/s to 1.05 km/s per synodic period. This translates to an average total usage of about 2.43 km/s per synodic period. It is interesting to note that this value agrees remarkably well with the estimation using a circular-coplanar analysis, described in the next section.

Circular-Coplanar Analysis

To calculate the Cycler in the circular-coplanar model we use a patched-conic method and assume that the Earth-to-Earth transfer is 1.5 years, the Mars-to-Mars transfer is 3.762 years (3:2 resonance), and the Earth-Mars and Mars-Earth transfers are both 0.572 years. This trajectory has a total repeat time of 6.406 years, or three synodic periods. We then minimize the V_∞ at Earth and Mars to reduce taxi requirements. The resulting trajectory (see Table 4) requires a V_∞ of 3.63 km/s with a flyby altitude of 5,060 km at Earth and a 4.25 km/s V_∞ with a –645 km (sub-surface) altitude flyby at Mars to continue ballistically. Since sub-surface flybys are not physically realizable, we

TABLE 1. VEHICLE 1 TRAJECTORY SUMMARY

Body	Date (mm/dd/yyyy)	V_∞ (km/s)	Altitude (km)	TOF (days)
E-1	05/18/2007	9.893	---	---
M-2	01/10/2008	4.351	300[a]	237
M-3	11/05/2011	4.664	300	1395
E-4	06/06/2012	4.161	10426	214
E-5	05/28/2013	4.164	3255	356
E-6	11/29/2013	4.281	18579	185
M-7	07/16/2014	5.463	300	229
M-8	05/05/2018	4.673	300	1389
E-9	12/14/2018	6.147	12471	223
E-10	12/14/2019	6.130	300	365
E-11	06/12/2020	5.943	9995	181
M-12	01/20/2021	4.445	300	222
M-13	10/05/2024	5.534	488	1354
E-14	06/03/2025	4.383	300	241
E-15	06/03/2026	4.397	21966	365
E-16	12/05/2026	4.526	2894	185
M-17	08/27/2027	2.980	300	265
M-18	01/18/2031	3.933	16971	1240
E-19	08/16/2031	4.302	300	210
E-20	08/15/2032	4.287	300	365
E-21	02/12/2033	4.397	300	181
M-22	12/10/2033	4.754	300	301
M-23	09/06/2037	6.426	300	1366
E-24	03/23/2038	4.453	22448	198
E-25	09/26/2038	4.427	1914	187
E-26	09/26/2039	4.408	1012	365
M-27	04/15/2040	4.714	17185	202
M-28	08/01/2043	2.747	300	1203
E-29	07/28/2044	4.253	10383	362
E-30	07/29/2045	4.243	30957	366
E-31	01/26/2046	4.376	35809	181
M-32	11/12/2046	4.761	300	290
M-33	08/17/2050	5.176	300	1374
E-34	03/26/2051	6.948	37279	221
E-35	03/25/2052	6.941	300	365
E-36	09/28/2052	6.909	413	187
M-37	07/29/2053	3.635	---	304
Average Earth V_∞ =	**5.180 km/s**		**Total ΔV =**	**12.31 km/s**
Average Mars V_∞ =	**4.550 km/s**		**ΔV Per Synodic Period =**	**0.58 km/s**

[a]An altitude constraint of 300 km is assumed at both Earth and Mars.

