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ANALYSIS OF A BROAD CLASS OF EARTH-MARS CYCLER TRAJECTORIES

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Earth-Mars cycler trajectories (cyclers) repeatedly encounter Earth and Mars. A systematic approach to constructing and evaluating such trajectories is decribed. The approach rediscovers known cyclers such as the Aldrin cycler and the VISIT cyclers. It also reveals some previously unknown cyclers. One extension of our approach is illustrated and other extensions are suggested.

Nomenclature

$\mathcal{P}$	orbit period, years
$\mathbf{R}$	position vector, AU
R_a	aphelion radius, AU
R_p	perihelion radius, AU
S	Earth-Mars synodic period, years
T	time to repeat the cycler trajectory, years
V_∞	hyperbolic excess speed, km/s
a	semi-major axis, AU
e	eccentricity
n	number of Earth-Mars synodic periods before repeating
p	parameter of the cycler orbit, AU
r	number of complete revs before repeating
r_1	number of complete revs on first leg
r_2	number of complete revs on second leg
$\Delta\Psi$	angle between initial Earth position and Earth's position after n synodic periods
ϕ	clockwise angle from Earth to Mars, rad
τ	time from beginning of repeat interval to intermediate Earth flyby, years
ω	argument of periapsis, rad

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Introduction

IN the 1960's, the utility of gravity-assist maneuvers was finally understood, and missions using multiple gravity assist flybys were shown to be possible.[1,2] In the late 1960's, Hollister[3,4] and Hollister and Menning[5,6] showed that ballistic trajectories exist which repeatedly encounter Venus and Earth.

Trajectories that repeatedly encounter the same planets on a regular schedule without stopping are now known as cycler trajectories, or cyclers. We distinguish cyclers that require large propellant expenditures from those that don't by referring to the former as powered cyclers and the latter as ballistic cyclers.

It seems that Rall[7] and Rall and Hollister[8] were the first to show that there exist cycler trajectories between Earth and Mars. Their method of finding Earth-Mars cyclers was essentially heuristic, so they wrote, "Because of the cut-and-try nature of the method, one cannot be certain that all periodic [cycler] orbits have been found—even among the types of periodic orbits considered."[8] The cyclers they did find repeat every four synodic periods or more.

In 1985, Aldrin suggested that an Earth-Mars cycler may exist which repeats every synodic period.[9] This was subsequently confirmed by Byrnes et al.[10] Also in 1985, Niehoff first proposed the VISIT 1 and VISIT 2 Earth-Mars cyclers.[11–13] These cyclers were investigated further and compared to the Aldrin cycler by Friedlander et al.[14]

A natural question that arises is whether or not there are any other Earth-Mars cycler trajectories. In this paper, we describe a method of constructing

such trajectories. This method might be described as "patching consecutive collision orbits".

Assuming conic orbits, several researchers have investigated the families of trajectories that leave Earth (or any other orbiting body) and return at a later date.[15–22] Such trajectories are known as consecutive collision orbits. Howell and Marsh provide an excellent historical overview in Ref. 22.

If only the Earth is used for gravity-assist maneuvers, then Earth-Mars cycler trajectories can be constructed by patching consecutive collision orbits together at the Earth encounters so that the entire trajectory repeats after an integer number of Earth-Mars synodic periods. We elaborate on this method of constructing Earth-Mars cyclers.

Interestingly, Poincaré knew about periodic solutions of this sort (i.e. consecutive collision orbits patched together at planetary encounters).[23] Such orbits are known as Poincaré's "second species" periodic orbits and have been studied quite extensively.[24–29] However, as far as we know, Poincaré's second species periodic orbits have not previously been considered as potential Earth-Mars cycler trajectories.

Methodology

In order to construct Earth-Mars cycler trajectories, we begin by making a number of simplifying assumptions:

1. The Earth-Mars synodic period S is $2\frac{1}{7}$ years.
2. Earth's orbit, Mars' orbit, and the cycler trajectory lie in the ecliptic plane.
3. Earth and Mars have circular orbits.
4. The cycler trajectory is conic and prograde (direct).
5. Only the Earth has sufficient mass to provide gravity-assist maneuvers.
6. Gravity-assist maneuvers occur instantaneously.

We note that assumption 1 is equivalent to assuming that the orbital period of Mars is $1\frac{7}{8}$ years (whereas a more accurate value is 1.881 years).

Assumptions 2 and 3 allow us to set up a planar coordinate system with the Sun at the origin and the Earth on the positive x-axis at the launch date. (After we find a cycler trajectory, we choose the launch date so that the spacecraft encounters Mars.) An initial Earth-Mars configuration is illustrated in Fig. 1.

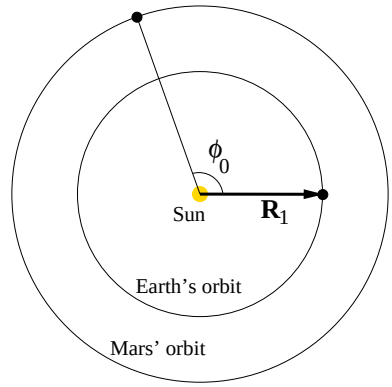


Fig. 1 Example initial configuration.

Now we must determine what conditions must be met if the spacecraft orbit is to be a cycler trajectory. At the initial time, $t_0 = 0$, the clockwise angle ϕ_0 from Earth to Mars is chosen so that the spacecraft will encounter Mars after leaving Earth. Following the Mars encounter, the spacecraft may encounter Earth again. If an Earth encounter happens when the Earth-Mars angle $\phi = \phi_0$ again, then the spacecraft could return to Mars using the same (shape) Earth-Mars transfer orbit that it used initially. Moreover, the trajectory could be repeated indefinitely and hence it is a cycler trajectory.

Let T be the time to repeat a cycler trajectory. Then the above discussion implies that $\phi(T) = \phi(0) = \phi_0$. Since $\phi(t)$ (the clockwise angle from Earth to Mars) only repeats once per synodic period, T must be an integer number of synodic periods:

$$T = nS = n \cdot \left(2\frac{1}{7}\right) \qquad \text{where } n = 1, 2, \ldots \tag{1}$$

Since the angular velocity of the Earth is 2π radians per year, we also know that $\mathbf{R}_{Earth}(T) = (\cos(2\pi T), \sin(2\pi T))$. Therefore the conditions for the spacecraft orbit $\mathbf{R}(t)$ to be a cycler trajectory are:

$$\mathbf{R}(0) = (1, 0) \tag{2}$$
$$\mathbf{R}(nS) = (\cos(2\pi nS), \sin(2\pi nS)) \tag{3}$$

where $n = 1, 2, \ldots$. This is a Lambert problem. Given n, we want to find a solution $\mathbf{R}(t)$ to the two-body problem that connects $\mathbf{R}_1 = (1, 0)$ to $\mathbf{R}_2 = (\cos(2\pi nS), \sin(2\pi nS))$ in a time of flight $T = nS$.

For example, let us consider the case $n = 1$, which means we are looking for cycler trajectories that repeat every $T = nS = 2\frac{1}{7}$ years. In $2\frac{1}{7}$ years, the Earth orbits the Sun $2\frac{1}{7}$ times, so when the spacecraft returns to Earth after $2\frac{1}{7}$ years, the Earth will

be $\frac{1}{7}$ of a rev (51.43°) ahead of where it was when the spacecraft left. (This will also be true when $n = 8, 15, \ldots$.) The geometry of this Lambert problem is illustrated in Fig. 2.