TABLE 2. VEHICLE 2 TRAJECTORY SUMMARY

Body	Date (mm/dd/yyyy)	V_∞ (km/s)	Altitude (km)	TOF (days)
E-1	02/08/2003	5.596	---	---
M-2	09/22/2003	4.538	300[a]	226
M-3	06/23/2007	4.274	300	1370
E-4	02/18/2008	5.255	17706	240
E-5	02/17/2009	2.273	327	365
E-6	08/22/2009	5.153	300	186
M-7	04/08/2010	6.579	542759	229
M-8	08/30/2013	2.806	778	1240
E-9	09/07/2014	5.800	19429	373
E-10	09/07/2015	5.789	1138	365
E-11	03/05/2016	5.882	31308	180
M-12	10/13/2016	5.269	300	222
M-13	07/12/2020	5.179	5594	1368
E-14	02/23/2021	5.758	310	226
E-15	02/23/2022	5.775	9982	365
E-16	08/28/2022	5.655	16780	186
M-17	05/20/2023	3.446	300	265
M-18	07/04/2026	2.879	300	1141
E-19	05/27/2027	3.236	300	327
E-20	05/10/2028	3.152	10289	349
E-21	11/12/2028	3.217	19647	186
M-22	09/02/2029	3.063	11643	294
M-23	05/04/2033	3.858	300	1340
E-24	12/09/2033	6.197	16470	219
E-25	12/19/2034	3.329	9021	375
E-26	06/19/2035	3.224	5369	182
M-27	01/07/2036	2.685	8729	202
M-28	08/20/2039	3.891	300	1321
E-29	04/26/2040	3.956	3487	250
E-30	04/15/2041	3.278	9115	354
E-31	10/19/2041	3.301	1541	187
M-32	08/05/2042	2.910	6157	290
M-33	04/16/2046	4.293	300	1350
E-34	12/22/2046	9.960	300	250
E-35	12/22/2047	9.975	300	365
E-36	06/21/2048	9.654	5441	182
M-37	4/21/2049	8.029	---	304
Average Earth V_∞ =	**5.246 km/s**		**Total ΔV =**	**22.03 km/s**
Average Mars V_∞ =	**4.247 km/s**		**ΔV Per Synodic Period =**	**1.05 km/s**

[a]An altitude constraint of 300 km is assumed at both Earth and Mars.

TABLE 3. VEHICLE 3 TRAJECTORY SUMMARY

Body	Date (mm/dd/yyyy)	V_∞ (km/s)	Altitude (km)	TOF (days)
E-1	03/23/2005	8.422	---	---
M-2	11/03/2005	4.391	300[a]	225
M-3	08/29/2009	3.243	5147	1395
E-4	05/10/2010	3.250	17144	254
E-5	05/11/2011	3.232	3820	366
E-6	11/12/2011	3.296	17278	185
M-7	07/08/2012	4.335	6343	239
M-8	01/14/2016	3.226	300	1285
E-9	11/24/2016	7.994	9053	315
E-10	11/25/2017	8.008	303	366
E-11	05/24/2018	7.806	19702	180
M-12	02/12/2019	5.895	300	264
M-13	10/14/2022	7.297	300	1340
E-14	05/09/2023	5.364	1086	207
E-15	05/08/2024	5.363	191947	365
E-16	11/10/2024	5.468	853	186
M-17	08/12/2025	2.484	84256	275
M-18	02/10/2029	3.926	300	1278
E-19	11/14/2029	8.629	21976	277
E-20	12/02/2030	5.901	300	383
E-21	05/31/2031	5.739	300	180
M-22	04/05/2032	7.321	300	310
M-23	08/24/2035	5.633	1932	1236
E-24	03/28/2036	6.209	2313	217
E-25	03/28/2037	6.205	131919	365
E-26	10/01/2037	6.189	300	187
M-27	06/16/2038	2.560	300	258
M-28	11/06/2041	4.573	2062	1239
E-29	08/09/2042	6.665	18927	276
E-30	08/07/2043	6.977	300	363
E-31	02/04/2044	7.178	300	181
M-32	11/14/2044	5.682	1192	284
M-33	06/12/2048	4.423	300	1306
E-34	02/03/2049	8.881	307	236
E-35	02/04/2050	8.880	4735	366
E-36	08/07/2050	8.631	5636	184
M-37	06/11/2051	6.410	---	308
Average Earth V_∞ =	**6.559 km/s**		**Total ΔV =**	**17.62 km/s**
Average Mars V_∞ =	**4.760 km/s**		**ΔV Per Synodic Period =**	**0.83 km/s**

[a]An altitude constraint of 300 km is assumed at both Earth and Mars.

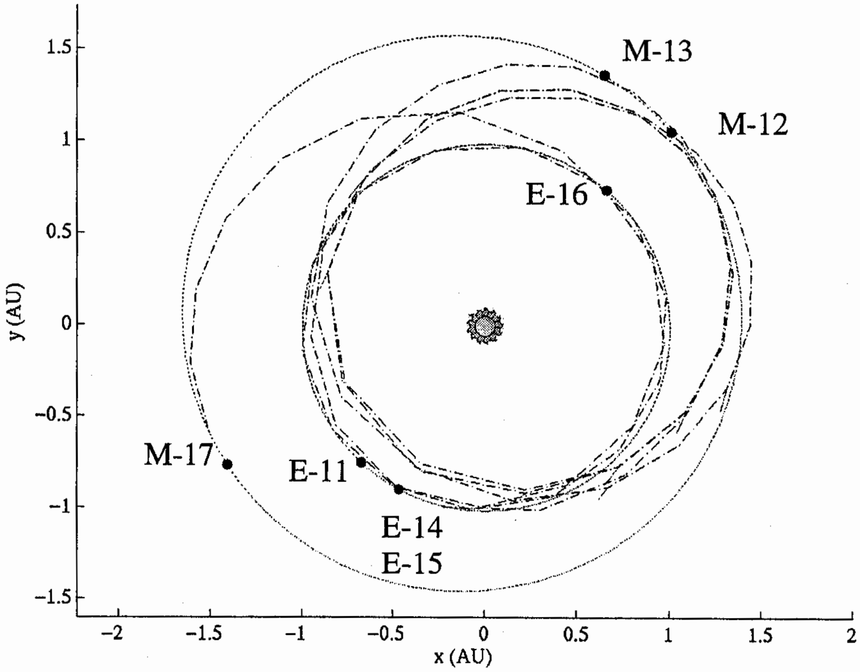

Figure 1. Representative Partial Trajectory Plot (E-11 through M-17 for Vehicle 3).

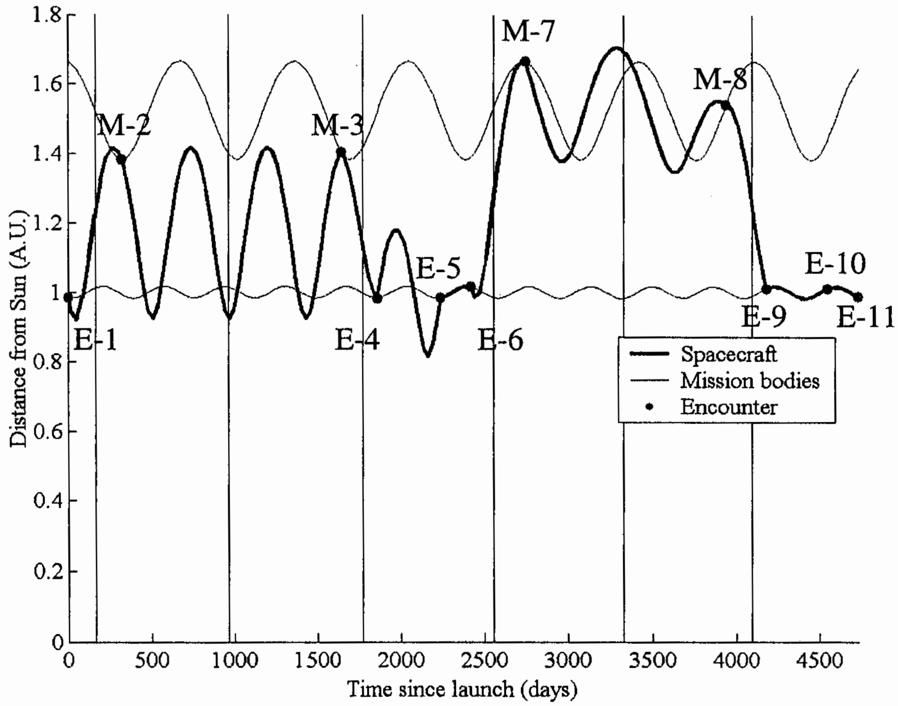

Figure 2. Partial Radial Distance Plot (E-1 through E-11 for Vehicle 2).