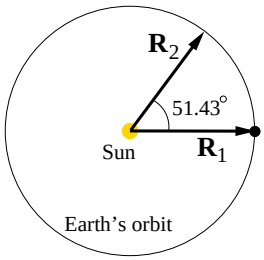


Fig. 2 Departure/arrival geometry when $\mathbf{n = 1, 8, 15, \ldots}$.

The $n = 1$ case has multiple solutions (i.e. there are many different trajectories that connect $\mathbf{R}_1$ to $\mathbf{R}_2$ in $2\frac{1}{7}$ years). These solutions can be illustrated by a plot that shows the orbital periods of solutions with various times of flight (see Fig. 3). The solutions to the $n = 1$ case correspond to those solutions with a time of flight of $2\frac{1}{7}$ years (2.143 years). In Fig. 3, we see that there are seven solutions, corresponding to the seven points on the solution curves with a time of flight of 2.143 years. In fact, one of the solutions is the Aldrin cycler, which has an orbital period of 2.02 years.

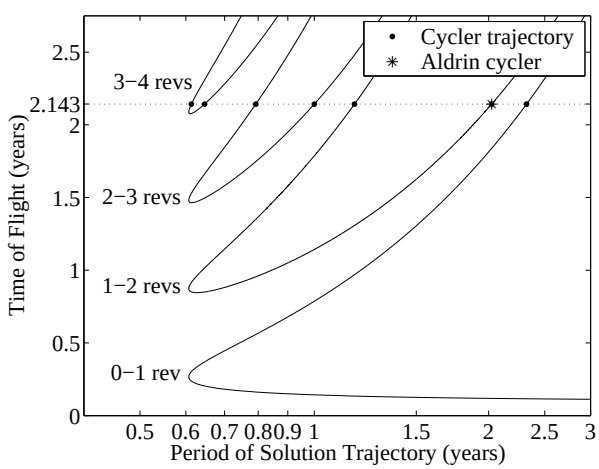


Fig. 3 The seven cycler trajectories that repeat every synodic period ($n = 1$).

Categorizing Cycler Trajectories

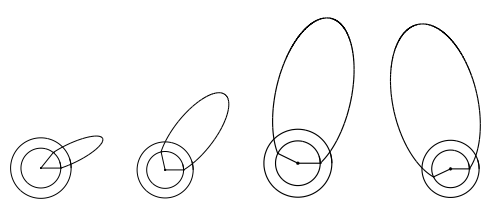
As seen in the above example, there can be multiple solutions for a given choice of n (the time-to-repeat in synodic periods). For each n, there is one solution which makes less than one revolution. There are two solutions that make between one and two revs, two solutions that make between two and three revs, and so on. Once the number of revs is large enough, there are no solutions because the time of flight is not long enough to accommodate all of the revs. When there are two solutions for a given number of revs, they are referred to as the short-period and long-period solutions. Every solution can be uniquely identified by specifying:

1. n, the time-to-repeat in synodic periods. $n = 1, 2, 3, \ldots$.
2. Whether the solution is long-period, short-period, or unique-period (i.e. when $r = 0$).
3. r, the number of revs, rounded down to the nearest integer. $r = 0, 1, 2, \ldots, r_{max}(n)$.

We denote a solution by a three-element expression of the form nPr, where P is either 'L', 'S', or 'U' depending on whether the solution is long-period, short-period, or unique-period, respectively. For example, the seven solutions in the $n = 1$ case are 1U0, 1L1 (the Aldrin cycler), 1S1, 1L2, 1S2, 1L3, and 1S3. Tables 1, 2, and 3 show the form of the unique-period, long-period, and short-period cyclers for n=1–4. The initial and final position vectors ($\mathbf{R}_1$ and $\mathbf{R}_2$) are indicated by line segments. It is interesting to note that cyclers 1L2, 2L4, 3L6, and 4S8 are all equivalent to Earth's orbit.

When n is a multiple of seven, $\mathbf{R}_1 = \mathbf{R}_2$ (i.e. a resonant transfer), so the Lambert problem becomes degenerate. In these cases, the nPr notation must be extended to accommodate the larger variety of solutions. We discuss this extension later.