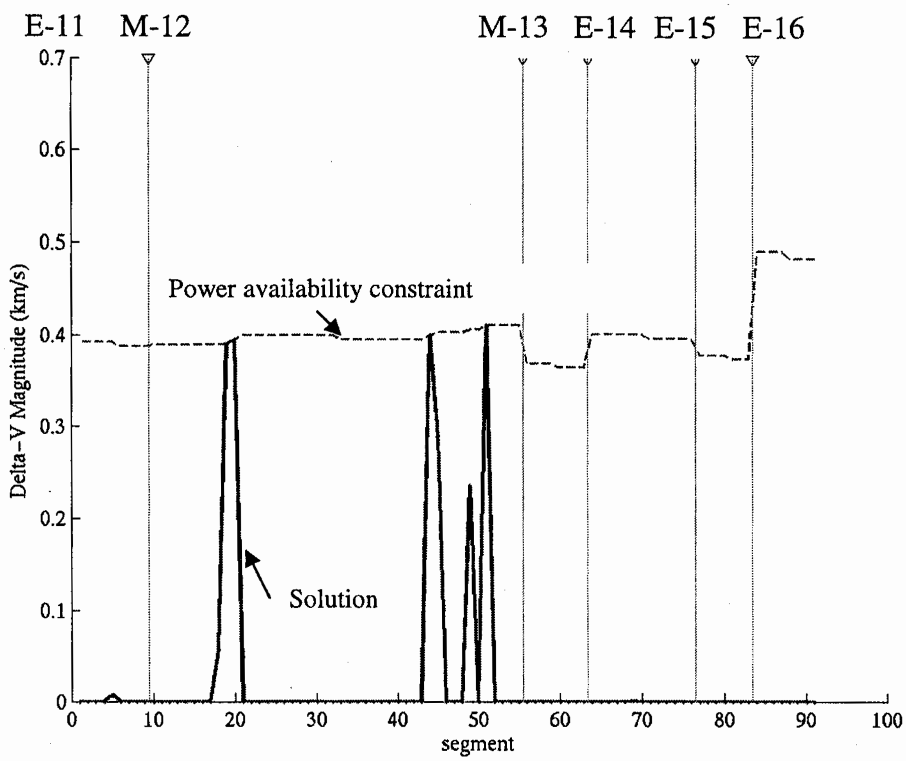

Figure 3. Partial Thrust Profile (E-11 through E-16 for Vehicle 3) where each Segment is about 30 days long.

TABLE 4. CIRCULAR-COPLANAR TRAJECTORY

Body	V_∞ (km/s)	ΔV (km/s)	Altitude (km)	TOF (days)
E-1	3.63	0	5060	---
M-2	4.25	1.24	300	209
M-3	4.25	1.24	300	1374
E-4	3.63	0	5060	209
E-5	3.63	0	∞[a]	365
E-6[b]	3.63	0	5060	183

[a]May be lowered by adjusting E-4 flyby altitude.

[b]Cycler trajectory repeats (E-6 = E-1).

replace the ballistic Mars encounter with a powered flyby that has a minimum altitude of 300 km and a ΔV of 1.24 km/s. This results in a cost of 2.48 km/s per synodic period for the entire system, which is very close to the average value of 2.43 km/s per synodic period found in Tables 1-3.

CONCLUSION

Designing Cyclers is a formidable task, particularly in the powered case described in this paper. Even with the latest software techniques for low-thrust gravity-assist trajectory optimization, this numerically-challenging problem was still not solvable without compromise.

The new powered Cycler that we have presented is most likely not optimal (though we think it is close to optimal), but the trajectory is certainly feasible and flyable, with reasonable expenditure of propellant. We have shown that there is good reason to believe that this Cycler can be propagated into the distant future. We hope that this powered Cycler, which requires only three vehicles to provide complete coverage of all synodic transfer opportunities, will prove a useful benchmark in future Mars transportation architecture studies.

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