Table 1 Unique-period solutions (nU0)

	n=1	n=2	n=3	n=4
$r = 0$				

Evaluating Solutions

Not all cycler trajectories are practical in applications. In this section we describe some criteria for evaluating their usefulness in potential missions.

Table 2 Long-period solutions (nLr)

	n=1	n=2	n=3	n=4
$r = 1$	[a]			
$r = 2$				
$r = 3$				
$r = 4$	N.S.[b]			
$r = 5$	N.S.	N.S.		
$r = 6$	N.S.	N.S.		
$r = 7$	N.S.	N.S.	N.S.	
$r = 8$	N.S.	N.S.	N.S.	
$r = 9$	N.S.	N.S.	N.S.	N.S.

[a] Aldrin cycler.

[b] No solution.

Table 3 Short-period solutions (nSr)

	n=1	n=2	n=3	n=4
$r = 1$				
$r = 2$				
$r = 3$				
$r = 4$	N.S.[a]			
$r = 5$	N.S.	N.S.		
$r = 6$	N.S.	N.S.		
$r = 7$	N.S.	N.S.	N.S.	
$r = 8$	N.S.	N.S.	N.S.	
$r = 9$	N.S.	N.S.	N.S.	N.S.

[a] No solution.

Number of Cycler Vehicles Required

When $n = 1$ and the cycler trajectory crosses Mars' orbit, it crosses Mars' orbit at two points. By launching the cycler spacecraft at the correct time, it will encounter Mars at the first Mars-orbit crossing. This minimizes the time of flight from Earth to Mars. A cycler trajectory used in this way is called an 'outbound cycler' because it is used to travel from Earth out to Mars.

Similarly, the cycler spacecraft can be launched at a different time so that it encounters Mars at the last Mars-orbit crossing before returning to Earth. This minimizes time of flight from Mars to Earth. When the cycler trajectory is used in this way it is called an 'inbound cycler'. We note that the difference between an inbound cycler and an outbound cycler is the launch date, not the shape of the cycler trajectory.

If we assume that there is one short Earth-Mars trip and one short Mars-Earth trip every synodic period, then we can estimate the number of cycler spacecraft required as $2n$, a number we want to keep small. The value $2n$ is an upper bound on the number of vehicles required because sometimes there is

more than one short-duration Earth-Mars (or Mars-Earth) leg per repeat interval (T). This occurs, for example, with the Rall-Hollister cyclers and the VISIT cyclers.

Aphelion Radius

In order for a cycler trajectory to be used for traveling between Earth and Mars, it should cross the orbit of Mars (i.e. the aphelion radius should be greater than the orbital radius of Mars). A quick glance at Tables 2 and 3 reveals that many cycler trajectories don't pass this test. However, if the aphelion radius is just slightly below the orbital radius of Mars, then the eccentricity of Mars' orbit combined with some small ΔV maneuvers may be enough to make up for a shortfall.

V_∞ at Earth and at Mars

Since taxi spacecraft must rendezvous with the cycler spacecraft as it passes Earth and Mars, we want the Earth V_∞ and the Mars V_∞ to be as small as possible. This typically rules out trajectories with a small number of revs (r) per repeat interval. The orbit that achieves the lowest possible sum of V_∞ at Earth and V_∞ at Mars is the Hohmann transfer orbit (2.95 km/s at Earth and 2.65 km/s at Mars). Unfortunately, the Hohmann orbit is not a cycler trajectory.

Required versus Maximum Possible Turn Angle

In order for the spacecraft to return to Mars on the same-shape orbit it used originally, the orbit's line of nodes must be rotated by $\Delta\Psi$ degrees, where $\Delta\Psi$ is the angle between the initial and final Earth positions ($\mathbf{R}_1$ and $\mathbf{R}_2$):

$$\Delta\Psi = \frac{n}{7} \cdot 360^\circ \pmod{360^\circ} \tag{4}$$

We note that when n is a multiple of seven, the line of nodes does not need to be rotated ($\Delta\Psi = 0^\circ$). This means that all cycler trajectories with n a multiple of seven are ballistic cyclers. The VISIT 1 and VISIT 2 cyclers are examples of $n = 7$ solutions.

If the line of nodes must be rotated, an Earth gravity-assist may accomplish this without propellant. This is not always possible, however, since the required flyby radius may be too close to the Earth's center (e.g. some solutions require subsurface flybys). We assume that Earth flybys are constrained to altitudes greater than or equal to 200 km. Turning that can't be accomplished by an Earth gravity-assist maneuver must be made up by doing ΔV maneuvers.

Rotating the line of nodes is equivalent to rotating the V_∞ vector at Earth. If the required V_∞ turn angle is less than the V_∞ turn angle obtainable with a 200 km flyby, then no ΔV maneuver is required (i.e. the cycler is ballistic). Otherwise, a ΔV maneuver is required (i.e. the cycler is powered).

The Most Promising Solutions

Table 4 lists characteristics of the most promising cycler trajectories with $1 \leq n \leq 6$. We note that the Aldrin cycler (1L1) is among the most practical, despite the fact that the Earth flybys can't provide all of the required turning.

Cycler 2L3 is noteworthy, even though its aphelion is slightly below the orbit of Mars. It is one of the two-synodic-period cyclers analyzed by Byrnes et al.[30]

Some of the cyclers with $n = 6$ are also promising. The 6S7, 6S8, and 6S9 cyclers have required turn angles that are less than the maximum possible turn angles. This implies that they are ballistic cyclers. They also have low V_∞ at Earth and at Mars. Unfortunately, these cyclers would require twelve vehicles in order to provide short Mars-Earth and Earth-Mars trips every synodic period.

Table 5 lists some characteristics of the $n = 7$ cyclers. These cyclers are special because they repeat every $T = nS = 7 \cdot \left(2\frac{1}{7}\right) = 15$ years (i.e. an integer number of years). This means that the Earth is at the same point in inertial space at the beginning and the end of the repeat interval (T), so the spacecraft orbit line of nodes doesn't need to be turned. Therefore all $n = 7$ cyclers are ballistic cyclers. The VISIT 1 and VISIT 2 cyclers are $n = 7$ cyclers.

Many of the $n = 7$ cyclers encounter Earth and Mars more often than once every 15 years (see Table 5). For example, the VISIT 1 cycler encounters Earth every 5 years and Mars every 3.75 years. This means that fewer than 14 spacecraft are required to ensure frequent short Earth-Mars transfers.

Also, because of their simple geometry, the orbital characteristics of the $n = 7$ cyclers can be found analytically. Since each cycler makes r revs during the 15-year repeat time, the orbit period is $15/r$ years. In fact, an estimate of the period, $\mathcal{P}$, of any nPr cycler is:

$$\mathcal{P} \approx \frac{15n}{7r + [n \pmod 7]} \quad \text{(years)} \tag{5}$$

Since the orbit period of an $n = 7$ cycler is $15/r$ years, the semi-major axis, a, is:

$$a = \left(\frac{15}{r}\right)^{2/3} \quad \text{(AU)} \tag{6}$$

The cycler orbit perihelion radius, R_p, isn't uniquely determined by r, however. If $r < 15$,

Table 4 The most promising cyclers that repeat every 1 to 6 synodic periods

Cycler (nPr)	Aphelion radius, AU	V_∞ at Earth, km/s	V_∞ at Mars, km/s	Shortest transfer time, days	Required turn angle, degrees	Max. possible turn angle, degrees
1L1[a]	2.23	6.54	9.75	146	84	72
2L2	2.33	10.06	11.27	158	134	44
2L3	1.51[b]	5.65	3.05[c]	280[d]	135	82
3L4	1.89	11.78	9.68	189	167	35
3L5	1.45[b]	7.61	2.97[c]	274[d]	167	62
3S5	1.52[b]	12.27	5.45[c]	134[d]	167	33
4S5	1.82	11.23	8.89	88	167	38
4S6	1.53	8.51	4.07	157	167	54
5S4	2.49	10.62	12.05	75	134	41
5S5	2.09	9.08	9.87	89	134	50
5S6	1.79	7.51	7.32	111	135	62
5S7	1.54	5.86	3.67	170	135	79
5S8	1.34[b]	4.11	0.71[c]	167[d]	136	103
6S4	2.81	7.93	12.05	87	83	59
6S5	2.37	6.94	10.44	97	84	68
6S6	2.04	5.96	8.69	111	84	78
6S7	1.78	4.99	6.66	133	85[e]	90[e]
6S8	1.57	4.02	3.90	179	85[e]	104[e]
6S9	1.40[b]	3.04	1.21[c]	203[d]	86[e]	120[e]

[a] Aldrin cycler.

[b] Note: the semi-major axis of Mars is 1.52 AU.

[c] Difference between Mars' speed and spacecraft aphelion speed.

[d] Time to transfer from Earth to aphelion.

[e] Ballistic cycler: required turn angle is less than maximum possible turn angle.

Table 5 Cyclers that repeat every seven synodic periods (15 years)

# Revs every 15 years (r)	Period ($15/r$), years	Aphelion radius,[a] AU	Years between Earth encounters	Years between Mars encounters
1	15	[11.16, 12.16)	15	15
2	7.5	[6.66, 7.66)	15	7.5
3	5	[4.85, 5.85)	5	15
4	3.75	[3.83, 4.83)	15	3.75
5	3	[3.16, 4.16)	3	15
6	2.5	[2.68, 3.68)	5	7.5
7	2.143	[2.32, 3.32)	15	15
8	1.875	[2.04, 3.04)	15	1.875
9	1.667	[1.81, 2.81)	5	15
10[b]	1.5	[1.62, 2.62)	3	7.5
11	1.364	[1.46, 2.46)	15	15
12[c]	1.25	[1.32, 2.32)	5	3.75
13	1.154	[1.20, 2.20)	15	15
14	1.071	[1.09, 2.09)	15	7.5

[a] Range given corresponds to perihelion range $R_p \in (0, 1]$ AU.

[b] VISIT 2 cycler.

[c] VISIT 1 cycler.

then the semi-major axis of the cycler orbit is larger than the semi-major axis of Earth's orbit, so all $R_p \in (0, 1]$ AU are possible. If we choose a value for R_p, then we can calculate the cycler orbit eccentricity, e, using:

$$e = 1 - \frac{R_p}{a} = 1 - \frac{R_p}{(15/r)^{2/3}} \tag{7}$$

which in turn lets us calculate the cycler aphelion radius, R_a:

$$R_a = a(1 + e) = 2 \cdot (15/r)^{2/3} - R_p \quad \text{(AU)} \tag{8}$$

When n is a multiple of seven, r and R_p determine the semi-major axis and eccentricity of the cycler orbit, but they don't uniquely determine its argument of periapsis, ω (i.e. the angle from the x-direction to the cycler orbit periapsis direction). Since the radius of the spacecraft must be 1 AU when crossing the x-axis, the argument of periapsis can have two possible values, given by:

$$\omega = \pm \arccos\left(\frac{p-1}{e}\right) \tag{9}$$

where $p = a(1 - e^2)$ is the parameter of the cycler orbit. Therefore, when n is a multiple of seven, we denote a cycler using an expression of the form $n(R_p)r\pm$, where R_p is the perihelion radius in AU and the sign indicates whether the argument of periapsis is positive or negative, respectively. Figures 4–6 illustrate the use of this notation for various VISIT 2 cyclers. In Fig. 4, the argument of periapsis is zero, so the sign is not needed.

Extending our Method of Constructing Cyclers

So far, we have been assuming that the spacecraft remains on the same orbit during each repeat interval (of length T). An Earth gravity assist occurs only at the end of each repeat interval (if at all). We now consider dropping this restriction because there is no reason why the spacecraft can't perform multiple gravity assists per repeat interval. As long as the trajectory returns to Earth after an integer number of synodic periods, it is still a cycler.

Indeed, multiple gravity assists could be very useful. If one gravity assist can't adequately turn the line of nodes, then more gravity assists might be able to. Also, more Earth encounters may imply more short Earth-Mars transfers per repeat interval.

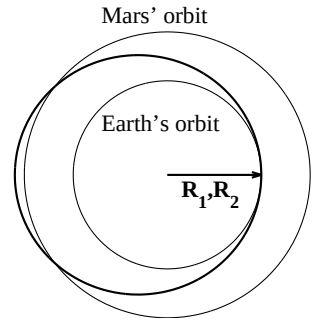


Fig. 4 The VISIT 2 cycler 7(1.0)10. When n is a multiple of seven, the cycler is represented by the expression $n(R_p)r\pm$, where R_p is the perihelion radius and the + or − sign (as needed) indicates whether the argument of periapsis is positive or negative, respectively.

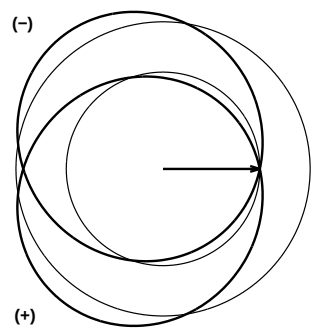


Fig. 5 The VISIT 2 cyclers 7(0.9)10+ and 7(0.9)10–.

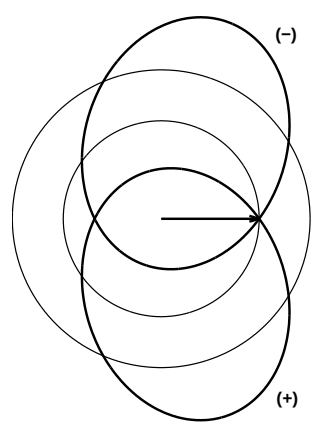


Fig. 6 The VISIT 2 cyclers 7(0.5)10+ and 7(0.5)10–.

It is clear that there are many possibilities to investigate. Here we consider a specific one. Namely, we assume that the repeat interval is two synodic periods (i.e. $n = 2$) and that there is one extra Earth gravity assist during the repeat interval. Let τ be the time from the beginning of the repeat interval to the intermediate gravity assist. Let r_1 be the number of revs on the first leg and r_2 be the number of revs on the second leg. This setup is illustrated schematically in Fig. 7.

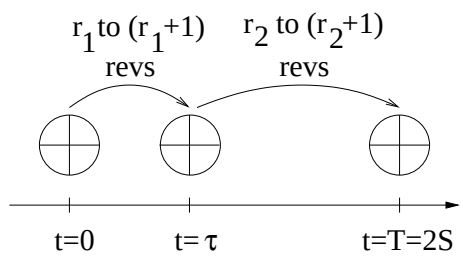


Fig. 7 Extension of the method to include an intermediate Earth gravity assist at time $t = \tau$.

For now, we ignore the cases where the first or second leg lasts an integer number of years (i.e. we ignore resonant transfers). Any cycler that meets all of the above conditions can be described completely by its values of τ, r_1, r_2, and whether the first and second legs are short-period, long-period, or unique-period.

To evaluate these cyclers, we test all possible combinations of these five parameters. Fortunately, we don't need to consider all $\tau \in (0, T)$, since reversing the order of the first and second legs doesn't change the characteristics of the trajectory. That is, a trajectory with first leg of length τ has the same characteristics as a trajectory with first leg of length $T - \tau$. Therefore we only need to consider $\tau \in (T/2, T) = (2\frac{1}{7}, 4\frac{2}{7})$ years.

The most promising of these cyclers are listed in Table 6. They are all ballistic, they all have V_∞ at Earth less than 12 km/s, and they all cross Mars' orbit at least once. The third trajectory listed has remarkably low V_∞ at Earth and Mars (4.7 km/s and 5.0 km/s, respectively). We will refer to it as the ballistic S1L1 cycler. A plot of the ballistic S1L1 cycler is shown in Fig. 8. Only the first leg crosses Mars' orbit, so a short (153-day) transfer is only available once every two synodic periods. We note that this cycler is very similar to the Case 3 cycler in Ref. 30.

Other Possible Extensions

To complete the extension we have described, resonant transfers must also be investigated. Other directions that remain to be pursued include:

- The use of multiple intermediate gravity assists.
- Using Venus, Mars, or other planets for intermediate gravity assists.
- Repeat times other than two synodic periods (when using intermediate gravity assists).

Other known cyclers (such as the Rall-Hollister cyclers[7,8] and the cyclers identified by Byrnes et al.[30]) can be constructed within this more general framework. In particular, the use of one-year Earth-to-Earth transfers and half-year Earth-to-Earth transfers (known as "backflips") should be investigated further.

An estimate of the total ΔV required by powered cyclers would also be useful for identifying nearly-ballisitic cyclers.

For each cycler trajectory constructed in our simplified solar system model, we hope to find a corresponding cycler trajectory in a more accurate model.

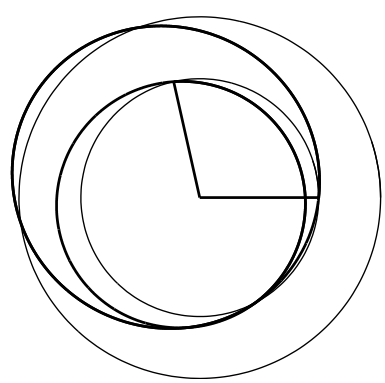


Fig. 8 The ballistic S1L1 cycler (with an intermediate Earth gravity assist). S1 indicates that the first leg is short period and makes between one and two revs. L1 indicates that the second leg (after the intermediate Earth gravity assist) is long period and makes between one and two revs.

Conclusions

We have developed a method for constructing Earth-Mars cycler trajectories. Previously known cyclers, such as the Aldrin cycler and the VISIT cyclers, can be constructed using this method.

Table 6 Promising ballistic two-synodic-period cyclers which use an intermediate Earth gravity assist

$P_1r_1P_2r_2$[a]	Time τ of intermediate Earth flyby, years	V_∞ at Earth, km/s	Leg 1 Mars orbit crossing times, years	V_∞ at Leg 1 Mars orbit crossing, km/s	Leg 2 Mars orbit crossing times, years	V_∞ at Leg 2 Mars orbit crossing, km/s
U0L1	2.754	11.3	0.188, 2.567	14.0	3.449, 3.592	5.4
L2U0	2.541	8.8	NC[b]	NC	2.781, 4.046	10.3
S1L1	2.828	4.7	0.419, 0.920, 1.908, 2.409	5.0	NC	NC

[a] P_1 is 'L', 'S', or 'U', depending on whether the solution on leg 1 is long-period, short-period, or unique-period, respectively. Similarly for P_2 on leg 2. The r_1 and r_2 refer to the number of revs on leg 1 and leg 2 (rounded down to the nearest integer), respectively.

[b] Leg does not cross Mars' orbit.

However, our construction method is not completely general. We investigated a simple extension and discovered some remarkable, previously-unknown cyclers. Many other possible extensions were identified and still remain to be explored.

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